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Trajectory Optimization for a Supersonic Air-Breathing Missile System Using Pseudo-Spectral Method

Jung-Woo Park* and Min-Jea Tahk**
Department of Aerospace Engineering,
Korea Advanced Institute of Science and Technology,
Daejeon, South Korea, 305-701

Hong-Gye Sung***
The School of Aerospace and Mechanical Engineering,
Korea Aerospace University,
Kyunggi-Do, South Korea, 412-791

Abstract

This paper deals with supersonic air-breathing missile system. A supersonic air-breathing missile system has very complicated and incoherent thrust characteristics with respect to outer and inner environment during operation. For this reason, the missile system has many maneuver constraints and is allowed to operate within narrow flight envelope.

In this paper, trajectory optimization of the missile is accomplished. The trajectory optimization problem is formulated as a discrete parameter optimization problem. For this formulation, Legendre Pseudo-Spectral method is introduced. This method is based on calculating the state and control variables on Legendre-Gauss-Lobatto (LGL) points. This approach helps to find approximated derivative and integration quantities simply.

It is shown that, for this trajectory optimization, trend analysis is performed from thrust characteristics on various conditions so that the trajectory optimization is accomplished with fine initial guess with these results.

Key Word : Air-breathing missile system, Trajectory Optimization, Legendre Pseudo-Spectral Method

Introduction

There are many studies and contribution to supersonic air-breathing propulsion system from its first appearance at 1900s. In present, guidance weapon is developed with a trend of concept of “fast” and “efficient” to improve survival probability and long distance flight. A supersonic air-breathing missile is one of the fastest vehicles and has good efficiency to consuming fuel. For this reason, this kind of missile system has been one of main issues on aerospace field till today.

For a supersonic air-breathing missile trajectory optimization problem, we have to consider the missile engine characteristics exactly. In this study, it is shown that what the engine properties are and how to consider the characteristics on trajectory optimization problem.

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* Student in the Ph. D Course
** Professor
E-mail : mjtahk@fdcl.kaist.ac.kr Tel : +82-42-350-3718 FAX : +82-42-350-3710
*** Professor

Over the past decades, there are abundant researches (materials and papers) for this kind of engine characteristics but there are few researches for trajectory optimization which are considerable. Most of researches are concern with the internal combustion mechanisms of combustion chamber and investigation of inlet and nozzle structure [1~4]. However, from these studies we can know the thrust feature of air-breathing engine and the tendency of it so that the results from trajectory optimization can be verified to be matched as expected.

Due to the complex characteristics of thrust, direct optimization method will be adopted. Out of various direct methods, Legendre Pseudo-Spectral method is applied to transform the continuous optimal problem to the discrete parameter optimization. This method is one of collocation methods. The states and the control variables at each node are dealt with as parameters to be optimized.

After the optimization is completed, the results from the optimization are verified whether they track the trend or not.

Missile Specification

Aerodynamic Characteristics

From the missile geometry, aerodynamic data base can be extracted. Non-dimensional lift and drag coefficient are obtained from the geometry. The aerodynamic coefficients are generally given as functions of Mach number and angle of attack for drag and lift.

$$C_L = f_{cl}(M, \alpha) \tag{1}$$

$$C_D = f_{cd}(M, \alpha) \tag{2}$$

Since air-breathing missile does not maneuver agilely due to the risk of flame out, the angle of attack (or side slip) is limited below3 6 degree. This allowance is due to model thrust characteristics. Over the 6 degree for angle of attack, thrust is dropped dramatically with same condition. This reason help the limit of missile maneuver and it is automatically assumed that the missile given in this paper does not exceed the angle of attack of 6 degree.

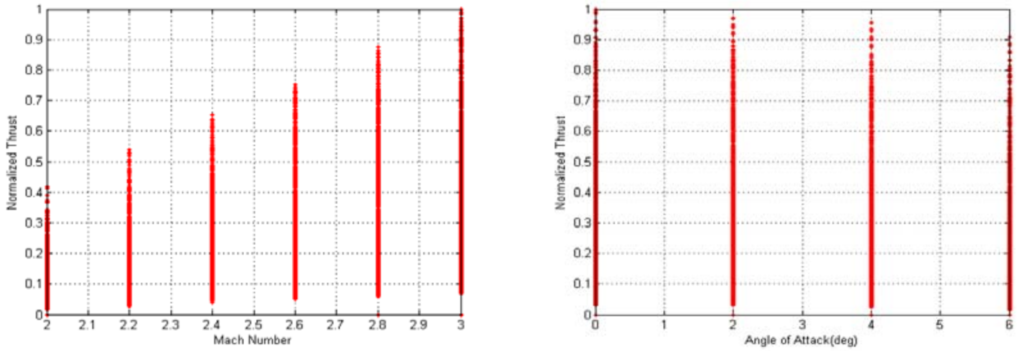
Thrust Characteristics

In this paper, we consider an air-breathing missile which has no air compressor, just compresses air by its own intake shape. As taking air through shocks (oblique shocks and final normal shock) at the intake region, air flow which is slew down can be achieved. Due to this characteristic, airflow injected cannot be sustainable. Generally, airflow rate at the intake depends on the conditions of altitude, Mach number, and angle of attack. When altitude changes air density changes, when Mach number changes airspeed changed, and when angle of attack changes shock locations and the area of intake changes. This air flow rate definitely affects to thrust. Thrust characteristics for Mach, angle of attack, and altitude are shown fig. 1. The graphs for thrust deal with whole data and are projected drown for each parameter.

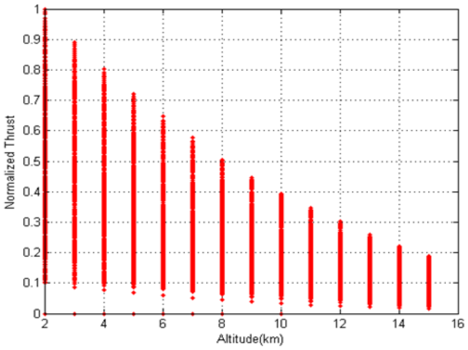
On the other hand, thrust through the engine combustor is affected by fuel injection rate and the nozzle throat area change. It is related to internal dynamics of the engine. Therefore thrust is defined as a function of 5 different conditions(Mach, angle of attack, altitude, fuel injection rate, and nozzle throat area).

It is important to find the physical meaning of missile thrust with respect to each parameter or condition since this consideration gives the relation between missile flight dynamics and engine combustion mechanism so that finally it helps the optimization problem to obtain a solution.

$$Thrust = f_{th}(M, H, \alpha, \dot{m}_f, A_{th}) \tag{3}$$



(a) Mach vs Thrust (b) AoA vs Thrust



(c) Altitude vs Thrust

Fig. 1. Thrust Characteristics w.r.t Mach, AOA, and Altitude

From these results with flight conditions, the tendency of engine or thrust characteristics can be denoted. Generally when Mach number increases, high thrust is obtained. However after passed through a threshold thrust decreases. This is due to the reduction of total pressure recovery ratio after through the intake [5]. The engine which is used in this study is designed to output the best performance at Mach 3. Therefore the reduction with Mach number does not appear. When angle of attack increases, the leak of airflow occurs and the total pressure recovery ratio decreases with improper shock position. It makes thrust decrease with high angle of attack. And for the altitude change, as expected, higher altitude, lower thrust due to the low air flow rate from low air density. It is expected that, with not abundant air, no more thrust increase.

On the whole results with flight conditions, altitude and Mach number have particularly influence on the value of thrust. Under high dynamic pressure, plenty of air flow is guaranteed and the total pressure recovery is attained with adequate amount of fuel flow. Therefore if missile needs high thrust level, it is proper to fly at low altitude with high velocity. But it is considerable that, in such a case, the missile faces with high drag. We come to terms with this thing.

Flight Mechanism

With the aerodynamic data base and the thrust results with respect to flight and operational conditions, flight mechanism can be displayed. Fig. 2 shows the mechanism. From internal dynamics and environmental conditions, the data on the look-up table for aerodynamic coefficients and the thrust are called.

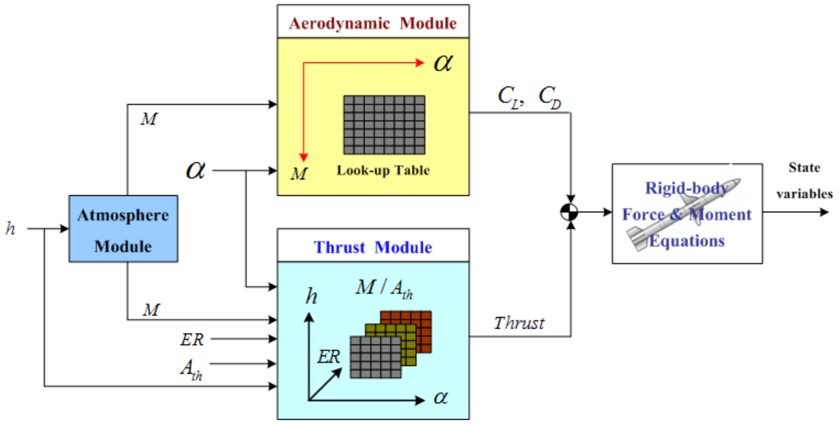


Fig. 2. Data Acquisition Logic and Flight Mechanism

Scheme for Trajectory Optimization

Optimization Method

Generally there are two types of methods for optimization, indirect and direct method.

Indirect methods (dynamic optimization) find a trajectory satisfying the necessary conditions (the Euler-Lagrange equations or the Pontryagin's minimum principle), which pose a TPBVP. Lagrange multiplier (co-states or adjoint variables) is introduced to augment the system governing equation, and explicitly employ the necessary conditions for optimality.

On the other hand, direct methods (static optimization) update the control variables (including design parameters) and the trajectory minimizes (or maximizes) the value of cost function while satisfying the boundary conditions and the other state and control constraints.

Supersonic air-breathing missile has lots of complex inequality constraints for control, and it is hard to be applied to the indirect methods. As described before, the missile considered in this paper is very passive system with respect to flight and engine operational environments and there are restricted maneuvers. For instance, to prevent the flame out at combustion chamber, airflow rate is kept over a certain level so that angle of attack or side slip angle, altitude, Mach number should be under designated boundaries every moment. This fact makes the missile system has more narrow and arbitrary flight envelope than other aero-vehicle. Therefore to solve the trajectory optimization problem, direct methods via certain optimization tool (or algorithm) such as CFSQP and CEALM [6] are proper. In this paper, both of CFSQP and CEALM will be applied for a problem to find an optimal trajectory with a performance index which denotes minimizing the consumption of fuel to climb. They will be applied to a problem but with different uses of them. CEALM will be used to find the tendency expected and it helps for initial guessing for CFSQP which is mainly used for the trajectory optimization problem in this paper. This difference application of them will be demonstrated later.

Sub-choice for Optimization Method

Due to the complexity of air-breathing propulsion system, we carefully decide the use of direct method for the trajectory optimization which is employed in this paper. Here, sub-choice for optimization method remains. Generally a continuous optimization problem can be transformed to a discrete problem. It is parameter optimization problem to treat certain trajectories for states, controls or either. Out of many studies for parameter optimization, what will be used for our problem is the choice.

One of many ways for parameter optimization is collocation method. Collocation method takes all states and control variables as unknown parameters. In this method, static constraints are added on every node after discretization. In this manner, there's no integration procedure but equality constraints which is called as 'defect' are loaded. All trajectories of system's states are expressed as user defined polynomials and if the polynomials are proper enough to account for the complexity of real system, it is one of solutions. However it is suboptimal solution also. Despite a glance of no improvement, it is useful when user has a marginal accuracy level for all system's states and control variables. The order of polynomials will express all parameters with the accuracy level of its order. And it takes much less time to optimize.

As one of the collocation methods, Pseudo-Spectral (PS) method [7~10] for optimal control have been developed rapidly. It is known that PS method is a powerful computational method for solving complex nonlinear control problems and various algorithms have been developed since 1990s. Differed with any other collocation methods which collocate just two node points, PS method collocates all parameters over the nodes for one state or control with Nth order basis interpolation functions. In PS method, the continuous functions are approximated at a set of carefully selected quadrature nodes. These nodes are called Gauss points, Gauss-Radau points or Gauss-Lobatto points. The quadrature nodes are determined by the corresponding orthogonal polynomial basis used for the approximation. It is known that, with just N nodes, quadrature integration achieves zero error for any polynomial integrand of degree less than or equal to 2N-1 [15~19]. Furthermore, compared with Hermite-Simpson method which is one of representative collocation methods, Hermite-Simpson induces totally N equality constraints for each state on N+1 node points, on the other hand PS method induces N+1 equality constraints for each state on N+1 node points. In spite of this a little disadvantage, PS algorithm adopts Gauss type integration rule. It helps simple and accurate integration for states or controls much more than Hermite-Simpson's. That's why the PS method is adopted for optimization in this paper. In present, PS method is employed to many applications for trajectory optimization, which include launch vehicle trajectory optimization, orbit transfer mission for satellites, and etc. In this paper, to use the advantage and to get fancy results, trajectory optimization will be accomplished using PS method.

Furthermore, Legendre polynomials are tried to be basis functions and an optimal problem is solved with quadrature nodes called as LGL points.

Gross Structure to Optimize

Fig. 3 shows the whole process of choice of optimization method.

Since it is hard to construct the necessary conditions or Hamiliton-Jacobi-Bellman equation, direct method is used in this paper. Additionally the numerical error and computational burden (taking long time) due to the integration process, we choose collocation method which interpolates both of state and control variables. Out of collocation methods, PS method is chosen finally. Using PS method, it is expected to get a fine solution and the advantage of problem formulation.

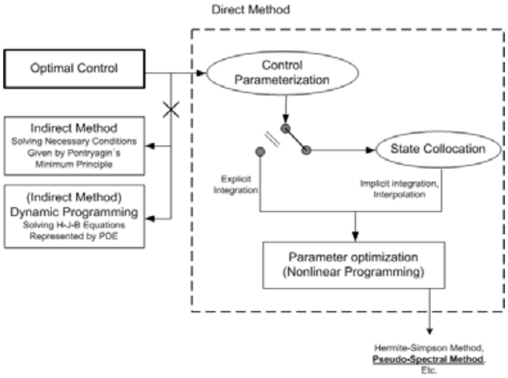


Fig. 3. Optimization Diagram and Choice of Optimization Method

Trajectory Optimization of Supersonic Air-breathing Missile

Problem Definition and Performance Index

The purpose of this study is to trajectory optimization for minimization fuel consumption till reach designed altitude and Mach after boosting phase. It is important to save fuel since saved fuel can be used for an optional operation or long range (or endurance) flight. Therefore minimizing fuel consumption is taken to be performance index in this paper. The mathematical expression is as followed.

$$\min \;\; J(u,x,\tau_f) = \int_{\tau_0}^{\tau_f} \dot{m}_f \, d\tau, \;\; \tau_f \; \textit{is free} \tag{4}$$

Additionally, boundary conditions are given as;
1) Initial Boundary Conditions:
Mach number, altitude, flight path angle = (2.1, 3 km, 0 deg)
2) Terminal Boundary Conditions:
Mach number, altitude, flight path angle = (2.9, 14km, 0 deg)

These boundary conditions imply that the flight from land to air is dealt with. It is assumed that a booster can help till the initial conditions.

Governing Equations

When it is assumed to be 2-D equation for simplicity, the dynamic equations are expressed like below.

$$\begin{aligned} \dot{X} &= V\cos\gamma \\ \dot{Z} &= -V\sin\gamma \\ \dot{V} &= \frac{T\cos\alpha - D}{m} - g\sin\gamma \\ \dot{\gamma} &= \frac{(T\sin\alpha + L)/m - g\cos\gamma}{V} \end{aligned} \tag{5}$$

In this governing equations control variables are angle of attack, equivalence ratio (fuel injection rate) and nozzle throat area. With angle of attack change, lift and drag differ and with equivalence ratio and nozzle throat area rate change, thrust differs.

Problem Reformulation into PS method

From using PS method, the problem is transformed into PS formulation on discrete region, that is

$$\min \;\; J^N = M[a_N, \tau_f] + \frac{\tau_f - \tau_o}{2} \sum_{k=0}^{N} L(a_k, b_k) w_k \tag{6}$$

$$= \frac{\tau_f - \tau_o}{2} \sum_{k=0}^{N} \dot{m}_f w_k$$

subject to

$$\frac{\tau_f - \tau_o}{2} f(a_k, b_k) - \sum_{l=0}^{N} D_{kl} a_l = 0 \;\; , \; k = 0, \ldots, N \tag{7}$$

$$\phi_0 = \begin{bmatrix} Z_0 + 3000 \\ M_0 - 2.1 \\ \gamma_0 \end{bmatrix} = 0 \; , \; \phi_f = \begin{bmatrix} Z_f + 14000 \\ M_0 - 2.9 \\ \gamma_f \end{bmatrix} = 0 \tag{8}$$

where a_k and b_k are state and control variable vector at 'k'th node. a_k and b_k are the parameters in this parameter optimization problem.

Trend Analysis

The problem for trajectory optimization is to save fuel during the flight to a designation position and direction. As described, an air-breathing missile takes a flight from low altitude and low velocity to high altitude and high velocity. It can be differently expressed as a flight from low energy level to high energy level.

Therefore the problem can be defined newly as energy change with possibly small consumption of fuel during the flight. A specific energy level for an aircraft is defined as follows

$$E_s = h + \frac{V^2}{2g} \tag{9}$$

In this paper, it is required that energy level increases as highly as possible with unit fuel consumption. For this, Eq. (9) is modified to be useful to solve the problem.

$$\frac{dE_s}{dt}\frac{dt}{dm_f} = d\left(h + \frac{V^2}{2g}\right)/dt\,/\dot{m}_f = \left(\dot{h} + \frac{V\dot{V}}{g}\right)/\dot{m}_f = \frac{V(T\cos\alpha - D)}{W\dot{m}_f} \tag{10}$$

where $W = mg$ is weight.

The final equation (Eq. (10)) is used to find the best combination of equivalence ration and nozzle throat area which induce the maximum value of energy change with unit fuel mass. If altitude, Mach, and angle of attack are assumed to be fixed, then drag is determined. Therefore with fixed drag, the problem is to find two parameters which make maximum value of Eq. (10).

To obtain the parameters, Co-Evolutionary Augmented Lagrange Method (CEALM) is used. The reason of this choice of use is that since CEALM employs genetic algorithm and stochastic process in it, therefore it searches wider region than the other engines which use gradient method. By using CEALM, the risk is reduced to be fallen at the bad local minimum. From the results, we can find a proper initial guess for trajectory optimization via CFSQP. Fig. 4 indicates the concept drawing of CEALM processing on the region of equivalence ratio and nozzle throat area.

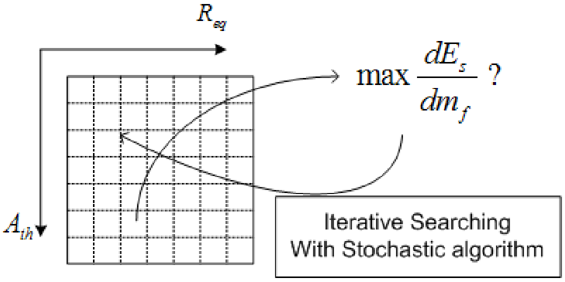


Fig. 4. Concept Drawing for CEALM

Numerical Results

Trajectory optimization using Pseudo-Spectral is accomplished by using CFSQP. All dynamic and boundary constraints are satisfied in a tolerance bound which user designates. The results are obtained from initial guessing with the values for time (30 sec), equivalence ratio (0.9 for whole node points) and nozzle throat area (0.9 for most flight and 0.75 for the end of flight).

This initial guess comes from the results of trend analysis using CEALM.

- ★ Time of flight: 33.44 sec
- ★ Fuel consumption: 14.59kg
- ★ Flight Distance : about 22.5 km
- ★ State and Control Trajectories (Fig. 5 and Fig. 6)

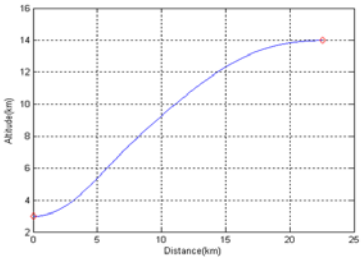
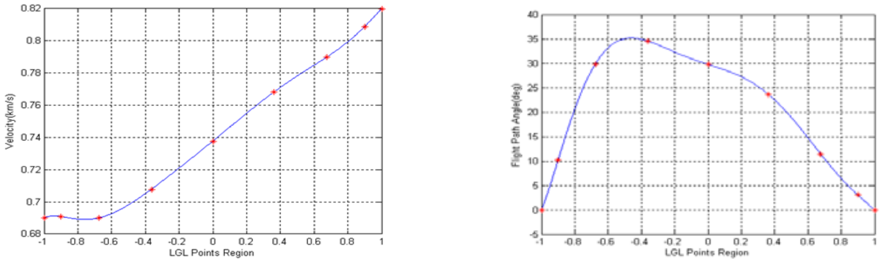
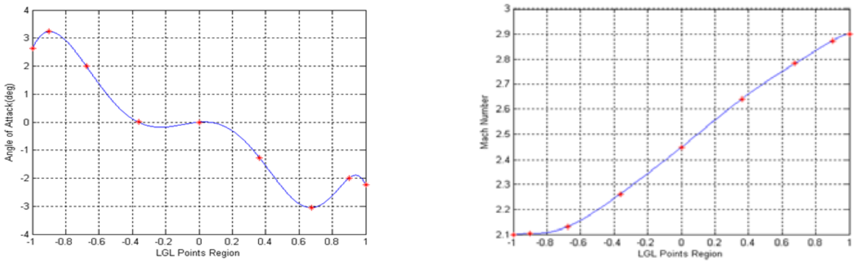


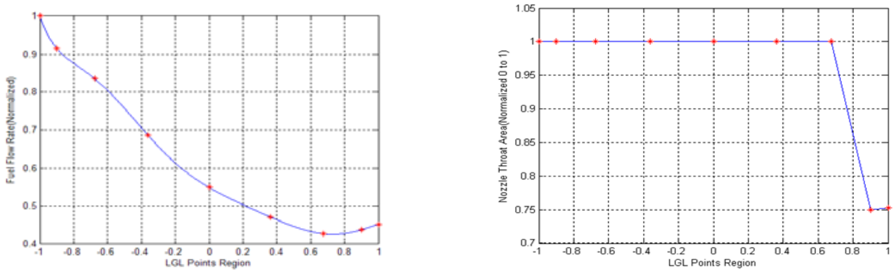
Fig. 5. Trajectory of Missile(2-D)



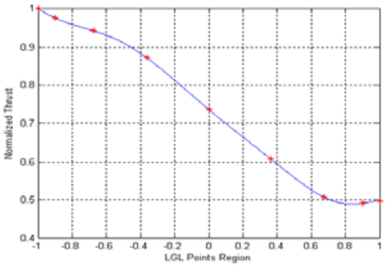
(a) Velocity (km/s) (b) Flight Path Angle (deg)



(c) Angle of Attack (deg) (d) Mach number



(e) Fuel Flow Rate (normalized) (f) Nozzle Throat Area (normalized)



(g) Thrust (normalized)

Fig. 6. Optimized States, Controls, and Other Physical Quantities

On each graph, (star) points are the values optimized on LGL points and a line is from Lagrange interpolation (Except for nozzle throat area). For optimization, 8th order Legendre polynomials are used as basis function (9 node points). Fig. 5 shows the 2-D path trajectory of missile and Fig. 6 shows the rest states, controls and physical quantities which can be obtained. Fuel consumption is about 14.59 kg and the flight distance is about 22.5 km during the flight (for 33.44 sec) as described.

One thing remarkable is the result in Fig. 6.(f). It shows the nozzle throat to be suddenly closed a little during the terminal flight. It means that, during the terminal flight near designated altitude(14 km) and Mach(2.9), narrower nozzle throat area is proper. The data set for nozzle throat area is divided with 0.25 normalized scale interval. Therefore one step narrower nozzle throat area setting is proper that wider nozzle throat area.

Cost Analysis with Alternative Initial Settings and Different Time

To verify whether the results from optimization are well optimized or not, it is required that optimizations should be performed with other initial settings and different flight time (fixed) and compared with obtained results above. Table 1 and 2 show costs with different initial setting for optimization not from trend analysis.

When initial settings for equivalence ratio and nozzle throat area differ, the obtained trajectories for states and controls differ slightly with the results from the trend analysis. It generates worse results. With different flight time which is fixed, a minor result is obtained, too. From these results, it is guaranteed that the cost value of 14.59 kg is the best solution out of the other initial settings and time for optimization.

Table 1. Optimization Results (Cost) with Alternative Initial Settings

Initial Settings (for whole flight)	Cost (kg)
Equivalence Ratio : 0.6 Nozzle Throat Area : 0.9	14.71
Equivalence Ratio : 0.7 Nozzle Throat Area : 0.9	14.61
Equivalence Ratio : 0.9 Nozzle Throat Area : 0.5	14.62
Equivalence Ratio : 0.9 Nozzle Throat Area : 0.75	14.61

Table 2. Optimization Results (Cost) with Different Flight Time (fixed)

Time (sec)	30	32	35	36
Cost (kg)	14.85	14.64	14.61	14.63

Conclusions

Using PS method, we try to find an optimal solution for a supersonic air-breathing missile trajectory in this paper. PS method is generally faster than a case when explicit integration procedure is involved. Due to the complexity of air-breathing propulsion missile characteristics, direct method (parameter optimization) approach is demonstrated. Among many kinds of parameter optimizations, collocation method is used, which parameterizes both of states and controls. Finally, one of collocation methods is adopted for problem formulation, and that is PS method.

It cannot be assured whether the solution is optimal or not. Therefore trend analysis is described to find a proper initial set of parameters. From this trend analysis, we can validate the obtained solution. The other initial settings not through the trend analysis result in worse performance. Therefore it gives denotation that, at least, the results from initial guessing via trend analysis is close to global solution though it is still local solution. Additionally, this gives a sense to set initial settings when optimization is performed using CFSQP.

Finally, through this study, it is hoped that similar trajectory optimizations with alternative performance index are accomplished and it is expected that there are applications of this research such as a guidance law to track the trajectory obtained by optimization.

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