Cross-flow Analogy and Euler Solutions for Missile Body Aerodynamics

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Abstract

For aerodynamic design of missile bodies of non-circular cross-section, the combination of the slender body theory and the cross-flow analogy can hardly be applied owing to the lack of experimental data. An alternative is to utilize the Euler solution in the design stage. For enhanced accuracy, however, an adequate viscous correction is necessary to the Euler solution. In this work, such a procedure is examined to compensate the viscous effect by utilizing the concept of proportionality factor in cross-flow analogy. Predictions of aerodynamic coefficients combining the Euler solution and the viscous correction via proportionality factor are made for a missile body of elliptic cross-section. Results indicate that the present approach can be adopted in designing missile bodies of non-circular cross-sections.

Key Word : missile aerodynamics, slender body theory, cross-flow analogy, Euler solution, proportionality factor

Introduction

In earlier phase of aerodynamic design of missiles, cross-flow analogy is often employed for its simplicity. In cross-flow analogy, slender body theory is used for potential flow region and viscous effect is taken into consideration by cross-flow theory [1]. This method readily provides us with the normal force and pitching moment coefficients, which are the deciding factors used in conceptual and preliminary aerodynamic design of missile bodies. The cross-flow analogy can be extended to bodies of non-circular cross-sections if drag data for such bodies are available, which enables the cross-flow method applicable to the design of non-circular missile bodies. A definite drawback of the cross-flow analogy, however, is that it employs an empirically adjusted coefficient, termed as the proportionality factor, for increased accuracy. The proportionality factor necessary for accurate predition of the desired normal and pitching moment coefficients for practical body [2] is hard to estimate, if detailed wind-tunnel data are not available. This hampers the adoption of the cross-flow analogy in designing missile bodies with non-conventional cross-sections.

In the present work, we attempt to re-interpret the proportionality factor in view of Euler solutions. The availability of computing power nowadays makes it possible to get Euler solutions fast and cheap enough so that Euler solver may efficiently be used in early stage of design. This can be achieved by first calculating the inviscid region by an Euler solver and then modifying the results with cross-flow analogy to account for viscous effects. Computations are carried out for the case of bodies having both circular and elliptic cross-sections.

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Crossflow Analogy

At small angle of attack, the slender body theory works well for the bodies of high fineness-ratio like a missile body. However, when the angle of attack becomes large, aerodynamic coefficients predicted by the slender body theory deviate considerably from the experimental data. It is known that this is caused by the flow separation in the cross-flow plane, that is the flow separation occurring in the direction perpendicular to the body axis. The flow separation in the cross-sectional plane results in increase of drag component force, which also affects to the normal force and pitching moment [1]. Allen [2] introduced a correction term to include this viscous effect. For bodies of circular cross-section, the normal force and the pitching moment formula put forward by Allen are given by

$$C_N = \text{normal force}/(q_\infty A_R) = C_N^P + C_N^\nu = \frac{\sin 2\alpha}{A_R}\int_0^l \frac{dA}{dx}dx + \frac{2\eta C_{d_0}\sin^2\alpha}{A_R}\int_0^l R\,dx \quad (1)$$

$$C_M = \text{pitching moment}/(q_\infty A_R X) = \frac{\sin 2\alpha}{A_R X}\int_0^l \frac{dA}{dx}(x_m - x)\,dx + \frac{2\eta C_{d_0}\sin^2\alpha}{A_R X}\int_0^l R(x_m - x)\,dx \quad (2)$$

In the equations above, q_∞ is the dynamic pressure of the free stream, A_R the reference area (normally base area of the missile body). C_N^P is the normal force coefficient due to potential flow, C_N^ν the normal force coefficient due to viscous cross-flow. l is the length of the body, C_{d_0} the two dimensional drag coefficient of circular cross-section, η the proportionality factor, X the reference length (normally the diameter of the missile base), R the radius of the circular body, and α the angle of attack. x and x_m are the coordinates along the body axis and of the center of gravity, respectively. For proper application of the equations above, the body has to be slender, and the potential flow part and the viscous flow part should be independent.

Aerodynamic prediction method employing cross-flow analogy has been improved continually by Kelly [3], Sigal [1], and Jorgensen [4], among others. Jorgensen [4] extended the cross-flow analogy to predict the normal force and pitching moment coefficients for bodies having non-circular cross-sections and bodies with wings and tails. He adopted Newtonian impact theory to estimate the drag coefficient of non-circular cross-section. The proportionality factor in principle is the ratio of the two dimensional to the three dimensional drag coefficients and depends on the Mach number, the Reynolds number, the fineness ratio, and the cross-sectional shape. Jorgensen used the proportionality factor in a slightly different point of view to evaluate its value from the difference between the experimental data and the potential flow formula as given below for bodies of circular cross-section :

$$C_N = \frac{\sin 2\alpha\cos(\alpha/2)}{A_R}\int_0^l \frac{dA}{dx}dx + \frac{2\eta C_{d_0}\sin^2\alpha}{A_R}\int_0^l R\,dx = \sin 2\alpha\cos(\alpha/2) + \frac{\eta C_{d_0} A_p \sin^2\alpha}{A_R} \quad (3)$$

$$\eta = \frac{A_R((C_N)_E - C_N^P)}{C_{d_0} A_p \sin^2\alpha} \quad (4)$$

Here, A_P denotes the plan-form area. As Eqn. (4) indicates, the proportionality factor η is function of the measured value of the normal force coefficient $(C_N)_E$. Since the proportionality factor depends on several variables, the proportionality factor is estimated first for different bodies at various angles of attack and then a representative value is obtained for general use as a function of cross-flow Mach number. The proportionality factor so obtained is applied for the prediction of aerodynamic coefficients for the bodies of non-circular cross-section. Jorgensen used this proportionality factor for the calculation of aerodynamic coefficients of a missile body having elliptic cross-section as follows.

$$C_N = \frac{\sin 2\alpha \cos(\alpha/2)}{A_R} \int_0^l \left(\frac{C_n}{C_{n_0}}\right)_{SB} \frac{dA}{dx} dx + \eta C_{d_0} \frac{\int_0^l \left(\frac{C_n}{C_{n_0}}\right)_{New} 2R dx}{A_R} \sin^2\alpha$$
$$= \sin 2\alpha \cos(\alpha/2) \left(\frac{C_n}{C_{n_0}}\right)_{SB} + \eta C_{d_0} \frac{A_P}{A_R} \sin^2\alpha \left(\frac{C_n}{C_{n_0}}\right)_{New} \quad (5)$$

$$C_M = \frac{\sin 2\alpha \cos(\alpha/2)}{A_R X} \int_0^l \left(\frac{C_n}{C_{n_0}}\right)_{SB} \frac{dA}{dx}(x_m - x)\, dx + \frac{\eta C_{d_0} \sin^2\alpha}{A_R X} \int_0^l \left(\frac{C_n}{C_{n_0}}\right)_{New} 2R(x_m - x)\, dx \quad (6)$$

$$x_{cp} = \left(\frac{x_m}{X} - \frac{C_M}{C_N}\right) X \quad (7)$$

The ratios of the normal force for elliptic cross-section to that for circular cross-section are respectively as given differently for Newtonian impact theory and slender body theory [4],

$$\left(\frac{C_n}{C_{n_0}}\right)_{New} = \frac{3}{2}\sqrt{\frac{a}{b}} \left\{ \frac{-b^2/a^2}{(1-(b^2/a^2)^{(3/2)})} \log\left[\frac{a}{b}\left(1+\sqrt{1-\frac{b^2}{a^2}}\right)\right] \frac{1}{1-(b^2/a^2)} \right\} \quad (8)$$

$$\left(\frac{C_n}{C_{n_0}}\right)_{SB} = \frac{a}{b}\cos^2\phi + \frac{b}{a}\sin^2\phi \quad (9)$$

The constants a and b are the semi major and minor axes of an ellipse, and the angle ϕ is the angle between the flow and the semi minor axis. For the case shown in Fig. 1, $\phi = 0°$

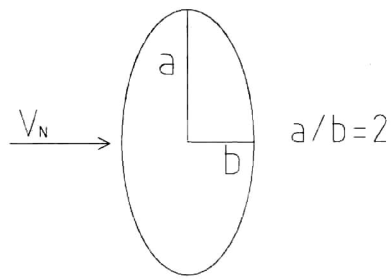

Fig. 1. Cross-flow and elliptic cross-section

Euler Solutions

As is well known, application of slender body theory is limited to high fineness ratio and small angle of attack. On the contrary, Euler equations accurately depict non-viscous flow without such limitations. It is thus efficient to compute potential flow region by Euler solver and compensate the solution for viscous effect using cross-flow analogy. Euler equations may be used to simulate viscous flows with adequate separation modeling as discussed in Ref. 5 or may be solved together with the boundary layer equations for the computation of weakly interacting viscid-inviscid flows. In the present case, such were not attempted since the purpose of the present work was to assess Euler solutions in view of cross-flow analogy.

The Euler code used in the present work was developed by Kim [6] based on finite volume scheme and LU-SGS time integration. Using the computational results of the inviscid flow, the proportionality factor necessary to account for the viscous effect were obtained in the same manner as in Jorgensen [4]. The potential flow part of Eqn. 4 is replaced by the value from Euler solution.

Similarly, for bodies with non-circular cross-sections, we also followed the procedure of Jorgensen. The first terms in Eqns. 5 and 6 are replaced by the corresponding values from Euler solution. The values of $(C_N)_E$ are adopted from Jorgensen. The two-dimensional drag coefficients for circular cross-section is shown in Fig.2 taken from Ref.1.

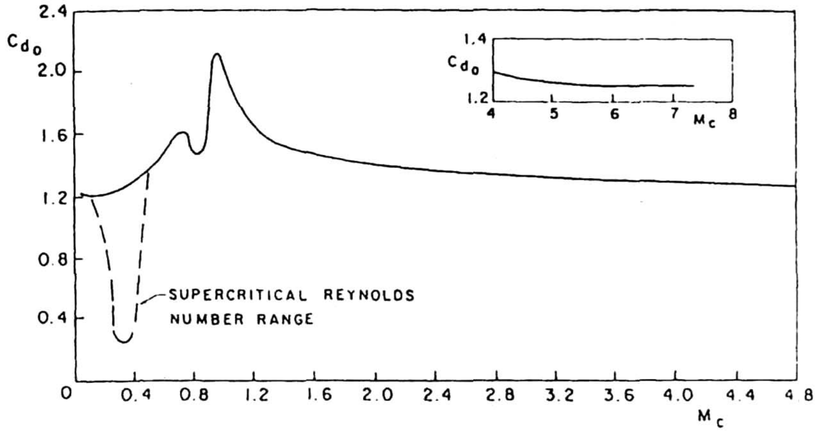

Fig. 2. Two-dimensional drag coefficient of circular cross-section [1]

Missile bodies subjected to computation are the ones of L/D=10 and 12.5 as sketched in Fig. 3. Cross-section of the body is either circular or elliptic. The computation was carried on 75 (axial direction) x 55 (radial direction) x 55 (azimuthal direction) grid for all the cases. A typical grid set-up is depicted in Fig. 4. The far boundary in the radial direction is located at 30-diameter distance from the axis of the body. Since the incoming flow had no side slip angle, the computation was performed for the symmetric half body.

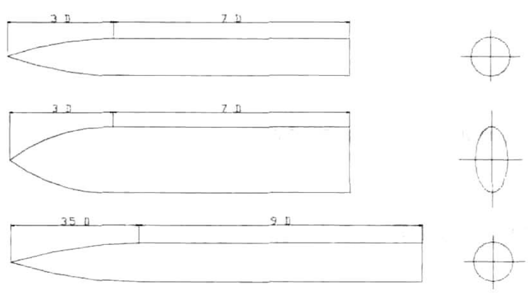

Fig. 3. Missile body models for computation

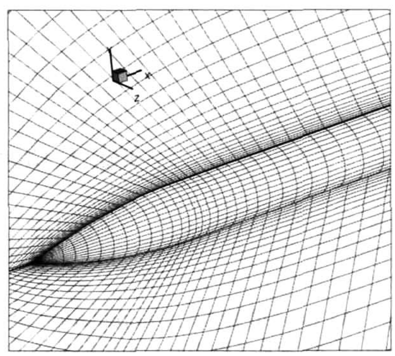

Fig. 4. Computational Grid

Results and Discussion

The aerodynamic coefficients predicted by the classical cross-flow analogy are compared with those obtained by the Euler solution plus the viscous correction suggested in the present work. This, in a way, helps us evaluate the use of Euler solution at the preliminary design phase. Specifically, the proportionality factor, C_N and X_{cp} are focused here. The pitching moment coefficient, C_M was not compared, since C_M was used in evaluating X_{cp} as given by Eqn. 7.

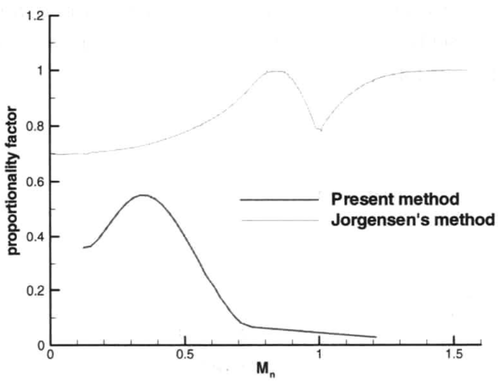

Fig. 5. Proportionality Factor

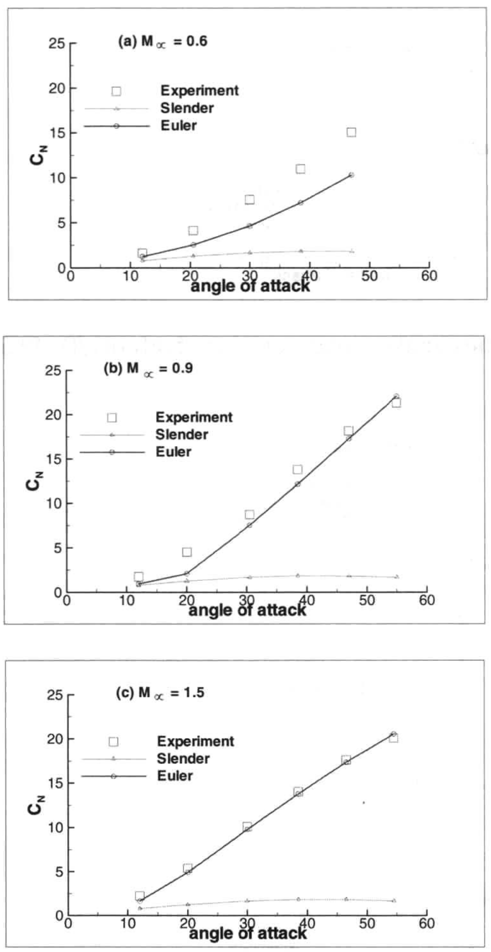

Fig. 6. Normal force coefficient for elliptic cross-sectional body

Figure 5 compares the proportionality factor of Jorgensen with that of the present method for bodies with circular cross-section. From Eqn. (4), the difference between the two curves is attributed solely to the difference in the normal force coefficients. When slender body theory is used, the proportionality factor increases with cross-flow Mach number, Mn, from 0.7 to a value close to 1. When Euler solution is adopted to estimate the inviscid part of C_N, the proportionality factor decreases with Mn. As is evident from Fig. 5, the proportionality factor for the case of Euler solution is much smaller than that of slender body theory. The proportionality factor curve of Fig. 5 hence indicates that the importance of cross-flow component or viscous correction to the Euler solution becomes small. This is more so when Mn is large. This signifies that the normal force coefficient from Euler solution increases much faster than that from slender body theory. Moore [7] pointed out that asymmetric vortex shedding dominates the cross-sectional flow when the angle of attack is greater than 25 ° in subsonic free stream, and this diminishes when Mn exceeds 0.5.

Figure 6 compares the inviscid normal force coefficients obtained from Euler solution (C_N^i) and slender body theory (C_N^p). This explains the difference between the two proportionality factor curves. C_N^i is already in fairly good agreement with the experimental data. The agreement improves as the free stream Mach number increases. As expected, C_N^p from the slender body theory fails to approximate normal force coefficient when the angle of attack is large. This discrepancy is in fact reflected throuth the large value of proportionality factor as shown in Fig. 5.

Predicted values of C_N and X_{cp} are compared using the proportionality factors given in Fig. 5. Predictions were made for the body of circular cross-section with L/D = 12.5 and for the body of elliptic cross-section with L/D = 10. Comparisons are given in Figs. 7 through 9. Figs. 7 and 8 compare respectively the normal force and the center of pressure for the circular cross-sectioned body of L/D = 12.5. Figure 8 clearly indicates that the center of pressure is closely predicted by the present Euler solution approach while the predictions of the normal force are about the same for both methods. This implies that the prediction of the pitching moment by the slender body theory is somewhat more erroneous than by the Euler solution. Figure 9 compares the normal force and the center of pressure for the body of elliptic cross-section. Evidently, satisfactory agreement with the experimental data is achieved by the Euler solution approach.

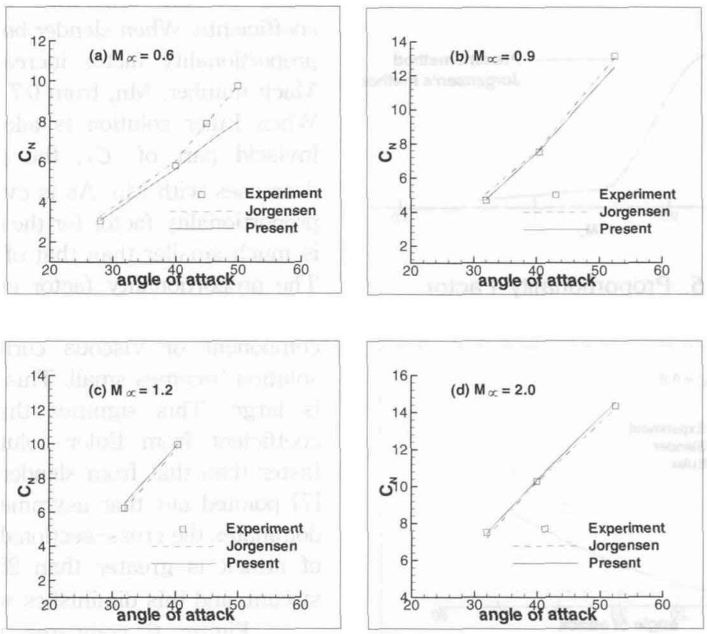

Fig. 7. Predictions of the normal force coefficients for the circular cross-sectioned body of L/D=12.5

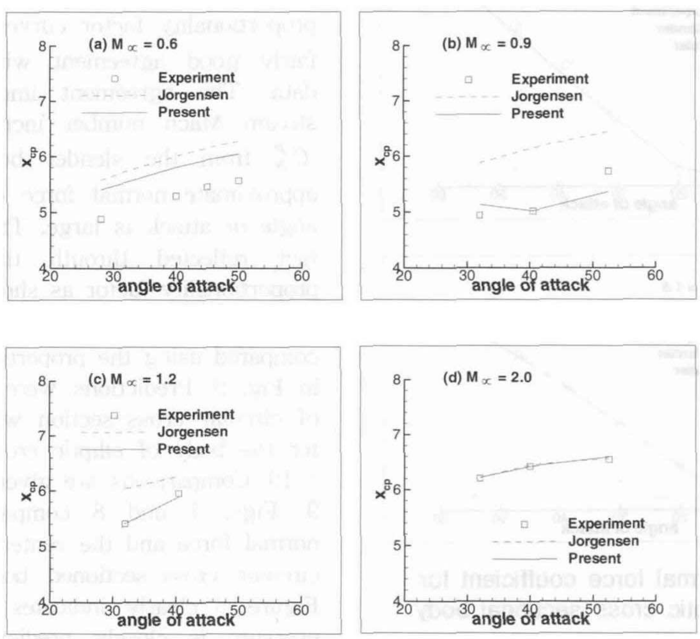

Fig. 8. Predictions of the center of pressure for the circular cross-sectioned body

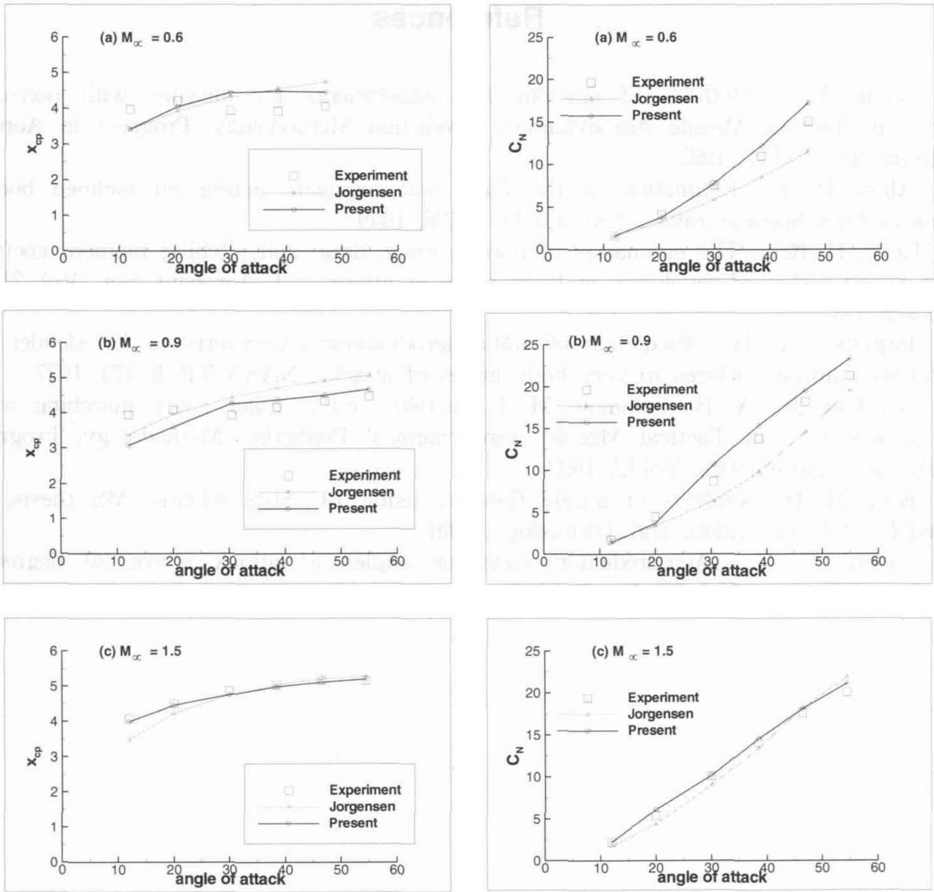

Fig. 9. Predictions of the center of pressure and the normal force coefficient for the elliptic cross-sectioned body of L/D=10

Conclusion

Use of cross flow analogy combined with the Euler solution in determining the normal force coefficient and the center of pressure for a missile body of elliptic cross-section is examined. To provide for the viscous correction to the Euler solution, the proportionality factor was found first using the experimental data of a circular cross-sectioned body. The normal force coefficient and the center of pressure by the slender body theory and the present Euler solution approach are compared. It is shown that the present Euler solution approach can be a viable tool in aerodynamic design of missile bodies of non-circular cross-section.

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References

1. Sigal, A., Methods of analysis and experiments for missiles with non-circular fuselages , in Tactical Missile Aerodynamics: Prediction Methodology, Progress in Aeronautis and Astronautics, Vol.12, 1992

2. Allen, H. J., Estimation of the force and moments acting on inclined bodies of revolution of high fineness ratio , NACA RM A9126, 1949

3. Kelly, H. R., The estimation of normal force, drag, and pitching moment coefficients for blunt-based bodies of revolution at large angle of attack , J. Aeronaut. Sci., Vol. 21, No.8, 1954, pp.549-555

4. Jorgensen, L. H., Prediction of static aerodynamic characteristics for slender bodies alone and with lifting surfaces to very high angles of attack , NASA TR-R 474, 1977

5. Wardlaw Jr., A. B., Solomon, M. J., Baltakis, F.P., Euler space marching methods applied to missiles , in Tactical Missile Aerodynamics: Prediction Methodology, Progress in Aeronautis and Astronautics, Vol.12, 1992

6. Kim, M. H., Analysis of nozzle flow by using LU-SGS scheme, MS thesis, Korea Advanced Institute of Science and Technology, 1991

7. Moore, F. G., Aeroprediction code for angles of attack above 30 degrees , J. Spacecraft and Rockets, Vol. 33, No. 3, 1996