Formation Geometry Center based Formation Controller Design using Lyapunov Stability Theorem

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Abstract

New formation flight controller for unmanned aerial vehicles is proposed. A behavioral decentralized control approach called formation geometry center control is adopted. Trajectory tracking as well as formation geometry keeping are the purpose of the formation flight, and therefore two controllers are designed: a trajectory tracking controller for reference trajectory tracking, and a position controller for formation geometry keeping. Each controller is designed using Lyapunov stability theorem to guarantee the asymptotic stability. Formation flight controller is finally obtained by combining the trajectory tracking controller and the formation geometry keeping controller using a weighting parameter that depends on the relative distance error between unmanned aerial vehicles. Numerical simulations are performed to validate the performance of the proposed controller.

Key Word : formation flight, behavioral approach, formation geometry center, Lyapunov stability theorem

Introduction

Recently, the use of Unmanned Aerial Vehicle (UAV) in military applications has increased significantly. Especially, UAVs play important roles in both military and civil operations for the purpose of surveillance, reconnaissance, monitoring the dangerous area, etc. The development of UAV formation flight system will increase the probability of mission completion via cooperative control in uncertain environments.

In general, formation flight control schemes are classified into several approaches: leader-follower approach, virtual structure approach, behavior based approach, and so on. Leader-follower approach is the most popular formation flight scheme in which the leader flies along the reference trajectory and the other UAVs follow the leader while keeping a certain distance. However, in case that the leader has a problem, the formation can be easily broken down. Also, there exists error propagation and time delay in the communication which may degrade the performance of formation flight. Virtual structure approach was developed to overcome the weak point of the leader-follower method. It treats all the formation members as a single agent and they maintain the geometric relationships. However, virtual structure based

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formation flight method has difficulties in the mathematical approach, and requires a lot of computation. Unlike the above two approach, the behavior based approach has been developed based on the natural behavior of emigrational birds. In this approach, each UAV has a decision capability and does not refer to the preceding one so that poorer transient response or other troubles caused by loosing information can be prevented.

Giulietti et al. proposed a Formation Geometry Center (FGC) control method which is a kind of behavioral based approach.[1] In FGC method, the aircraft in a formation does not refer to leader UAV or any other UAVs anymore. However, each UAV in formation flight is required to keep a specified distance from the imaginary point called FGC whose dynamics is related to each UAV. Controlling FGC does not require the path generation for each aircraft, because imaginary point inside the geometry is the control objects. Therefore, it is much easier to control a formation and to track a reference trajectory. In Ref. [1], point mass model was considered, and the FGC control is derived for a linearized model around a selected flight condition. In addition, linear quadratic formed regulator in the inertial frame was designed, but the controller cannot deal with the heading angle change.

In this study, three-dimensional point mass model is considered, and coordinate transformation regarding the heading angle variation is adapted for better performance. This leads to the equations of nonlinear dynamics so that nonlinear control method using Lyapunov stability theorem is used to generate the control input. Two separate controllers are designed, one is for the formation keeping and the other is for the trajectory tracking. Finally, the formation keeping controller and the trajectory tracking controller are combined by a weighting factor which depends on the relative distance error between UAVs.

This paper is organized as follows. In section 2, UAV model and the concept of FGC is briefly introduced. Formation controller using Lyapunov stability theorem is designed in section 3. Numerical simulation results are shown and discussed in the following section. Finally, conclusions are made in the section 5.

Preliminaries

UAV model

In this study, three-dimensional point mass model is considered. The equations of motion of the i-th UAV are given by the following equation.

$$x_i = V_i \cos\gamma_i \cos\chi_i \quad (1)$$

$$y_i = V_i \cos\gamma_i \sin\chi_i \quad (2)$$

$$z_i = -V_i \sin\gamma_i \quad (3)$$

The state variables that describe the aircraft motion are the velocity (V), flight path angle (γ), heading angle (χ), and three position variables (x, y, z) in the inertial frame, respectively.

To consider the time delay of aircraft dynamics and autopilot, it is assumed that (V_i, γ_i, χ_i) are modeled by the following first order dynamics.

$$\dot{V}_i = -\frac{1}{\tau_V}(V_i - V_{ci}) \quad (4)$$

$$\dot{\gamma}_i = -\frac{1}{\tau_\gamma}(\gamma_i - \gamma_{ci}) \quad (5)$$

$$\dot{\chi}_i = -\frac{1}{\tau_\chi}(\chi_i - \chi_{ci}) \quad (6)$$

where $\tau_V, \tau_\gamma, \tau_\chi$ are the time constants of each control variables, and V_c, γ_c, χ_c are the guidance command inputs.

Formation Geometry Center

Among various multi-vehicle formation structures, decentralized management approach which gives a certain level of decision capability is catching attentions these days. Giulietti et al. suggested FGC control method to overcome the problems of leader-follower approach and virtual structure approach.[1,2] Figure 1 shows the concept of the FGC, and the definitions of the inertial frame and body-fixed frame.

The position of FGC depends on the relative distances between the aircrafts, and it can be computed as

$$X_{FGC} = \frac{1}{N}\sum_{i=1}^{N} X_i \tag{7}$$

where N denotes the number of UAVs, and $X_i = [x_i \; y_i \; z_i]^T$.

Unlike the other formation methods, each UAV does not depend on the leader or the preceding UAV in the FGC control approach. Instead, FGC, an imaginary point, whose dynamics is related to all the UAV positions, becomes a control object. UAVs are required to track the prescribed path while keeping a certain distance from the FGC. For this reason, each UAV has its formation controller composed of two different controllers. One is the position controller to keep the formation geometry of certain shape, and the other is the trajectory controller to track the prescribed trajectory. Note that it is necessary to rotate the inertial frame to body-fixed frame of each UAV to calculate the relative distance. The rotation matrix from the inertial frame to the body-fixed frame are defined as

$$T_r = \begin{bmatrix} \cos\gamma_r \cos\chi_r & \cos\gamma_r \sin\chi_r & -\sin\gamma_r \\ -\sin\chi_r & \cos\chi_r & 0 \\ \sin\gamma_r \cos\chi_r & \sin\gamma_r \sin\chi_r & \cos\gamma_r \end{bmatrix} \tag{8}$$

After the position of the FGC in the inertial frame (X_{FGC}) is calculated, the relative distance vector can be derived by using the current position from the FGC position. And the rotation matrix in Eq. (8) is used to compute the relative distance between i-th UAV from the FGC as

$$\begin{aligned} d_i &= T_r^i \left(X_{FGC} - X_i\right) \\ &= T_r^i \, C_d^i X \end{aligned} \tag{9}$$

where $C_d^i = \frac{1}{N}[I^1 \; I^2 \cdots \; (1-N)I^i \cdots \; I^N]$, and I^i denotes a 3x3 identity matrix corresponding to the i-th UAV.

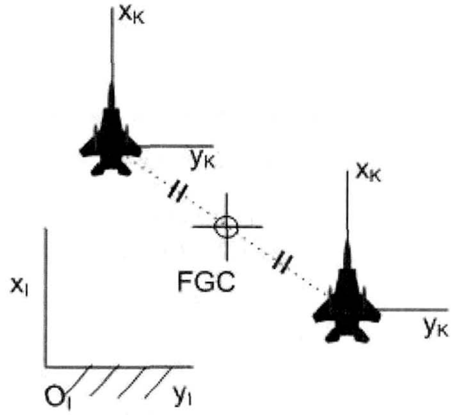

Fig. 1. Formation Geometry Center

Because formation consists of N-UAVs, Eq. (9) can be rewritten as

$$\begin{aligned} d &= [d_1 \cdots d_N] \\ &= T_r\, C_d\, X \end{aligned} \tag{10}$$

where $T_r = diag(T_r^{\ i})$ and $C_d = col(C_d^{\ i})$.

In addition to the formation keeping controller, trajectory tracking controller is required for tracking the reference trajectory. For this, let us define the position of FGC as

$$y = C_y\, X \tag{11}$$

where $C_y = \frac{1}{N}[I^1 \cdots I^N]$.

Also, let us define a state vector as follows

$$\dot{X}_i = \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{z}_i \end{bmatrix} = \begin{bmatrix} V_i \cos\gamma_i \cos\chi_i \\ V_i \cos\gamma_i \sin\chi_i \\ -V_i \sin\gamma_i \end{bmatrix} \tag{12}$$

Differentiation of Eq. (12) with respect to time yields

$$\ddot{X}_i = \begin{bmatrix} \cos\gamma_i \cos\chi_i & -\sin\gamma_i \cos\chi_i & -\cos\gamma_i \sin\chi_i \\ \cos\gamma_i \sin\chi_i & -\sin\gamma_i \sin\chi_i & \cos\gamma_i \cos\chi_i \\ -\sin\gamma_i & -\cos\gamma_i & 0 \end{bmatrix} \begin{bmatrix} \dot{V}_i \\ V_i \dot{\gamma}_i \\ V_i \dot{\chi}_i \end{bmatrix} \tag{13}$$

Substituting Eqs. (4)-(6) into Eq. (13), the second derivative of X_i can be written as the following equations. Note that the command input variables V_{ci}, γ_{ci}, χ_{ci} are separated, and therefore controller can be designed using those equations.

$$\begin{aligned} \ddot{X}_i &= \begin{bmatrix} \cos\gamma_i \cos\chi_i & -\sin\gamma_i \cos\chi_i & -\cos\gamma_i \sin\chi_i \\ \cos\gamma_i \sin\chi_i & -\sin\gamma_i \sin\chi_i & \cos\gamma_i \cos\chi_i \\ -\sin\gamma_i & -\cos\gamma_i & 0 \end{bmatrix} \begin{bmatrix} -\frac{V_i}{\tau_V} \\ -\frac{V_i}{\tau_\gamma}\gamma_i \\ -\frac{V_i}{\tau_\chi}\chi_i \end{bmatrix} \\ &\quad - \begin{bmatrix} \cos\gamma_i \cos\chi_i & -\sin\gamma_i \cos\chi_i & -\cos\gamma_i \sin\chi_i \\ \cos\gamma_i \sin\chi_i & -\sin\gamma_i \sin\chi_i & \cos\gamma_i \cos\chi_i \\ -\sin\gamma_i & -\cos\gamma_i & 0 \end{bmatrix} \begin{bmatrix} -\frac{1}{\tau_V} & 0 & 0 \\ 0 & -\frac{V_i}{\tau_\gamma} & 0 \\ 0 & 0 & -\frac{V_i}{\tau_\chi} \end{bmatrix} \begin{bmatrix} V_{ci} \\ \gamma_{ci} \\ \chi_{ci} \end{bmatrix} \\ &= R_{1i} - R_{2i} u_i \end{aligned} \tag{14}$$

Now, the time derivatives of the relative distances and FCG can be obtained as

$$\dot{d} = T_r C_d \dot{X} + \dot{T}_r C_d X \tag{15}$$

$$\begin{aligned} \ddot{d} &= T_r\, C_d \ddot{X} + \ddot{T}_r\, C_d X + 2\dot{T}_r C_d \dot{X} \\ &= \ddot{T}_r\, C_d X + 2\dot{T}_r C_d \dot{X} + T_r\, C_d (R_1 - R_2 U) \end{aligned} \tag{16}$$

$$\dot{y} = C_y\, \dot{X} \tag{17}$$

$$\ddot{y} = C_y (R_1 - R_2 U) \tag{18}$$

where $R_1 = col(R_{1i})$, $R_2 = diag(R_{2i})$ and $U = [u_1^{\ T} \cdots u_N^{\ T}]^T$.

Lyapunov Stability Theorem Based Control Design

Error Dynamics

In order to design the trajectory tracking controller and the formation keeping controller, let us define error variables that are differences between a current state and a desired state. Similar to the equations in the previous section, the error dynamics of formation keeping controller and trajectory tracking controller can be derived as follows.

$$e_y = r_y - y \tag{19}$$

$$\dot{e}_y = \dot{r}_y - C_y \dot{X} \tag{20}$$

$$\ddot{e}_y = \ddot{r}_y - C_y R_1 + C_y R_2 u \tag{21}$$

$$e_d = r_d - d \tag{22}$$

$$\dot{e}_d = \dot{r}_d - T_r C_d \dot{X} - \dot{T}_r C_d X \tag{23}$$

$$\ddot{e}_d = \ddot{r}_d - 2\dot{T}_r C_d \dot{X} - \ddot{T}_r C_d X - T_r C_d R_1 + T_r C_d R_2 u \tag{24}$$

Equations (19)-(21) are the trajectory tracking error dynamics of the current states and reference trajectory while Eqs. (22)-(24) are the formation keeping error dynamics of the desired relative distance and current distance error. Note that the second derivatives of error dynamics should be calculated in both controllers.

Controller Design

By considering the heading variable, the system dynamics become nonlinear so that linear quadratic regulator cannot be applied anymore. Therefore, nonlinear control method using Lyapunov stability theorem is adopted in this study.

Lyapunov candidate function for tracking error is considered as

$$V_y = \frac{1}{2} e_y{}^T Q_y e_y + \frac{1}{2} \dot{e}_y{}^T Q_{\dot{y}} \dot{e}_y \tag{25}$$

where Q_y, $Q_{\dot{y}}$ are positive definite matrices.
Differentiating Eq. (25) and using (21), we have

$$\begin{aligned} \dot{V}_y &= e_y^T Q_y \dot{e}_y + \dot{e}_y^T Q_{\dot{y}} \ddot{e}_y \\ &= e_y^T Q_y \dot{e}_y + \dot{e}_y^T Q_{\dot{y}} (\ddot{r}_y - C_y R_1 + C_y R_2 U_y) = -c_1 \dot{e}_y^T Q_{\dot{y}} \dot{e}_y \end{aligned} \tag{26}$$

where $c_1 > 0$.
To design a trajectory tracking controller, let us choose the control input as

$$U_y = R_2^{-1}\left\{R_1 - C_y^T (C_y C_y^T)^{-1} \left\{\ddot{r}_y + Q_{\dot{y}}^{-1} Q_y e_y + c_1 \dot{e}_y\right\}\right\} \tag{27}$$

Substituting Eq. (27) into Eq. (26), we have

$$\dot{V}_y = -c_1 \dot{e}_y^T Q_{\dot{y}} \dot{e}_y \le 0 \tag{28}$$

Equation (28) shows that the stability of the trajectory tracking controller is guaranteed in the Lyapunov sense.

Formation keeping controller is also designed in the same way. In order to reduce the relative distance error, the following Lyapunov candidate function is considered.

$$V_d = \frac{1}{2} e_d{}^T Q_d e_d + \frac{1}{2} \dot{e}_d{}^T Q_{\dot{d}} \dot{e}_d \tag{29}$$

where Q_d, $Q_{\dot{d}}$ are positive definite matrices.

Differentiating Eq. (29) and using (24), we have

$$\begin{aligned}\dot{V}_d &= e_d^T Q_d \dot{e}_d + \dot{e}_d^T Q_{\dot{d}} \ddot{e}_d \\ &= e_d^T Q_d \dot{e}_d + \dot{e}_d^T Q_{\dot{d}} (\ddot{r}_d - 2\dot{T}_r C_d \dot{X} - \ddot{T}_r C_d X - T_r C_d R_1 + T_r C_d R_2 U_d) \\ &= - c_2 \dot{e}_d^T Q_{\dot{d}} \dot{e}_d\end{aligned} \tag{30}$$

where $c_2 > 0$.

Choose the control input for formation keeping as

$$U_d = R_2^{-1}\left\{R_1 - C_d^T(C_d C_d^T)^{-1} T_r^T\left\{\ddot{r}_d - 2\dot{T}_r C_d \dot{X} - \ddot{T}_r C_d X + Q_{\dot{d}}^{-1} Q_d e_d + c_2 \dot{e}_d\right\}\right\} \tag{31}$$

Substitution of Eq. (31) into Eq. (30), we have

$$\dot{V}_d = - c_2 \dot{e}_d^T Q_{\dot{d}} \dot{e}_d \leq 0 \tag{32}$$

Equation (32) shows that the stability of the formation keeping controller is guaranteed in the Lyapunov sense.

In order to control both formation keeping and trajectory tracking, the control input for the FGC can be obtained by combining two stable controllers as follows

$$U = \eta U_y + (1-\eta) U_d \tag{33}$$

where $\eta = \exp(-\frac{e_d^T e_d}{c_3})$. The formation keeping controller and the reference trajectory tracking controller are designed to be stable in the Lyapunov sense. However, the combined input with η cannot guarantee the stability of the closed-loop system. Note that weighting factor η is chosen as an exponential function of the distance between each UAV and the formation geometry center. When the UAVs are far apart, they are forced to be controlled by the formation keeping controller which has heavy weighting on it compared to the tracking controller. Similarly, once the UAVs are kept in the desired formation, the weighting moves to track the trajectory so that tracking controller is fully activated. That means each stable controller works separately in each case, and the stability of the combined controllers make sense. Rigorous proof of the stability of the proposed combined formation controller is remained as a future work. If the stability is not guaranteed due to the underactuated problem, then other form of combined formation controller should be considered.

Numerical Simulation

In order to validate the proposed formation controller, numerical simulations were performed. Simulation is conducted for the two-dimensional case of two UAVs flying the straight trajectory under the scenario summarized in Table 1. and Figs. 2-5 show the simulation result. As shown in Figs. 3-5, each UAV starts from the different initial position with different velocity, and finally converges to the position satisfying the desired relative distance from FGC and reference velocity. Figure 6 shows the responses of formation keeping controller and trajectory tracking controller with respect to η. Initial relative distance is set to 10 m, and it is required to converge to 5 m. Also, initial heading angle is 30 deg, and it needs to be 0 deg. This result shows how the value of the η influences the UAV's behavior.

Table 1. Simulation Scenario

	Relative distance(m)	Velocity(m/s)
AC1	(10, 20)	32
AC2	(-10, -20)	30
Reference	(5, 10)	30

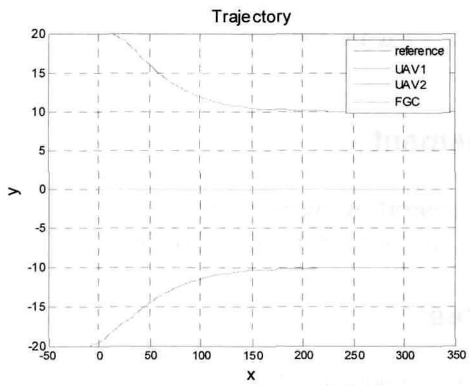

Fig. 2. Trajectory of two UAVs

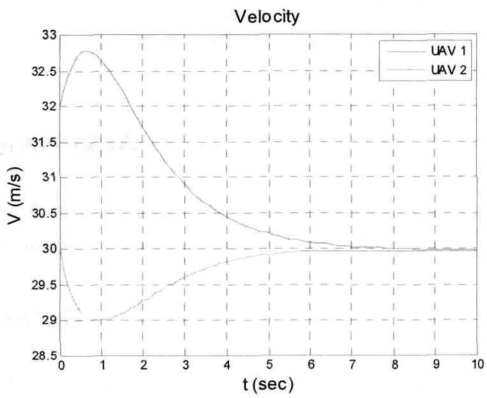

Fig. 3. Time histories of velocity

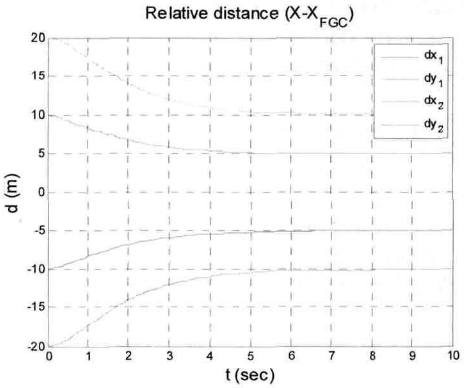

Fig. 4. Time histories of relative distance

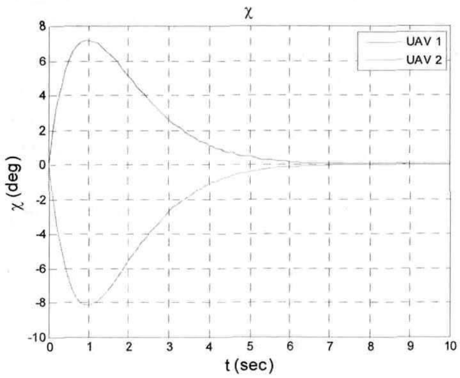

Fig. 5. Time histories of heading angle

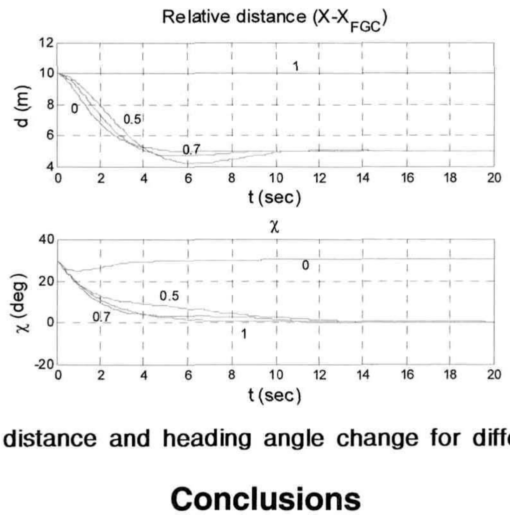

Fig. 6. Relative distance and heading angle change for different values of r

Conclusions

In this paper, a new formation flight method where the control object is the center of the formation geometry. Two individual controllers, tracking controller and formation controller, are designed based on the nonlinear controller using Lyapunov stability theorem. And the formation control input is designed by combining two controllers with weighting factor. Simulation results show that each UAV achieves the trajectory tracking as well as the formation keeping successfully. The proposed formation controller can be applied to the formation flight of multi satellite systems and formation driving of ground mobile robot systems.

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