# Paper

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# Spatial Decorrelation of SBAS Satellite Error Corrections in the Korean Peninsular

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#### Abstract

The characteristics of the SBAS satellite orbit and clock corrections are highly affected by the narrow network size in the Korean peninsula, which is expected to have an important role in the future dual frequency SBAS. The correlation between satellite corrections can be analyzed in terms of the spatial decorrelation effect which should be analyzed to keep the service area as wide as possible. In this paper, the characteristics of satellite error corrections for the potential Korean dual frequency SBAS were analyzed, and an optimal filter design approach is proposed to maximize the service area.

Key words: spatial decorrelation, SBAS, satellite orbit and clock corrections

# 1. Introduction

WADGPS (Wide Area Differential GPS) or its international standard the SBAS (Satellite Based Augmentation System) is designed to overcome the spatial decorrelation problem of the LADGPS (Local Area Differential GPS) by enhancing the number of reference stations and to provide integrity for avionic applications. However this does not mean that the SBAS removes the spatial decorrelation perfectly from the system. Actually we can observe rapid performance degradation at the reference station network boundary in various papers [1], [2]. Technically it comes from a strong correlation between the orbit and clock corrections as we can expect from the SISRE (Signal-In-Space Range Error) equation previously described [3]. By processing the WAAS (Wide Area Augmentation System) and IGS (International GNSS Service) data, we can observe that the WAAS corrected UREs (User Range Error) have better performance than individual WAAS corrected satellite orbit or clock errors described elsewhere [4].

The spatial decorrelation effect in the SBAS corrections will be a more notable issue in the Korean SBAS (e.g. Korea Augmentation Satellite System, a.k.a. KASS) due to the relatively narrow network size in the Korean peninsula. In the near future, SBAS implementations such as WAAS and EGNOS (European Geostationary Navigation Overlay Service) are expected to be upgraded to a dual-frequency system. Possibly, the KASS might have a plan to support the L1-L5 dual frequency after its successful certification. In the dual-frequency SBAS, the spatial decorrelation of satellite related corrections will be a major key to limiting the service area.

In this paper we analyzed the characteristics of the satellite orbit and clock corrections of the Korean SBAS and suggested a filter design approach to minimize the spatial decorrelation in the future dual-frequency application. For the simulation, we used the high rate GPS RINEX (Receiver Independent Exchange Format) data from the USUD IGS station in Japan and from several reference stations belonging to the National Geographic Information Institute (NGII) in Korea.

# 2. Characteristics of SBAS/GBAS corrections

#### 2.1 GBAS corrections

In the GBAS (Ground-based Augmentation System) such as the U.S. LAAS (Local Area Augmentation System) or Korean National wide DGPS, correction data are generated

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from a single reference station in the form of a scalar value assuming the user is collocated with the reference station. The corrections or common errors become spatially decorrelated as the user moves away from the collocated site.

As previously described [5], the error in the correction from the separation between the user and the reference station is simplified to equation (1), assuming an elevation angle greater than 10° and that the separation is less than 1,000 km.

$$\delta \varepsilon \leq \left(\frac{\varepsilon_s \sin \phi}{d}\right) \rho \sin \phi = \left(\frac{\varepsilon_s}{d}\right) \rho \sin^2 \phi \tag{1}$$

Here, the estimated satellite position error is  $\varepsilon_{sr}$  the distances of the user to the satellite is *d*, and the distance between the reference station and the user is p, and the elevation angle is  $\Phi$ . Equation (1) corresponds to  $\mathbf{e}_{user2} \cdot \delta \mathbf{x}_{sv}$  -  $\mathbf{e}_{user1} \cdot \delta \mathbf{x}_{sv}$  in Fig. 1. Based on this simplified model for the GBAS, the range accuracy degradation due to the spatial decorrelation is limited to a few centimeter in rms within the Korean peninsula. However it is impossible to uncorrelate the satellite orbit error from the others in the GBAS correction data; thus, the service area limitation due to the spatial effect is still the main drawback of the system.

#### 2.2 SBAS corrections in the Korean peninsular

#### 2.2.1 Minimum variance estimator

It is useful to apply the minimum variance estimator (MVE) to estimate the SBAS correction values for such reasons as the size limitation of the SBAS message frame, etc. previously described elsewhere [6]. In the SBAS MOPS (Minimum Operational Performance Standard), the effective range for the satellite orbit correction (i.e., long term correction) is defined as  $\pm 128$  m per axis in the ECEF (Earth-Centered Earth-Fixed) coordinate system. Therefore, it would be necessary to control the range of the orbital correction data considering the small size of the reference station network in the Korean peninsula, although the SBAS standard does not include the use of the MVE. The minimum



Fig. 1. Satellite orbit correction in the GBAS

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the error in the a priori data, then from equation (3) and

equation (4) [7].

 $\mathbf{\Lambda}_{0}^{'-1} = \mathbf{\Lambda}_{0}^{-1} + \mathbf{E}$ 

variance estimator is defined as follows:

 $\delta \mathbf{x} = \left( \mathbf{\Lambda}_{0}^{-1} + \mathbf{H}^{\mathsf{T}} \mathbf{W}^{-1} \mathbf{H} \right)^{-1} \mathbf{H}^{\mathsf{T}} \mathbf{W}^{-1} \delta \rho$ 

also the maximum likelihood estimator.

 $\mathbf{\Lambda} = \left(\mathbf{\Lambda}_0^{-1} + \mathbf{H}^T \mathbf{W}^{-1} \mathbf{H}\right)^{-1}$ 

(2)

(3)

(4)

, where  $\Lambda_0$  is the true a priori covariance;  $\Lambda$  is the true a

posteriori covariance; **H** is the observation matrix; **W** is the weight matrix, and  $\delta \rho$  is the observation data vector. The

observation matrix is assumed as a known and nonrandom function of time. The observation vector corresponds to the residual pseudo range in the SBAS and is assumed as zeromean random noise. If the noise is the Gaussian, the MV is

The true a priori covariance matrix  $\Lambda_0$  has a relationship

Here, E is an inverse error term in the employed a priori

covariance matrix for which the size is p-by-p. Equation (3)

becomes the true a posteriori covariance matrix ignoring

with the employed a priori covariance matrix  $\Lambda'_0$  shown in

$$\boldsymbol{\Lambda}_{c} = \left( \mathbf{E} + \boldsymbol{\Lambda}_{0}^{-1} + \mathbf{H}^{T} \mathbf{W}^{-1} \mathbf{H} \right)^{-1}$$
(5)

As  $E \rightarrow \infty$ , the employed a priori covariance becomes zero and the calculated a posteriori covariance matrix reduces to

$$\Lambda_{c} = \mathbf{O} \tag{6}$$

In case equation (2) is estimated with the a priori covariance employed, the actual a posteriori covariance is defined as

$$\Lambda_{a} \equiv \left\langle \left( \delta \mathbf{x}^{*} - \delta \mathbf{x} \right) \left( \delta \mathbf{x}^{*} - \delta \mathbf{x} \right)^{\mathsf{T}} \right\rangle$$
(7)

As done in equation (6), if  $\mathbf{E} \to \infty$ , actual a posteriori covariance matrix is redefined as

$$\Lambda_a = \Lambda_0 \tag{8}$$

From equation (6) and equation (8) we can understand that as  $\mathbf{E} \to \infty$  satellite related errors are equivalent to zero i.e. the minimum variance estimator assumes the satellite navigation system has negligible broadcast orbit errors. In this case PRC (Pseudo-Range Correction) from the SBAS broadcast message is equivalent to that from GBAS.

The lower bound of the **E** is equal to  $-\Lambda_0^{-1}$  because the covariance matrix has a nonnegative definite nature. As **E** approaches to the lower bound,  $\Lambda'_0$  increases to an infinite value. This indicates that the minimum variance estimator works because it has no a priori data just like the least squares solution. Therefore the calculated a posteriori covariance is reduced as

$$\boldsymbol{\Lambda}_{c} = \boldsymbol{\Lambda}_{a} = \left( \mathbf{H}^{T} \mathbf{W}^{-1} \mathbf{H} \right)^{-1}$$
(9)

### 2.2.2 Spatial decorrelation in the SBAS

Although the SBAS is developed to overcome the spatial decorrelation effect explained in the previous GBAS correction section, correction messages from the SBAS still experience the spatial issue if users are not within the service area defined by system.

Figure 2 shows the satellite orbit correction concept to describe the spatial decorrelation in the SBAS.  $\delta \mathbf{x}_{sv}$ ,  $\delta \mathbf{x}_0$ , e and  $\delta C_{sv}$  are the estimated and true satellite orbit error, line of sight vector and the estimated satellite clock error. The true clock error is assumed as zero, and user1 is at the center of the reference station network (i.e., the service area). From Fig. 2 and equation (10), we can see the PRC of user1 calculated from the estimated corrections has the same value as that of the true error. However, in the case of user2 who is far away from the network, the two PRCs have different values which are proportional to the gap between the estimated orbit error and the true orbit error.

$$\delta \rho_{flt} = \mathbf{e}'_{user} \delta \mathbf{x} - \delta c_{user} \tag{10}$$

Here, flt means the fast and long term correction. The description above in Fig. 2 implies that the performance degradation from the spatial effect depends on the estimated satellite orbit error. Although equations (2) and (3) have the optimum solution when there is no error in the a priori data, it might be reasonable to use stochastically obtained data shown in Fig. 3 because it is impossible to get the true a priori data in real-time.



Fig. 2. Satellite orbit correction in the SBAS

However we know that the real correction data from the WAAS or MSAS does not match with the IGS final products or with even the statistical data as previously described [8]. Considering the IGS products are the most believable true references, we can say that the orbit and clock correction from the currently operational SBAS implementation does not follow the true ephemeris error in the physical sense. This implies there is a strong correlation between the clock correction and orbit correction in the SBAS. Considering PRC in equation (10), this mismatch is not a big issue because the clock terms can observe any error from the orbit correction. Actually, orbit determination is conducted with time synchronization together; it causes the correlation between them in the ODTS (Orbit Determination and Time Synchronization) problem. The main difference between the SBAS correction estimation and the ODTS problem is whether the reference station network is global.

The geometrical size of the reference station network of the SBAS is relatively small compared to that of the global GPS monitoring station network, which causes error in the estimated orbit and clock correction. Especially, the network size in the Korean peninsula is too narrow in terms of satellites orbiting in a 20,000 km radius, which causes an observation matrix that is extremely sensitive to noise, i.e., mathematically results in a bad DOP (Dilution of Precision) or bad condition number. Because the bad DOP can increase the correction data up to the km level, the use of the MV estimator is inevitable in the Korean network mentioned in section 2.2.1. However, we would like to highlight that the MV estimator output to reduce the very large correction data for the SBAS standard message size is absolutely not optimum and might be far from true error. As described in Fig. 2, the more different the estimated orbit parameters are from the true data, the greater the increase



Fig. 3. Statistical signal-in-space errors of GPS (cited from [8])

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is in the spatial error.

# 3. Kinematic filter for satellite orbit and clock error estimation

In this paper, we used the Kalman filter in the form of the full vector as suggested previously [9], [10] to estimate satellite related errors. The state vector is defined as follows:

$$\mathbf{X} = \begin{bmatrix} \partial \mathbf{R}^T & \partial \mathbf{b}^T & \partial \mathbf{B}^T \end{bmatrix}^T$$
(11)

where

$$\partial \mathbf{R} = \begin{bmatrix} \partial \mathbf{R}_1^T & \partial \mathbf{R}_2^T & \cdots & \partial \mathbf{R}_m^T \end{bmatrix}^T$$
$$\partial \mathbf{b} = \begin{bmatrix} \partial \mathbf{b}_1 & \partial \mathbf{b}_2 & \cdots & \partial \mathbf{b}_m \end{bmatrix}^T$$
$$\partial \mathbf{B} = \begin{bmatrix} \partial \mathbf{B}_1 & \partial \mathbf{B}_2 & \cdots & \partial \mathbf{B}_{n-1} \end{bmatrix}^T$$

The m and n denote the number of satellites visible and the ground reference stations. The system matrix in the full vector filter is defined as follows:

$$\mathbf{H} = \begin{bmatrix} \mathbf{E}_{1} & -\mathbf{I}_{mm} & \mathbf{I}_{1} \\ \mathbf{E}_{2} & -\mathbf{I}_{mm} & \mathbf{I}_{2} \\ \vdots & \vdots & \vdots \\ \mathbf{E}_{n} & -\mathbf{I}_{mm} & \mathbf{I}_{n} \end{bmatrix}$$
(12)

, where the  $\mathbf{I}_{mm}$  is a m-by-m identity matrix.  $\mathbf{I}_i$  has unit values at *i-th* column and zero at others. For the simulation a linear kinematic Kalman filter was used with equations (11) and (12). In equation (12),  $\mathbf{I}_n$  is defined as zero because the relative clock bias will be estimated in the filter.

Within the framework of this research, the FOM (Figure of Merit) to evaluate the spatial decorrelation effects upon the users is the positioning (horizontal) error. However, as an indirect evaluation method, the integrity information including the user location effects can be used, and we decided to use the UDRE (User Differential Range Error) related values.

$$\sigma_{UDRE}^{2} = \left(\sum_{i=1}^{i=n} \frac{1}{\mathbf{P}_{UDRE,ii}}\right) = \left(\sum_{i=1}^{i=n} \frac{1}{\left(\mathbf{R} + \mathbf{H}\mathbf{P}_{sv}\mathbf{H}^{T}\right)_{ii}}\right)$$
(13)

$$\delta UDRE = \sqrt{\mathbf{e}^{\mathrm{T}} \cdot \mathbf{C} \cdot \mathbf{e}} \tag{14}$$

Equation (13) is defined to be broadcasted from a GEO satellite in the form of the UDREI, and a detailed description can be found in the SBAS standard [11]. The implementation and definition of equation (14) are based on a description

previously reported [12]. In equation (14) the covariance matrix C is defined to be broadcasted from a GEO as the message type 28.

# 4. Service area extension for the fast/long term corrections

The extreme cases explained in section 2.2.1 and the descriptions for Fig. 2 in section 2.2.2 are very helpful to understand the characteristics of the SBAS correction when implemented with the minimum variance estimator. If the a priori covariance has an error in the matrix, definitely the estimates from equation (2) are not optimum. The non-optimum correction data from the SBAS might cause a performance degradation due to the spatial decorrelation, which is dependent on the error in the a priori covariance. Because it is impossible to know the true a priori data for all epochs, we defined two representative scenarios in this paper to check the effect of the a priori data on the spatial decorrelation in the SBAS.

Scenario 1 : set the a posteriori covariance matrix similar to the real covariance Scenario 2 : set the a priori covariance matrix similar to the real covariance

Regarding scenario 1, the idea is that the range accuracy degradation due to the spatial decorrelation is small in the GBAS discussed in section 2.1. In section 2.2.1, we described that as  $\mathbf{E} \rightarrow \infty$ , the minimum variance estimator assumes that the satellite navigation system has a negligible broadcast orbit error. It implies that the a posteriori covariance in scenario 1 would be relatively small. Scenario 2 is designed to provide an a priori covariance similar to or larger than the statistical data if possible. We expect that the currently operational SBAS implementation such as the WAAS uses the latter configuration.

To compare the two scenarios above, we used real GPS observation data to simulate possible Korean SBAS messages. The observation data from five NGII reference stations were downloaded to estimate the correction data and to encode the standard SBAS message. The selected stations were located at Jindo, Sejong, Busan, Inje, and Ganghwa. Our concern was to extend the service area of the SBAS user by minimizing the certified SBAS algorithm modification when it starts the dual frequency service in the near future. To analyze the effect on the service area extension, an IGS station outside of the service area was selected as a user under test. Among the IGS stations in the East Asia area, the

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only IGS station providing high rate data is the USUD station in Japan. The selected station provides L1/L2 dual frequency observables as 1-Hz in the form of the RINEX.

In Fig. 4, the estimation results for the satellite orbit errors are plotted in meters assuming the scenario 1 configuration. A priori information for the orbit error is assumed approximately 0.05 m per each coordinate axis, which means that the filter assigned a very high confidence to the GPS broadcast ephemeris. The post processing results for 18 hours of RINEX data were used for all visible satellites. The results show that the a posteriori covariance element has a higher value than that of the a priori at approximately 0.4 m, which is similar to the rms error of the current block vehicles. In Fig. 4, a few outliers reaching up to about 4 ~ 5 m are the filter outputs from the rising satellites, i.e., values before the converging. Except for the peaks, the estimated orbit errors mostly stay within a two-meter level. The satellite orbit error estimate results in Fig. 5 follow the scenario 2 configuration. For the a priori covariance of the second configuration, the orbit errors are assumed to be approximately 3 m per each coordinate, which is a few times larger than the statistics of the GPS satellite orbit error shown in Fig. 3. In Fig. 5, the results show that the a posteriori error is 11.7 m, which is a very large error and is relative to that of the statistics in Fig. 3. Except for a few outliers corresponding to rising satellites, the estimated orbit errors are within about 100 m. Because the orbit



Fig. 4. Satellite orbit corrections estimated – Scenario 1



Fig. 5. Satellite orbit corrections estimated - scenario 2

correction value per each axis has an effective range of  $\pm 128$  m by the SBAS standard [11], the outliers should be cut off at the effective range limit before composing the correction message.

In Fig. 6 and Fig. 7, the estimated satellite clock corrections are shown for each scenario. The results show that the a posteriori errors are 0.1 m and 0.4 m each. For both of the cases, the clock error statistics are lower than those of the SIS statistics in Fig. 3. The erective range of the clock correction in the SBAS standard is -256 ~ 255.875 m. In both scenarios, there was no saturation in the clock estimates which can be seen in the figures. Although there were some outliers from Fig. 4 to Fig. 7, it does affect the SBAS operation because the SBAS correction message composer will not include corrections corresponding rising satellites for a given time.

The residual error associated with the fast and long-term corrections is characterized by the  $\sigma_{flt}$  which is defined as equation (15) in the SBAS standard [11].

$$\sigma_{iii}^{2} = \begin{cases} \left[ \left( \sigma_{UDRE} \right) \left( \delta UDRE \right) + \varepsilon_{ic} + \varepsilon_{irc} + \varepsilon_{ilc} + \varepsilon_{or} \right]^{2}, & \text{if } RSS_{UDRE} = 0 \\ \left[ \left( \sigma_{UDRE} \right) \left( \delta UDRE \right) \right]^{2} + \varepsilon_{ic}^{2} + \varepsilon_{irc}^{2} + \varepsilon_{ilc}^{2} + \varepsilon_{or}^{2}, & \text{if } RSS_{UDRE} = 1 \end{cases}$$
(15)

Here, the epsilon  $\epsilon$  denotes the degradation factor. As described in the previous sub-section, the Kalman filter is defined in the form of the full vector, which means the root square sum RSS<sub>UDRE</sub> is zero. In the simulation, the degradation factors are assumed as zero because integrity is outside of the scope of this paper. Assuming zero degradation factors, the variation  $\sigma_{nt}$  is a function



Fig. 6. Satellite clock corrections estimated - scenario 1



Fig. 7. Satellite clock corrections estimated - scenario 2

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of the broadcast UDRE  $\sigma_{UDRE}$  and the location specific modifier  $\delta$ UDRE. Therefore, we can use equation (15) as an indirect measure to analyze the spatial decorrelation effects of the SBAS correction messages in this paper.

In Fig. 8 and Fig. 9 snapshot of the residual  $\sigma_{nt}$  is plotted for each scenario. We can see that there is negligible residual variation in Fig. 8 while the estimated correction error in and out of the network has a larger value by about 0.1 m in Fig. 9. Please note that a change in covariance of the observables and satellite geometry could affect the values of  $\sigma_{nt}$ , and the variation might be larger than this snapshot example. From Fig. 8 and Fig. 9 we can infer that the position error of scenario 2 could be larger than that of scenario 1.

In Fig. 10 and Fig. 11, the horizontal errors are plotted to directly compare the performance. The legend written as L1/L2 SBAS in the two figures means that the ionospheric delay is removed by the dual frequency technique without applying the SBAS ionospheric correction. Because the ionospheric delay can be assumed to be canceled out by 99% or more, we can infer that the performance difference between the two figures comes from the spatial decorrelation effect of the satellite orbit and clock correction only. The horizontal errors (CEP) in scenario 1 and scenario 2 are 0.62 m and 1.09 m each. The elevation mask is set to 10 degrees for user.

Detailed positioning performance is listed in Table 1 for



Fig. 8.  $\sigma_{flt}$  w/o degradation factor – scenario 1



Fig. 9.  $\sigma_{\rm flt}$  w/o degradation factor – scenario 2

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comparison. Due to the carrier smoothing process in the user navigation module and the high quality of the RINEX observables, the USUD IGS station standalone performance is quite good in the horizontal. However, we can see that the vertical position error is relatively large due to the ionospheric delay error of the Klobuchar model. For both of the scenarios, the performance of the SBAS positioning is clearly better than that of the Standalone positioning. From the table, we can see that the positioning errors of scenario 2 are poor and have twice the w.r.t. compared to the scenario 1 results in all directions. From results, we can conclude that the a priori covariance of the filter should be configured to make the a posteriori covariance close to the SIS statistics to reduce the effects of the spatial decorrelation for the SBAS of the Korean peninsula.

# 5. Conclusion

In this paper, the characteristics of the satellite orbit and clock corrections of the WADGPS were analyzed, and



Fig. 10. Horizontal error in ENU - scenario 1



Fig. 11. Horizontal error in ENU - scenario 2

Table 1. Positioning performance comparison

RMS	East [m]	North [m]	Up [m]
Standalone	1.12	1.52	5.42
SBAS (scenario1)	0.56	0.54	1.45
SBAS (scenario2)	0.93	1.00	3.29

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a filter design approach was proposed to minimize the spatial decorrelation effect of the satellite error corrections. A simulation was conducted with the high rate RINEX data from the National Geographic Information Institute and the IGS. Positioning and integrity results show that dual-frequency users can keep accurate navigation performance in the service area boundary by optimizing the a priori information of the filter. In case the a priori covariance is close to the SIS statistics, SBAS corrections experience large spatial decorrelations. However, the spatial effects are reduced well when the a posteriori covariance is close to the SIS statistics.

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