Large-Scale Vortical Structures in The Developing Plane Mixing Layer Using LES

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Abstract

Study of turbulent mixing layers has been a popular subject from the point of view of both practical application and phenomenological importance in engineering field. Turbulent mixing layers can be applied in many fields where rapid transition to turbulence is desirable in order to prevent boundary layer separation or to enhance mixing. The ability to control mixing, structure and growth of the shear flow would obviously have a considerable impact on many engineering applications. In addition to practical applications, free shear flows are one of the simplest flows to understand the fundamental mechanism in the transition process to turbulence. After the discovery of large-scale vortical structure in free shear flows many researchers have investigated the physical mechanism of generation and dissipation processes of the vortical structure.

This study investigated the role of the large-scale vortical structures in the turbulent mixing layer using LES(Large-Eddy Simulation). The result shows that the pairing interaction of the vortical structure plays an important role in the growth rate of a mixing layer. It is found that the turbulence quantities depend strongly on the velocity ratio. It is also found that the vorticity in the high-velocity-side can extract energy from the mean flow, while the vorticity in the low-velocity-side lose energy by the viscous dissipation. Finally the results suggest the guideline to obtain the desired flow by control of the velocity ratio.

Key Word : Vortical Structure, Mixing Layer, LES, Velocity Ratio

Introduction

This paper considers a developing plane mixing layer that are formed by the merging of two free streams initially separated by a splitter plate sketched in Figure 1. This simple flow geometry is a generic model arising in natural phenomena and in industrial applications. The ability to control mixing, structure and growth of the mixing layer would obviously have a considerable impact on many engineering application. In addition to practical application, mixing layers are one of the simplest flows to understand the fundamental mechanism in the transition process to turbulence.

During the past three decades, mixing layers have attracted increasing attention after the discovery of the large-scale vortical structures which are the prominent features of such flows during their early development and transition to fully turbulent flow. Winant and Browand[1)] observed the repeated formation and pairing of vortices in the developing mixing layer. They

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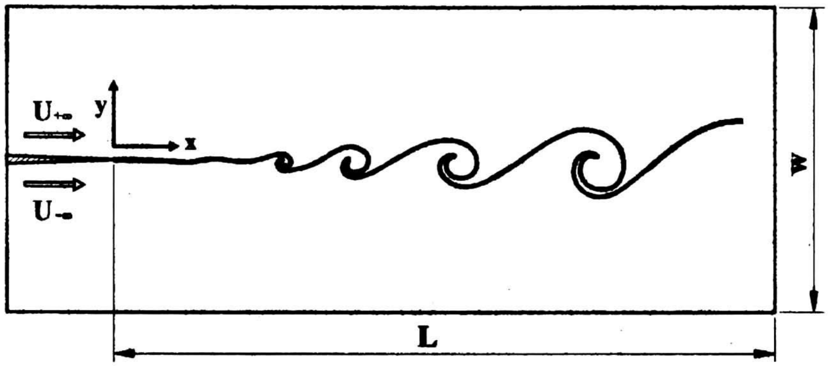


Fig. 1. Schematic diagram of a plane mixing layer

also found the growth of a mixing layer is controlled by the pairing mechanism of these vortical structures. Monkewitz and Huerre[2)] have studied the theoretical dependence of spatially growing waves on free-stream velocity ratio for the hyperbolic tangent profile. Michalke and Hermann[3)] have investigated the equivalent problem for axisymmetric jets. Gaster, Kit and Wygnanski[4)] modeled the large-scale vortical structures that occur in a forced turbulent mixing layer by linear inviscid stability theory. Ashurst[5)] studied the evolution of two-dimensional structures in a temporally growing shear layer by numerical simulation based on the discrete vortex dynamics method. Maekawa, Mansour and Buell[6)] used direct numerical simulation to study the development and large-scale vortical structures of forced spatially developing wake. Strykowski and Nicuum[7)] have studied the influence of velocity and density ratio of spatially developing mixing layer.

Large-eddy simulation (LES) is an alternative technique for the numerical solution of turbulent flows. In LES the large-scale motions are resolved, whereas the effect of the small-scale motion is modeled by a subgrid model while all scales of the motion are resolved in the direct numerical simulation (DNS). Piomelli and Zang[8)] applied LES to compute the transition in the temporally developing boundary. Recently Lewellen[9)] investigated vortex-pair breakup in aircraft wakes using LES and Vreman et al[10)] tested the subgrid models for turbulent stress tensor in the turbulent mixing layer.

Seo and Nikitopoulos[11)] investigated the role of three-dimensional large-scale structures in the developing shear layer subjected to external forcing based on the instability theory. In the present study we present the results of large-eddy simulation (LES) focused on the mixing region in the evolution of vortical structures dominated by the vortex pairing interaction.

Mathematical Formulation

The mixing layer simulated contains two stream separated by a splitter plate with free stream velocities U_{∞} and $U_{-\infty}$ respectively as shown in Fig. 1. To analyze the problem a number of simplifying assumptions are in order. According to the classical assumption, the mean flow will be considered as parallel with a zero spanwise component. Furthermore, the sole mean flow velocity component will have the typical self-similar hyperbolic tangent velocity profile[2)].

The governing equations employed for LES are obtained by filtering the time-dependent Navier-Stokes equations. The filtering process filters out the eddies whose scales are smaller than the filter width or grid spacing used in the computation. By applying the filtering operation in the Navier-Stokes equations, one can obtain

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho \overline{u_i}}{\partial x_i}=0 \tag{1}$$

$$\frac{\partial}{\partial t}(\rho \overline{u_i})+\frac{\partial}{\partial x_j}(\rho \overline{u_i}\ \overline{u_j})=\frac{\partial}{\partial x_j}\left(\mu \frac{\partial \overline{u_i}}{\partial x_j}\right)-\frac{\partial \overline{p}}{\partial x_i}-\frac{\partial \tau_{ij}}{\partial x_j} \tag{2}$$

where τ_{ij} is the subgrid-scale stress defined by

$$\tau_{ij} \equiv \rho\ \overline{u_i u_j} - \rho \overline{u_i}\ \overline{u_j} \tag{3}$$

The subgrid-scale stresses resulting from the filtering operation are unknown and require modeling. In this study the Smagorinsky-Lilly model as one of the eddy viscosity models of the following form is employed to model the subgrid-scale stresses;

$$\tau_{ij}-\frac{1}{3}\tau_{kk}\delta_{ij}=-2\mu_t \overline{S_{ij}} \tag{4}$$

where μ_t is the subgrid-scale turbulent viscosity, and $\overline{S_{ij}}$ is the rate of strain tensor for the resolved scale defined by

$$\overline{S_{ij}} \equiv \frac{1}{2}\left(\frac{\partial \overline{u_i}}{\partial x_j}+\frac{\partial \overline{u_j}}{\partial x_i}\right) \tag{5}$$

In the Smagorinsky-Lilly model, the eddy viscosity is modeled by

$$\mu_t = \rho L_s^2 |\overline{S}| \tag{6}$$

where L_s is the mixing length for subgrid scales and $|\overline{S}|=\sqrt{2\overline{S_{ij}}\ \overline{S_{ij}}}$. In FLUENT, L_s is computed using

$$L_s = \min(kd,\ C_s V^{1/3}) \tag{7}$$

where k=0.42, d is the distance to the closest wall, and V is the volume of the computational cell. The default value of C_s in FLUENT is 0.1.

In the study we are going to investigate how the velocity ratio R defined as $(U_\infty - U_{-\infty})/(U_\infty + U_{-\infty})$, in cases of 0.4, 0.5, 0.6, and 0.7, affects the vortical structures in the turbulent mixing layer. Fig. 2 shows the computational unstructured grid system of the study. The filtered governing equations (1) and (2) are solved using time step with 0.02.

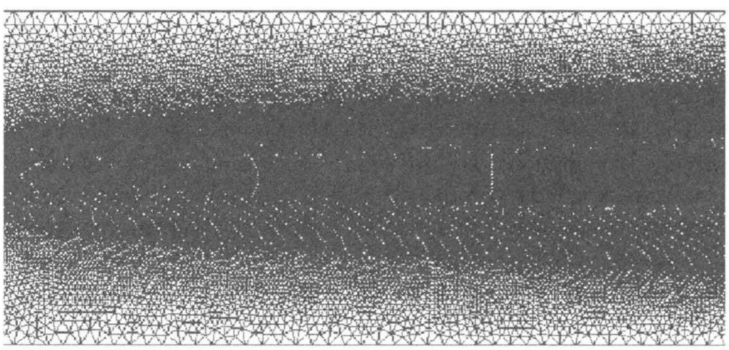
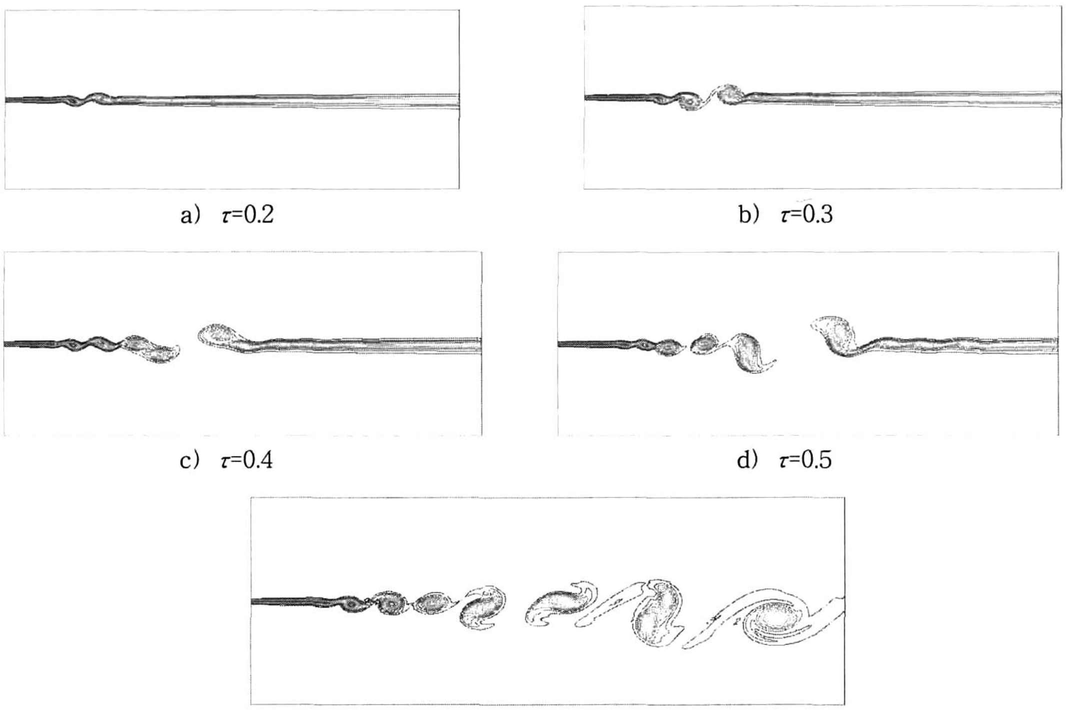


Fig. 2. Computational grid system of the flow field

Results and Discussion

Figure 3 shows the result of the time evolution of vorticity at velocity ratio R=0.6. At the nondimensional time τ=0.2 shown in Figure 3(a), the primary spanwise structure in the mixing layer begins to roll-up by the Kelvin-Helmholtz instability. As time increases, the large-scale vortical structure originated by the Kelvin-Helmholtz instability grows with the entrainment of surrounding fluid as shown in Figure 3(b). The vortical structure begins to cluster together neighboring vortical structure and to form larger vortical structure through pairing process (see Figure 3(c) and (d)). The successive merging of vortices is the primary process governing the growth of the mixing layer. As shown in Figure 3(e) the vorticity initially contained in the mean velocity gradient is constantly redistributed into larger and larger vortices and the size of such structures continues to increase with the distance from the splitter plate.

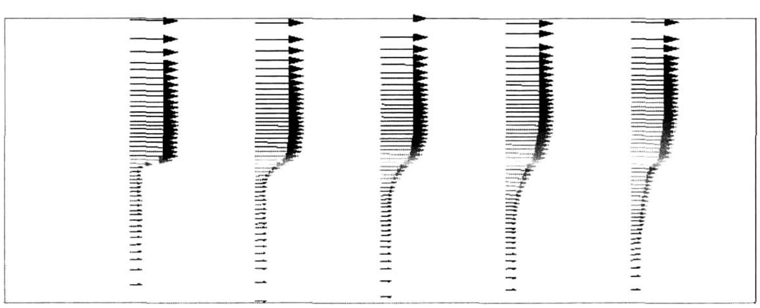


a) τ=0.2 b) τ=0.3

c) τ=0.4 d) τ=0.5

e) τ=2.0

Fig. 3. Time evolution of vortices at velocity ratio R=0.6



x/L = 0.17 0.33 0.50 0.67 0.83

Fig. 4. The streamwise velocity vectors at velocity ratio R=0.6

Figure 4 shows the streamwise velocity vectors at several downstream positions for velocity ratio R=0.6. As shown in Figure 4, the mean velocity profile has the hyperbolic tangent form and it is consistent with the numerical study of Monkewithz and Huerre[2].

The thickness of mixing layer is defined as $\delta = \Delta U/(du/dy)_{max}$ and Figure 5 represents the vorticity thickness for various streamwise position at velocity ratio R=0.6. As a result, the thickness of mixing layer gradually increases as x/L increases.

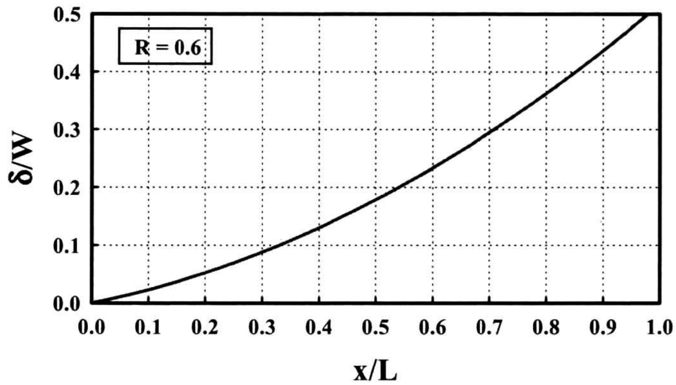


Fig. 5. The change of vorticity thickness for various streamwise position

In Figure 6 it is found that the mixing layer grows faster in the higher velocity region than in the lower velocity region. Thus the mixing layer develops asymmetrically from the center of the mixing layer. It is also found that the growth rate of the mixing layer is greater to the higher velocity region than to the lower velocity region. In the present study it is realized that the asymmetric development of the mixing layer depends strongly upon the velocity ratio R.

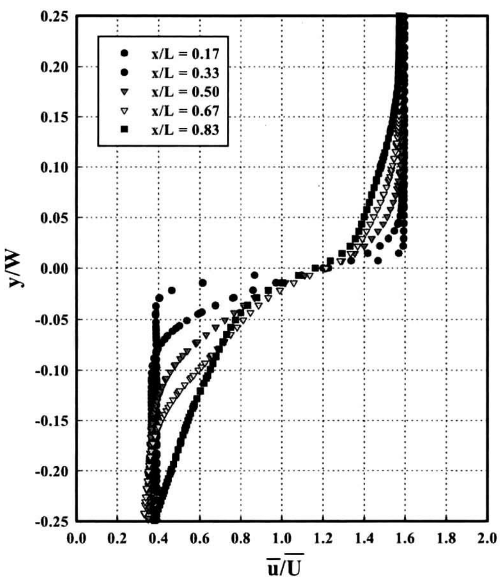


Fig. 6. The mean velocity profiles at velocity ratio R=0.6

Figures 7~9 show the time evolutions of vortical structures in the mixing layer. The roll-up originated by Kelvin-Helmholtz instability locates upstream as the velocity ratio R increases. It is shown that the growth rate of the mixing layer increases as the velocity ratio R increases.

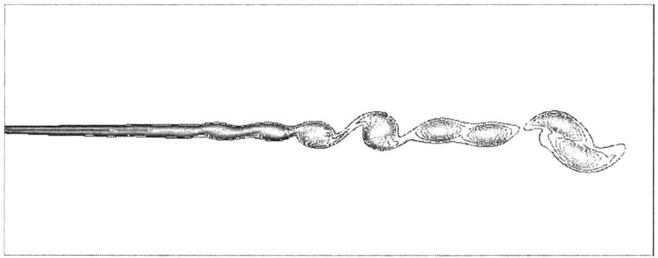


Fig. 7. Time evolution of vortices at velocity ratio R=0.4

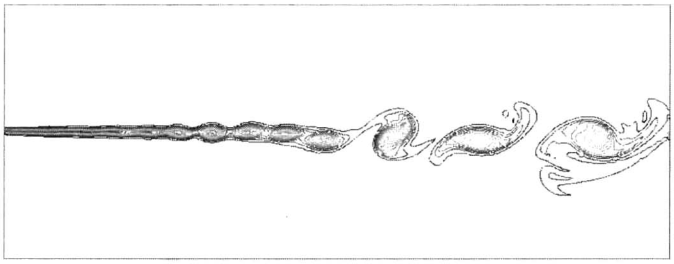


Fig. 8. Time evolution of vortices at velocity ratio R=0.5

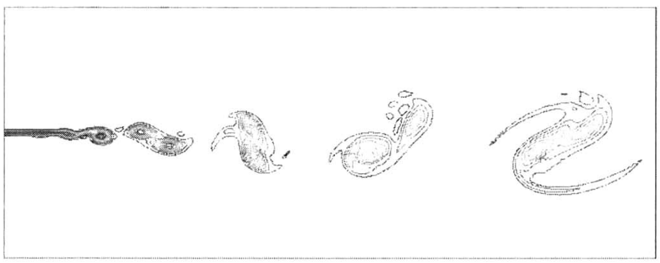


Fig. 9. Time evolution of vortices at velocity ratio R=0.7

In the study it is represented the location of the roll-up as X_R. Figure 10 shows the location of vortex roll-up in the mixing layer. As shown in Figure the roll-up position moves gradually upstream as the velocity ratio R increases. Therefore one can reduce the space required the mixing by moving the roll-up position to upstream.

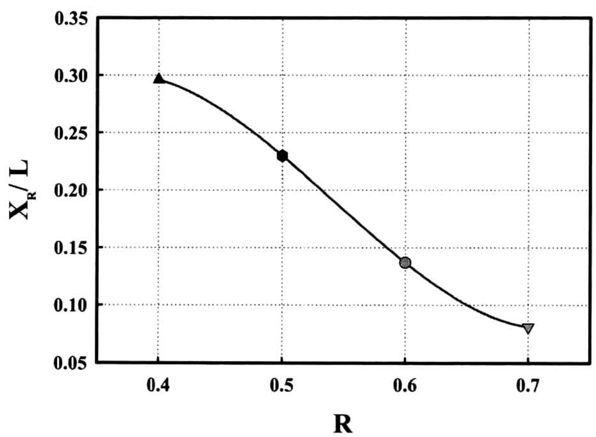


Fig. 10. The location of the roll-up of the mixing layer for several velocity ratio

Figure 11 shows the Reynolds stress induced by large-scale vortical structures at x/L=0.5. A negative Reynolds stress in the region of mean velocity gradient implies a feed of energy from the mean flow into the large-scale vortical structures while a positive Reynolds stress will result partly into a feed of energy from the large-scale vortical structures back into the mean flow and partly dissipate energy by viscosity.

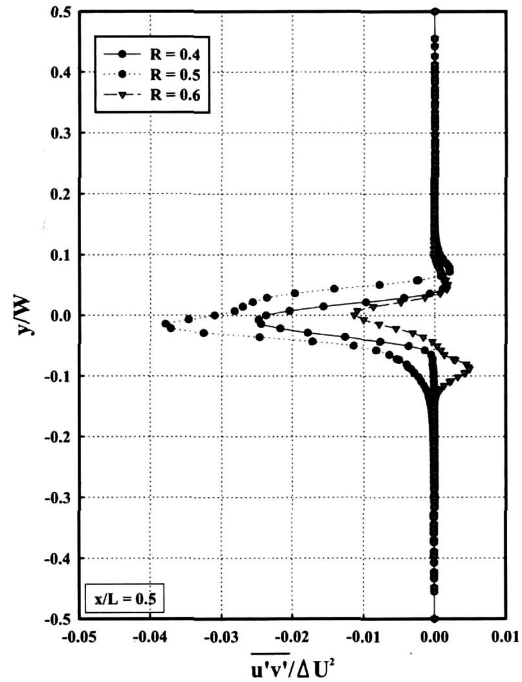


Fig. 11. rms uv for various velocity ratio at x/L=0.5

Conclusions

A numerical study has been performed in a two-dimensional turbulent mixing layer in order to investigate the effect of the velocity ratio on the large-scale vortical structure. The profiles of mean velocities agree reasonably well with Monkewitz and Huerre[2]. The result shows that the pairing interaction of the vortical structure plays an important role in the growth rate of a mixing layer. It is found that the turbulence quantities depend strongly on the velocity ratio. It is also found that the vorticity in the high-velocity side can extract energy from the mean flow, while the vorticity in the low-velocity-side lose energy by the viscous dissipation.

Acknowledgement

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