Nonlinear Structural Analysis of High-Aspect-Ratio Structures using Large Deflection Beam Theory

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Abstract

The nonlinear structural analyses of high-aspect-ratio structures were performed. For the high-aspect-ratio structures, it is important to understand geometric nonlinearity due to large deflections. To consider geometric nonlinearity, finite element analyses based on the large deflection beam theory were introduced. Comparing experimental data and the present nonlinear analysis results, the current results were proved to be very accurate for the static and dynamic behaviors for both isotropic and anisotropic beams.

Key Word : Nonlinear Structural Analysis, Geometric Nonlinearity, Material Nonlinearity, Large Deflection Beam Analysis, Isotropic Beam, Composite Box Beam

Introduction

The aspect ratio can be defined as the span divided by the mean or average chord of wing in aerospace engineering field. It is a measure of slenderness of wings or structures[1]. High aspect ratio wings are long and slender wings. Those high-aspect-ratio wings have many advantages such as less drag at low speeds. Helicopter rotor blades, high-altitude aircrafts and high-performance sailplanes are typical examples of high-aspect-ratio structures. Recently, as the unmanned high-altitude long-endurance (HALE) aircraft have gained interest, such high-aspect-ratio structures become more important in aircraft structural design.

In general, beam models have been used for structural analysis of wings from the early times of aviation. They are quite effective for the preliminary design of high aspect ratio wings. On the other hands, plate models have been considered for the case of low aspect ratio wing such fighter-type aircraft wing structures[2]. To investigate characteristics of high-aspect ratio structures, thus, beam models are used in the present research.

A high-aspect-ratio structure such as wing or rotor blade can have large deflections, which under certain loading conditions, can cause the geometric nonlinear range of the structure. Hence, the structural analyses of a high-aspect-ratio structure require nonlinear analyses that involve structural, inertial, and external loads. The research carried out during the past decades includes the treatment of the complete coupled flatwise-edgewise-torsion problem including large deflection effects. Geometrical nonlinearities due to large deflections play an important role in static and dynamic characteristics of high-aspect-ratio structures[3].

Most of the structural dynamic models for high-aspect-ratio structures are moderate deflection type beam theory[4] that are based on ordering schemes and are valid for moderate

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deflections. These models have frequently been applied to the aeroelastic stability and response analyses for isotropic and composite rotor blades[5]. However, a general purpose analysis needs a large deflection model without artificial restriction on displacements and rotations due to the deformation and nonlinearity. The ordering scheme, although it can be a useful theory in specific purpose research, is not desirable in a general purpose approach. To overcome these limitations, nonlinear models that are valid for large deflections and are not based on ordering schemes have been developed during the last few decades [6]. The only restriction on the deformation is that the strain is small compared with unity. There are no small angle approximations, and all kinematic nonlinear effects are included in the formulation.

In this paper, the static and dynamic characteristics of high-aspect-ratio structures including both an isotropic and anisotropic material undergoing large deflection have been investigated. The structural model used in the present analysis is based on Bauchau and Hong's beam model[7], which utilized orientation angles to represent large rotation. The finite element equations of motion derived from Hamilton's principle are solved for the static deflection using the Newton-Raphson method. A line search was combined with the Newton-Raphson method to accelerate the convergence of the iterative calculation in nonlinear structural analyses. After performing the coupled free vibration analysis of the structure about equilibrium position, modal analyses are conducted. The present research is verified by comparing static deflections and natural frequencies with other researchers' works. By comparing between linear and nonlinear analyses results, the influence of geometrical nonlinearity on high-aspect-ratio structures was investigated. Using these procedures, flutter analysis will be performed for HALE aircraft as well as rotating structures.

Theoretical Background

Large Deflection Beam Theory

To analyze a structure using the large deflection beam theory, following assumptions are introduced: There is no deformation on the cross sectional plane. Hence, one-dimensional model is available along the beam axis. The beam can have initial curvature and twist. Initial curvature, however, is small like a practical wing. The strain level remains low even if a large deflection occurs. And a wing can be modeled as a cantilever beam.

Using Euler angles, a deformed shape can be expressed with respect to a reference coordinate. The transformation matrix can be written as follows:

$$T=\begin{bmatrix} \cos\beta\cos\psi & \cos\beta\sin\psi & \sin\beta \\ -\sin\theta\sin\beta\cos\psi-\cos\theta\cos\psi & \cos\theta\cos\psi-\sin\psi\sin\beta\sin\theta & \cos\beta\sin\theta \\ -\cos\theta\sin\beta\cos\psi+\sin\theta\sin\psi & -\sin\theta\cos\psi-\sin\psi\sin\beta\cos\theta & \cos\beta\cos\theta \end{bmatrix} \tag{1}$$

To calculate the strain in the curvilinear coordinate, Green-Lagrange strain tensor is introduced. By the assumption of small initial curvature of the beam, high order terms in Green-Lagrange strain can be neglected. Using this, engineering strains are obtained as follows:

$$\begin{aligned} &\epsilon_{11}=\bar{e}_{11}+x_3\kappa_2-x_2\kappa_3+w_1{}',\ \epsilon_{22}=w_{2,2},\ \epsilon_{33}=w_{3,3} \\ &\gamma_{12}=2\bar{e}_{12}-x_3\kappa_1+w_{1,2}+w_2{}',\ \gamma_{23}=w_{2,3}+w_{3,2},\ \gamma_{13}=2\bar{e}_{13}-x_2\kappa_1+w_{1,3}+w_3{}' \\ &\kappa_i=K_{i-k_i} \end{aligned} \tag{2}$$

where x_1, x_2 and x_3 are curvilinear coordinates. w_1, w_2 and w_3 are the general warping displacements of an arbitrary point on the cross section. The force strains ($\bar{e}_{11}, 2\bar{e}_{12}, 2\bar{e}_{13}$) and moment strains ($\kappa_1, \kappa_2, \kappa_3$) components are given in Ref. 7. Herein, $(\ )'$ means the derivative with respect to x_1 and $(\ )_{,i}$ means the derivatives with respect to x_i, i = 2,3. Through a quasi-linear approximation, three-dimensional kinematics are divided into two-dimensional cross-sectionalanalysis and the one-dimensional global analysis. Using the Hamilton's principle, the equation of motion can be obtained as follows:

$$\int_{t_1}^{t_2} \sum_{i=1}^{m} (\delta U_i - \delta T_i - \delta W_i) dt = 0 \tag{3}$$

where δU_i, δT_i and δW_i are the variation of strain energy, the variation of kinetic energy and the virtual work done by external forces. The nonlinear finite element equation of motion is obtained in the matrix form,

$$[M(q)]\{\ddot{q}\} + [G(q)]\dot{q} + P(q) - P_C(q) = P_A(q) \tag{4}$$

where $M(q)$,$P(q)$, and $P_A(q)$ are mass matrix, internal elastic force vector and external force vector, respectively. $P_C(q)$ and $G(q)$ are centrifugal force vector and gyroscopic matrix if a rotating motion is included. Newton-Raphson method combined with the line search method to improve convergence and reliability is applied. To validate the developed nonlinear beam finite element model, two kinds of cases are chosen. The first one is for isotropic beam, and the second one is for anisotropic beam.

Numerical Analysis Results

Isotropic Beam Model

Firstly, to validate high-aspect-ratio structural analysis program, Princeton beam is chosen. The beam is straight with uniform rectangular cross-section. The Princeton beam experiments were conducted by E.H. Dowell and J.J. Traybar[8]. The experiments were a study of the flap, lag, and twist associated with geometrical nonlinear effect of a 7075 aluminum beam. The dimensions of the beam are illustrated in Fig. 1 and Table 1. The general idea was to determine the effect of point loads on a helicopter rotor blade by testing its static deflection and vibration modes at different pitch angle between 0 ~ 180 deg and to compare that data with a nonlinear deformation models performed by Hodges and Dowell.

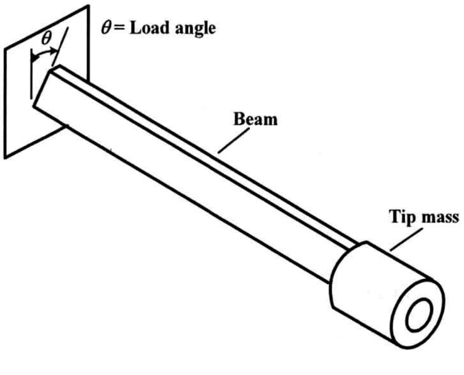
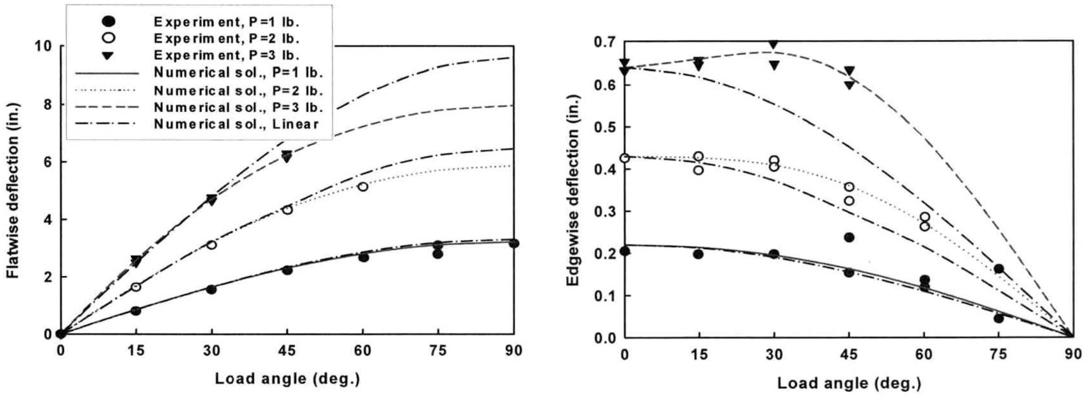


Fig. 1. Schematic of the Princeton beam experimental apparatus[6]

Table 1. Princeton beam properties[6]

Material	7075 Aluminum
Elastic modulus	1.04×10^{7} lb/in.
Poisson' Ratio	0.31
Length	20 in.
Thickness	0.125 in.
Width	0.5 in.
Mass density	2.626×10^{-4} lb-s^2/in.^{4}
Gravitational constant	386.089 in./s^2
Mass per unit length	1.6424×10^{-5} lb-s^2/in.^{2}
Mass moments of inertia (flatwise)	2.1420×10^{-8} lb-s^2
Mass moments of inertia (edgewise)	3.4204×10^{-7} lb-s^2
Axial stiffness (EA)	6.2856×10^{5} lb
Flatwise stiffness (EI)	8.4487×10^{2} lb-in.^{2}
Edgewise stiffness (EI)	1.2689×10^{4} lb-in.^{2}
Torsional stiffness (GJ)	1.0538×10^{3} lb-in.^{2}



(a) Flatwise +deflection (b) Edgewise deflection

Fig. 2. Comparisons of translational deflection for various mass at the tip

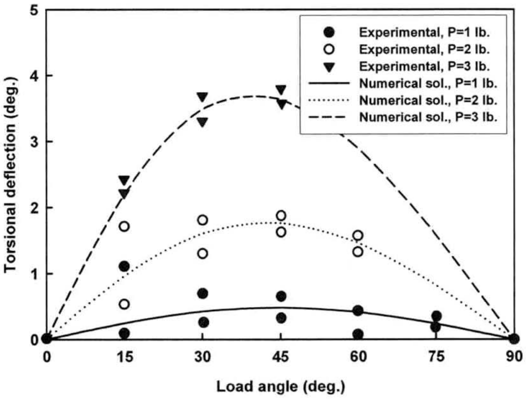
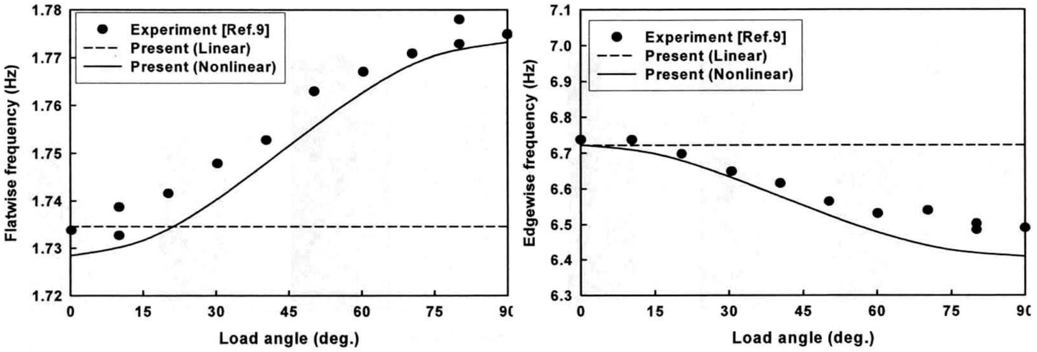


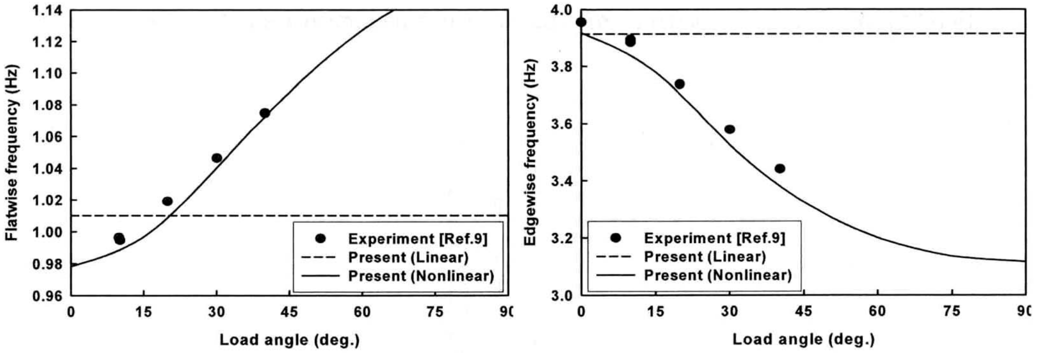
Fig. 3. Comparisons of rotational deflection for various mass at the tip

The present numerical analysis results and experimental measurements of tip deflection as a function of the load angle for several load level are shown in Figs. 2-3. The present nonlinear numerical results for the edgewise and flatwise components shows good agreement with the experimental data for all of the tip loads and load angles in Fig. 2a-b. Similar results for the torsional deflection are presented in Fig. 3. The nature of torsional deflection is discussed in Ref.9 and values of torsional deflections were calculated using the arc cos formulation. Static tip deflections show good agreement with experimental measurement data. It is notable that the torsional deflection of linear analysis is zero due to the absence of the coupling between the torsion and bending displacement. The Present analysis model based on large deflection beam theory, hence, can predict well structural nonlinearity effect in static analysis.

Fig. 4(a)-(d) illustrate the flatwise and edgewise frequencies as a function of load angles. In the case of 1-lb tip mass, the present analysis results are slightly offset from the experimental data but follow the trend exactly in both flatwise and edgewise frequencies. In the 2-lb force case, similar to 1-lb case, frequencies of calculated results are lower than experimental value but are omitted due to the page limitation. In the case of 3-lb tip force, the flatwise and edgewise frequency sweeps are shown for the experimental data and the present results. The correlation is excellent. From the comparisons, the present analysis results accurately predict the nonlinear static and dynamic behavior of isotropic beams.



(a) 1st flatwise (1-lb) (b) 1st edgewise (1-lb)



(c) 1st flatwise (3-lb) (d) 1st edgewise (3-lb)

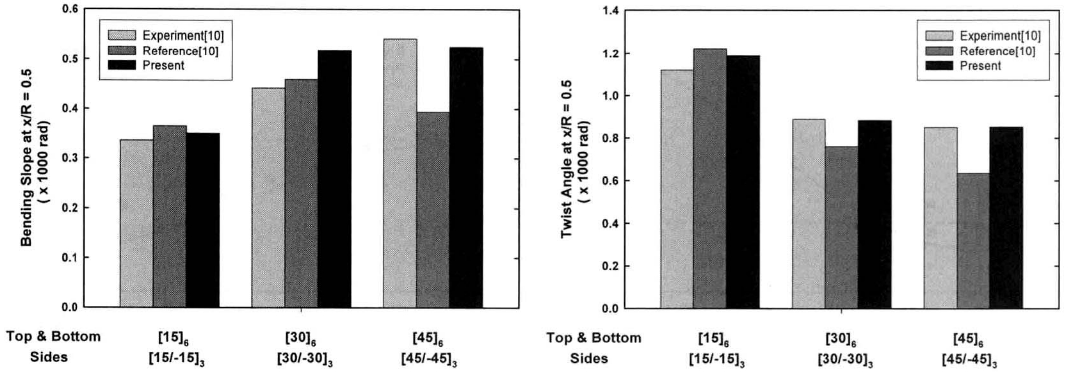
Fig. 4. Natural frequencies vs load angle for various tip mass

Anisotropic Beam Model

To validate anisotropic material case, a composite box beam is chosen. This model was used to investigate static and dynamic characteristics for the composite rotor blades of helicopter[10]. The beam has a symmetric lay-up sequence. The one-dimensional beam model was used. The beam has 5 elements. Each element has 4 nodes along the spanwise direction and 6 elements with 8 nodes on the cross-section. Physical properties of the beam for static and dynamic analysis are summarized in Table 2.

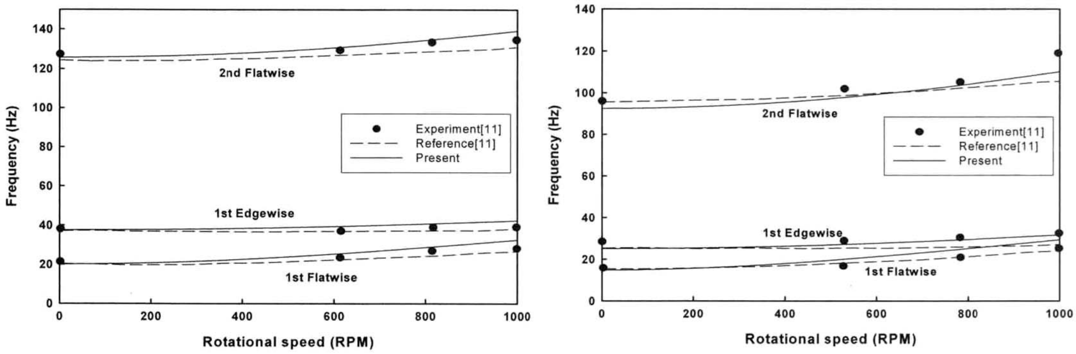
Table 2. Box-Beam Specimen Physical Properties for static/dynamic analysis of symmetric and anti-symmetric layup box beams [10, 11]

Rotor Characteristics		AS4/3501-6 Graphite/Epoxy	
L (in) - static	30	E_1 (msi)	20.59
- dynamic	33.25	E_2 (msi)	1.42
d (in)	0.530	G_{12} (msi)	0.87
c (in)	0.953	G_{23} (msi)	0.47
Ply Thickness (in)	0.005	ν_{12}	0.42
Wall Thickness (in)	0.030	ν_{23}	0.50
		Mass density ($lb\ s^2/in^4$)	0.1352e-3



(a) Bending slope (b) Twist angle

Fig. 5. Bending slope and twist angle at x/R=0.5 under unit tip torsion load of composite box beams



(a) ply angle θ = 30 deg (b) ply angle θ = 45 deg

Fig. 6. Natural frequencies of the composit box beam

Bending slope and twist angle at the middle of the beam with several ply angles under unit tip torsion load are shown in Fig. 5. The results using large deflection beam theory were compared with experimental data and analysis results in the Ref. 10. The results were well agreeable over most all of the validation cases in Fig. 5.

Chandra and Chopra performed dynamic tests to measure natural frequencies for the rotating beam[11]. The test was taken in the vacuum chamber to ignore aerodynamic effects. Changes of natural frequencies with increasing rotational speed were shown in Fig.6. The results were compared with experimental data and analysis results in the Ref.11. The results were well agreeable over most all of the validation cases in Fig. 6. The natural frequencies in Fig. 6(b) were smaller than those in Fig. 6(a). It was because of the reduction of bending stiffness as increasing ply angle.

Conclusion

In this study, nonlinear structural analyses of high-aspect-ratio structure were performed. For performing numerical analysis, the high-aspect-ratio structure was modeled as a beam. Finite element analyses based on the large deflection beam theory were performed. Comparing experimental data, linear analysis results, and nonlinear analysis results, the present nonlinear analysis results were proved to be very accurate on static and dynamic characteristics in both isotropic and anisotropic beams. From the current research, the nonlinear bending-torsion coupling effects were found. These phenomena may affect the static and dynamic characteristics of fluid-structure interaction problem of slender structures such as high-aspect-ratio wings.

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References

1. Kermode, A. C., Barnard, R. H., Philpott, D. R., The Mechanics of Flight, Prentice Hall, 2006.

2. Librescu, L., and Simovich, J., "A General Formulation for the Aeroelastic Divergence of Composite Swept Forward Wing Structures", Journal of Aircraft, Vol. 25, No. 4, 1988, pp. 364-371.

3. Ormiston, R. A., "Investigation of Hingeless Rotor Stability", Vertica, Vol. 7, No. 2, 1983, pp. 143-181.

4. Hodges, D. H., and Dowell, E. H., "Nonlinear Equations of Motion for Elastic Bending and Torsion of Twisted Non-Uniform Rotor Blades", NASA TN D-7818, 1974.

5. Hodges, D. H., and Ormiston, R. A., "Stability of Elastic Bending and Torsion of Uniform cantilever Rotor Blade in hover with Variable Structural Couplings" NASA TN D-8192, 1976.

6. Hinnant, H. E., and Hodges, D. H., "Nonlinear Analysis of a Cantilever Beam", AIAA Journal, Vol. 26, No. 12, 1988, pp. 1521-1527.

7. Bauchau, O. A., and Hong, C. H., "Nonlinear Composite Beam Theory", ASME Journal of Applied Mechanics, Vol. 55, 1988, pp. 156-163.

8. Dowell, E. H., and Traybar, J., "Experimental-Theorical Correlation Study of Non-Linear Bending and torsion Deformation of a Cantilever Beam", Journal of Sound and Vibration, Vol. 50, No. 4, 1977, pp. 533-544.

9. Rosen, A., Loewy, R., and Mathew, M. B., "Nonlinear Analysis of Pretwisted Rods Using 'Principal Curvature Transformation' - Part II: Numerical Results", AIAA Journal, Vol. 25, No. 4, pp. 598-604.

10. Smith, E. C. and Chopra, I., "Formulation and Evaluation of Analytical Model for Composite Box-Beams", Journal of American Helicopter Society, Vol. 36, No. 3, 1991, pp. 23-35.

11. Chandra, R. and Chopra, I., "Experimental and Theoretical Investigation of the Vibration Characteristics of Rotating Composite Box Beams", Journal of Aircraft, Vol. 29, No. 4, 1992, pp. 657-664.