Parametric Optimization Procedure for Robust Flight Control System Design

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Abstract

This paper is devoted to the parameter optimization of unmanned aerial vehicle's (UAV) flight control laws. Optimization procedure is based on the ideas of mixed H_2/H_∞ control of multi-model plants. By using this approach, some partial H_2-terms defining the performance of nominal and parametrically perturbed Flight Control System (FCS) responses to deterministic command signals in stochastic atmosphere as well as H_∞-terms defining robustness of the FCS can be incorporated in the composite cost function. Special penalty function imposed on the location of closed-loop system's poles keeps the speed of response and oscillatory properties for both nominal and perturbed FCS in reasonable limits. That is the reason why this procedure may provide reasonable trade-off between the performance and robustness of FCS that are very important especially for UAV. Its practical importance is illustrated by case studies of lateral and longitudinal control of small UAV.

Key Word : Unmanned Aerial Vehicle, H_2/H_∞ Multi-model Control, Longitudinal Autopilot, Lateral-Directional Autopilot

Introduction

Requirements for Flight Control System (FCS) of any aircraft could be formulated as follows: 1) suppressing exogenous stochastic disturbances produced by turbulent atmosphere, 2) providing required performance and stability in the presence of parametrical internal disturbances in all flight envelope of UAV. 3) In the case of unmanned aerial vehicles (UAV) these requirements have to be added with specific demands of simplicity and low prices, weight, power consumption, and size. Sometimes performance of flight control system, determined by first two requirements, is sacrificed in order to achieve the third one.

The usage of robust FCS is the most relevant way to satisfy the first two requirements. However, the application of modern robust control theory is restricted to some extent by the third requirement, insofar as some very important state space variables cannot be measured and full state feedback is not available. In this situation the most realistic way is the application of the known FCS structures with unknown parameters. These structures are based on real possibilities of acceptable UAV's sensor systems and positive experience in their practical implementation. By using the parametric optimization of FCS structures, their high performance and robustness in presence of external and internal disturbances can be achieved.

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One of the most powerful optimization methods, that yields the closed-loop system satisfying the aforementioned requirements, is the nominal performance with robust stability (NPRS) approach based on the mixed H_2/H_∞ control of multi-model plants [1], which can incorporate deterministic as well as stochastic criteria in one performance index, thus permitting reasonable trade-off between contradictory conditions of deterministic and stochastic performance indices minimization, meanwhile incorporation of H_2 and H_∞-norms allows to achieve compromise between requirements to suppress external and internal disturbances. The set of multi-model plants includes all parametrically disturbed models in different conditions of flight, which cover all flight envelope.

Control Law (CL) received as a result of this procedure is based on linearized UAV model. Meanwhile it is necessary to be aware that nonlinear functions, inherent to the UAV as itself and its control system, could seriously affect not only performance of system with aforementioned CL but even its stability. Therefore, it is necessary to make some adaptation of obtained linear CL to nonlinear environment. Some practical considerations how to do this are presented in a case study of UAV's longitudinal and lateral-directional CL.

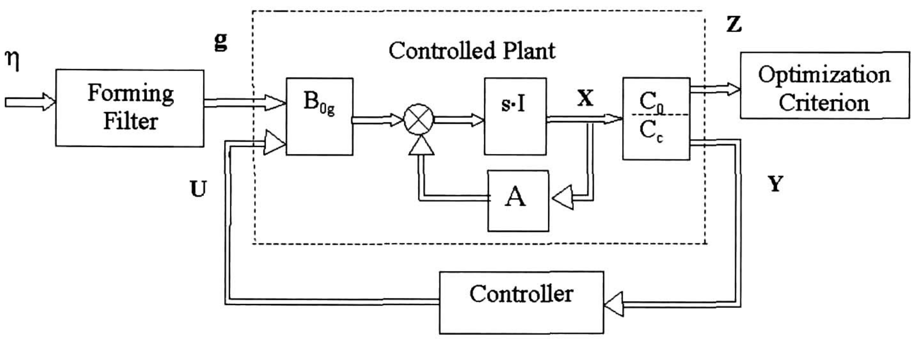

Fig. 1. Standard Form of the Control Optimization Problem

Mathematical Representation of System and Statement of the Optimization Problem

The standard form of the control system optimization problem is represented in Fig.1. In Fig.1, vector η represents the white noise exogenous disturbances, which along with forming filter creates vector g of stochastic wind velocities. This filter could be described by standard Dryden models. Transfer functions or state-space descriptions of forming filters associated with Dryden's spectral densities are known [4]. Matrix B_{0g} is designed to incorporates control input U and stochastic vector of wind gusts g in one input vector; matrix A is state-space matrix of UAV. There are two observation matrices: C_0, associated with output vector Z, that is used for computing system's performance index, and C_c, associated with other output vector Y for creating actual controller feedback. Forming filter is used for computing stochastic performance index, meanwhile for deterministic case it is omitted. Optimization procedure is based on the composite performance index (PI), which includes the following components:

1) H_2-norm for each model in deterministic case, which represents system sensitivity with respect to deterministic disturbances:

$$J_d=\sqrt{\int_0^\infty [X^TQX+2X^TNU+U^TRU]\,dt} \tag{1}$$

2) H_2-norm for each model in stochastic case:

$$J_s = \sqrt{E_M[X^TQX + 2X^TNU + U^TRU]} \tag{2}$$

3) H_∞-norm for each model:

$$\|G\|_\infty = \sup \bar{\sigma}(G(j\omega)), \quad 0 \le \omega \le \infty \tag{3}$$

In expressions (1) and (2), X stands for state vector, U is the control input vector, E_M stands for expectation operator, and Q, R, N are weighting matrices. In expression (3) $\bar{\sigma}$ is the maximal singular value of the closed-loop system transfer function matrix $G(j\omega)$ over frequency range: $0 \le \omega \le \infty$. Note that norms in expressions (1) and (3) are determined for closed-loop system only without forming filter, whereas norm in (2) is determined for series connection of forming filter and closed-loop system. Therefore, state space, control, and observation matrices A_s, B_s, C_s for stochastic case have larger dimensions than the same matrices A, B, C for deterministic case. These norms could be calculated [3] on the basis of corresponding controllability grammians for stochastic and deterministic cases, respectively :

$$G_s = \text{gram}\,(A_s, B_s) \tag{4}$$

$$G_d = \text{gram}\,(A, B) \tag{5}$$

Controllability grammians are solutions of corresponding Lyapunov equations [3] $AG_d + G_dA^T + BB^T = 0$, $A_sG_s + G_sA_s^T + B_sB_s^T = 0$, and the covariance matrix of vector Z could be expressed as follows[4]:

$$\text{cov}\, Z = C_sG_sC_s^T. \tag{6}$$

The diagonal elements of covariance matrix can be used for the system performance estimation, because it is possible to evaluate variance (and r.m.s.) of each state space vector component. Partial PI corresponding to Eq. (2) is equal to:

$$J_s = \text{trace}\,(C_s^wG_s(C_s^w)^T) \tag{7}$$

where C_s^w stands for weighted observation matrix. In accordance with expressions (1) and (2) elements of this matrix are weighted by diagonal matrix Q

$$C_s^w = QC_s. \tag{8}$$

Expressions for deterministic case are the same as (7) and (8):

$$J_d = \text{trace}\,(C^wG_d(C^w)^T) \tag{9}$$

taking into account that dimensions of matrices Q and C would be different. If matrix N in expressions (1),(2) is a zero matrix, then C_s^w (or C^w in deterministic case) become diagonal matrices, otherwise it has off-diagonal elements. For nonzero N case, obvious inequality has to be satisfied:

$$Q_0 = Q - NR^{-1}N > 0.$$

Now, it is possible to build up composite multi-model performing index. Consider a "nominal" model with nominal parameters and models with perturbed parameters, the number of these "perturbed" models is denoted as n_m. Each model represents certain flight conditions: velocity, altitude, and trim conditions. In this case composite PI could be written as follows:

$$J=\lambda_d J_{od}+\lambda_s J_{os}+\sum_{i=1}^{n_m}(\lambda_d^{(i)}J_d^{(i)}+\lambda_s^{(i)}J_s^{(i)}) \tag{10}$$

where $J_{od}, J_d^{(i)}$ stand for deterministic partial PI's, $J_{os}, J_s^{(i)}$ stand for stochastic parts; $\lambda_s, \lambda_s^{(i)}$ are weighting coefficients to make commensurable contributions of stochastic and deterministic parts to composite PI, $\lambda_d^{(i)}$ are weighting coefficients for "perturbed" deterministic models. Appropriate selection of these weighting coefficients could form desirable compromise between performance of perturbed and unperturbed models.

It is useful to add H_∞- norms Eq. (3) of nominal and perturbed systems' cosensitivity functions [1] to PI defined by Eq. (10):

$$J_c=J+\lambda_\infty\left[\|G_{nom}\|_\infty+\sum_{i=1}^{n_m}\|G_{per}\|_\infty\right] \tag{11}$$

where λ_∞ is weighting coefficient. With the redefined PI of Eq. (11), it is possible to reach the compromise between norms of sensitivity and complementary sensitivity functions. Let symbol $\overrightarrow{C_n}$ denote the vector of variable parameters of controller, which appears in quadruple of matrices $\{A_c, B_c, C_c, D_c\}$ in controller's description. So far as controllability grammian could be defined only for stable and fully controllable system, it is possible to find the minimal value of composite performance index Eq. (10) over the space of variable parameters $\overrightarrow{C_n}$, if and only if, in the process of performing optimization procedure closed-loop system would be stable. It denotes that searching of optimal value $\overrightarrow{C_n^*}$ of vector $\overrightarrow{C_n}$ must be made within stability domain of variable parameter's space. Therefore, total cost function for running optimization procedure has to include some penalty function (PF), restricting location's area of the closed-loop system poles in the complex plane. This area is represented in the Fig.2a with bold lines (for the case of two real and two complex poles) and could be characterized by 3 parameters: d_o, α(angle) or $K=\tan(\alpha)$, and D. These parameters restrict respectively: the minimal value of real part of poles to guarantee some minimal values of stability margins, the oscillatory properties of closed-loop system, and the bandwidth of closed-loop system. Therefore, penalty function for violation of the borders of area, shown in Fig.2a, must have at least 3 components for violation of vertical and slope borders. It must be zero inside the area shown at Fig.2a and must grow fast outside of this area.

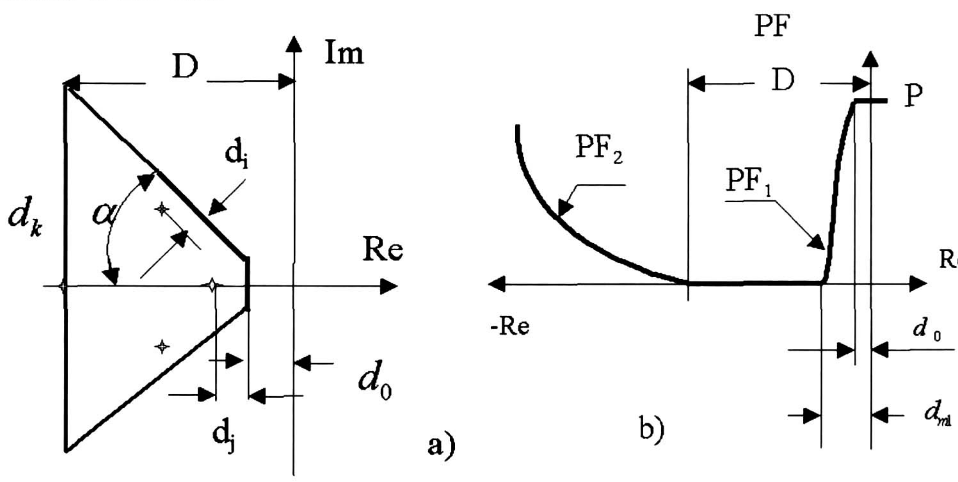

Fig. 2. Penalty function: a) preferable location of the closed-loop system's poles in the complex(horizontal) plane; b) cross-section of $PF(d_m)$-surface by vertical plane $PF-Re$

According to PI, determined by (10) or (11), the total cost function of optimization procedure would have the following form:

$$J_{\Sigma}=J+\sum_{i} PF_i \quad ;\text{or}\quad J_{\Sigma}=J_c+\sum_{i} PF_i \quad , \quad i=1,2,3 \tag{12}$$

From the viewpoint of stability, inequality $Re(P_j)\leq -d_o$, $\forall j: j=1,\dots,n$; where P_j are poles of closed-loop system (see Fig.2a), is the most important and the penalty for its violation must be the highest. Therefore, it is necessary to use in this case very steeply rising function like this:

$$\mathrm{PF}_1(d_m)=\begin{cases} 0 & , \text{ if } d_m\geq d_{ml} \\ \frac{P}{2}\left[1+\cos\left(\frac{\pi(d_m-d_0)}{d_{ml}-d_0}\right)\right] & , \text{ if } d_0 < d_m < d_{ml} \\ P & , \text{ if } d_m\leq d_0 \end{cases} . \tag{13}$$

Value d_m is defined as a minimum of all distances from the poles of closed-loop system to the imaginary axis for nominal and all n_m perturbed models, P being very large value (for example, $P=10^4 \sim 10^6$). This penalty function has very fast increasing of its gradient within borders $d_{ml}\sim d_o$. Other restrictions of poles' location are not so critical from the viewpoint of stability and in these cases softer penalty function could be chosen, for example:

$$PF_2=0, \text{ if } d_k\geq -D \text{ and } PF_2=(d_k+D)^2, \text{ if } d_k< -D,$$

where $d_k=\min(Re(P_k))$. Penalty functions PF_1 and PF_2 are shown in Fig.2b. Penalty function PF_3 for violation the linear border, tilted at angle α, is the same as PF_2, where in this case d_k is the minimal distance from the complex pole to the aforementioned border. Finally it is useful to add that in the optimization procedure some parameters of CL could be chosen unrealistically large. In this case it is useful to add to PF well-known restrictive term: $\mathrm{PF}_r=\sum_{r=1}^{l}\lambda_r p_r^2$, where l is number of parameters p_r, which need to be restricted, λ_r is weighting factor. However, including this term in PF is optional and it is useful in some special cases, when, for examples, system as itself has very large stability margins. Note, that pole placement in the aforementioned area is closely related to the robust properties of closed-loop system[6].

Design Procedure

The first stage of a design procedure consists of determination of structure and initial values of its parameters from the viewpoint of closed-loop pole placement in the prescribed area. As the first step to solve this task the standard LQR-procedure could be used for determination of signs and values of all CL components when full state vector could be measured.

Then, at the second stage, actual sensors are taken into account with compensators, for example, PD-controllers and so on, using some known structures[4,7]. Standard MATLAB procedures for determination of state space description of series and feedback connections can be used to find such values of controllers parameters, which place closed-loop system poles to the left half-plane inside the prescribed area or with some its minor violations. Well-known pole-placement methods[4,7] could be used as well. The result of this second step is finding CL parameters, which guarantee stability of closed-loop system. After this step it is possible to create and run the program for evaluation of performance and robustness of this system, based on the evaluation of PI(11) with unit weighting coefficients. As a result of this program all parameters of weighted performance index and penalty function could be evaluated:

- computing of eigenvalues of closed-loop system and evaluation of their location at the complex plane permits to estimate parameters d_o, d_{m1}, $K=\tan(\alpha)$, and D of penalty function PF,

- computing all H_2 and H_∞- norms permits to estimate all kinds of weighting coefficients λ in composite PI(10),(11) from the viewpoint of their commensuration,

- computing the diagonal elements of controllability grammians (4),(5) permits to estimate H_2 norms of each state space and control variables, and eventually elements of weighting matrix Q.

The simplest rule for adjustment of weighting coefficients is the following: if some component of performance index or H_2- norm of some specific signal is unacceptably large, then corresponding weighting coefficient in the total cost function must be increased. At the third stage aforementioned optimization procedure could be applied to find the optimal parameters, which deliver minimum to the total cost function (12). In our case simple but reliable Nelder-Mead optimization procedure was used, based on simplex method[8]. It requires more steps to be done for finding minimal value of cost function (12), but from other hand it doesn't require determination of gradients of cost function, which is not trivial task as itself. Eventually at the fourth stage it is necessary to evaluate the actual performance of designed system. It could be done analytically with evaluation program, created at the second step and experimentally on the basis of simulation using SIMULINK-program. In the simulation procedure it is possible to include some inevitable non-linearity (such as saturation of actuator) in combination with linear or full nonlinear model of UAV. At this stage the structure of control system, obtained on the basis of linearized model, has to be adapted to nonlinear environment.

It is important to mention that for the best adjustment of CL parameters it is necessary to run optimization program several times and evaluate its results. After each running and evaluation it is possible to change some weighting coefficients in the composite performance index in accordance with aforementioned rule to achieve better performance or/and robustness as well as restriction of some specific signals of control system.

Case Study 1: Robust Optimization of the UAV's Lateral-Directional Control

Consider the Heading-hold mode for small UAV with the following parameters: cruise speed $U_0=250$ km/hr, altitude $H_0=2$ Km; maximum take-off weight MTOW=146 Kg, moments of inertia: $J_{xx}=17.8$ Kg m^2, $J_{zz}=124$ Kg m^2, $J_{xz}=2.6$ Kg m^2; wings area: $S=1.84$ m^2. Control surfaces are elevator and ailerons only. The main perturbation, which causes the changing of all parameters of UAV model is the variation of air speed V_t from 250 (nominal) to 200 (perturbed) Km/hr. In this study 2 models are considered: nominal model (V_t^{nom} =69.44 m/sec) and perturbed model (V_t^{per}=55.56 m/sec). System matrix A_u(A_{up}) and control influence matrix B_u(B_{up}) for these 2 linearized models are:

$$A_u=\begin{bmatrix} -0.136 & 0.14 & 10^{-4} & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -56.12 & 0 & -11.25 & 3.3 & 0 & 0 \\ 1.2 & 0 & -0.21 & -0.24 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 69.4 & 0 & 0 & 0 & 69.4 & 0 \end{bmatrix},\quad A_{up}=\begin{bmatrix} -0.109 & 0.175 & 10^{-4} & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -36.28 & 0 & -9.2 & 2.8 & 0 & 0 \\ 0.86 & 0 & -0.17 & -0.185 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 55.6 & 0 & 0 & 0 & 55.6 & 0 \end{bmatrix}$$

$$B_u=[0\ \ 0\ \ 160\ \ 0\ \ 0\ \ 0\,]^T,\qquad B_{up}=[0\ \ 0\ \ 103.6\ \ 0\ \ 0\ \ 0\,]^T \tag{14}$$

where control input δa and state vector $x=[\beta,\phi,p,r,\psi,Y]^T$ (sideslip and roll angles, roll and yaw rates, yaw angle and track error respectively) are considered.

Structure of UAV roll and heading control system (along with track error control loop, inserted in chain-line rectangular) is represented in Fig.3a. In Fig.3a C_{roll}, C_{track}, *WF* stand for compensators in roll and track channels and washout filter respectively with the following transfer functions:

$$W_{Ci}(s) = \frac{T_{ni}s+1}{T_{di}s+1}$$
$$W_{wf}(s) = \frac{K_r s}{T_w s+1} \tag{15}$$

where $i=1$ for roll channel, and $i=2$ for track channel. Optimization procedure was applied for inner attitude control loop, consisting of roll compensator, washout filter, and gains K_ψ, K_ϕ, K_p. Therefore, vector of adjustable parameters $\overrightarrow{C_n}$ contains 7 components: $\overrightarrow{C_n} = [K_\Psi, K_\phi, K_p, K_{n1}, K_{d1}, K_r, T_w]$. After obtaining initial values of these parameters, placing poles of closed-loop system in the left half-plane, optimization procedure was performed with the following weighting coefficients for both models: $\lambda_\infty = 2$, $\lambda_s = 10$, $\lambda_d = 1$, and the following parameters of penalty function were chosen : $d_{m1} = 0.5$, $d_0 = 0.05$, $D = 25$, $K = \tan\alpha = 5$. These weighting coefficients were chosen to make all partial PI commensurable. Weighting coefficients in the norms (1), (2) for state variables ϕ, p, r, ψ and control variable δa, respectively, are [1, 1, 1.7, 1.7, 1.5]. In this particular case there are only 2 models (nominal and perturbed), so performance index is equal

$$J = \lambda_{0d}J_{0d} + \lambda_{0s}J_{0s} + \lambda_{pd}J_{pd} + \lambda_{ps}J_{ps} + \lambda_\infty H_\infty + \lambda_{p\infty}H_{p\infty} \tag{16}$$

where: J_{0d}, J_{0s} stand, respectively, for deterministic and stochastic partial PI of nominal system, λ_{0d}, λ_{0s} stand for corresponding weighting factors, while the symbols with subscripts p stand for the same values of perturbed system. $H_\infty(H_{p\infty})$ and $\lambda_\infty(\lambda_{p\infty})$ denote H_∞-norms and weighting factors for nominal and perturbed systems.

Optimization procedure produced optimal values of controller's parameters as follows: $K_\psi = 20$, $K_\phi = 15$, $K_p = 17$, $K_r = 50$; $T_{n1} = 0.06\,\text{sec}$, $T_{d1} = 0.02\,\text{sec}$, $T_w = 0.1\,\text{sec}$. Table 2 summarizes the main performance characteristics of nominal and perturbed systems (r.m.s. of state space and control variables in stochastic case; stability margins and norms Eqs. (1), and (3) in deterministic case). Comparison shows good combination of performance and robustness of UAV attitude control system. This system has 2 modes: bank angle-hold (*WF* and K_ψ are not used) and heading-hold, which use all components of attitude control. Symbols Ψ_c, ϕ_c, Y_c stand for heading, roll and track command signals.

Track control system contains the linear elements: gain K_Y and track compensator with transfer function $W_{C2}(s)$, which has the form of Eq. (15). Nonlinear element "Sat." (saturation) would be considered later. Track-hold system contains only 3 adjustable parameters: K_Y, T_{n2}, T_{d2}, which are components

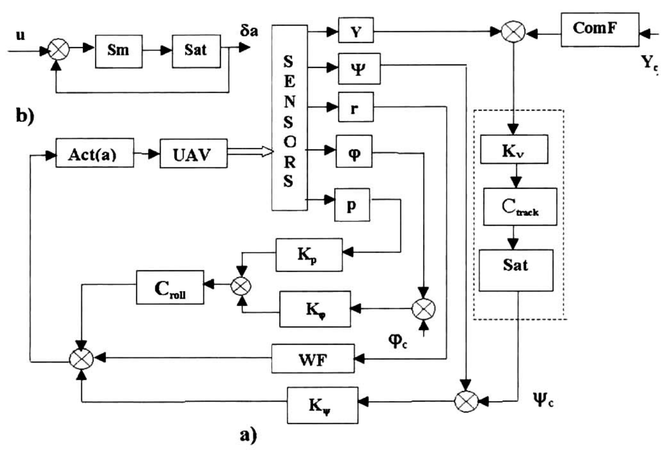

Fig. 3. Block diagrams of : a) the lateral channel of UAV flight control system b) the actuator

of vector $\overrightarrow{C_n}$. These parameters could be determined by the simpler optimization procedure, in which attitude control loop parameters are fixed as a result of aforementioned optimization procedure. Therefore it is necessary to determine with only 3 adjustable parameters: K_Y, T_{n2}, T_{d2} used in optimization procedure. They were found as : $K_Y = 0.0013$; $T_{n2} = 1.3$ sec , and $T_{d2} = 0.05$ sec .

Finally, it is necessary to mention that even with linear model of aircraft it is necessary to take into account the saturation of actuator [7]. Block diagram of actuator is represented in Fig. 3b, where block "Sm" is the servomotor with transfer function $W_{sm}(s) = \frac{1}{0.5s}$ and block "Sat" is the saturation, restricting the maximal angles of aileron deflection (about $20^o \sim 25^o$). Too large initial errors could cause transient "bang-bang" oscillations of actuator, thus causing instability of system. To avoid these undesirable phenomena it is necessary to restrict transient deflections of some state variables using the following (see Fig. 3a): 1) restriction of command signals from outer track loop to inner (heading and roll) loops using saturation block (see Fig. 3a): 2) using of smoothing filters in the command signals circuits, such as "ComF" with the following transfer function: $W_{CFi}(s) = \frac{1}{(1+T_1 s)(1+T_2 s)}$; $T_1 < T_2$. So far as command filter is not included in closed-loop systems, these time constants are not determined by aforementioned optimization procedure, but could be easily chosen on the basis of simulation. In our case $T_1 = 0.2$ sec , $T_2 = 0.5$ sec are chosen. Transient processes in nominal and perturbed systems in the "Track-select" Mode are shown in Fig. 4, and 5 by solid lines for nominal system and by dashed-line for perturbed one. Frequency response of closed-loop heading-hold system (see Fig6: amplitude-top, phase-bottom) shows robustness property of this system. Beside simulation of this system with linear model, which is presented in this paper, its simulation with nonlinear model of UAV were performed also. It demonstrated good matching with results of linear model simulation.

Table 1. Comparison of characteristics for nominal and perturbed plant.

Plant	R.m.s. of state space and control variables					Stability margin		H_2 -norm	H_∞ -norm
	$\varphi \frac{rad}{100}$	$p \frac{rad}{100}$	$r \frac{rad}{100}$	$\Psi \frac{rad}{100}$	$\delta a \frac{rad}{100}$	Phase (deg)	Ampl. (dB)		
Nominal	8.5	0.1778	3.92	3.47	0.89	22.1	∞	0.38	0.255
Perturb.	6.7	0.1107	3.83	4.29	1.07	25.6	∞	0.33	0.161

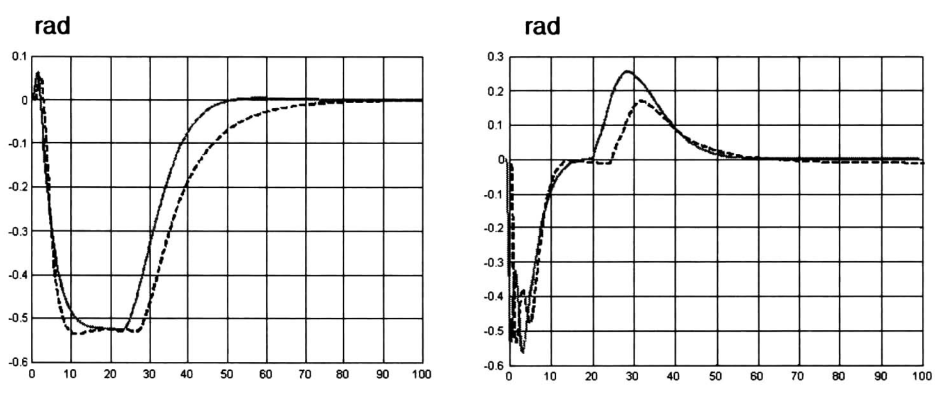

Fig. 4. Transients in the lateral-directional control channel : a) heading angle(rad), b) roll angle(rad)

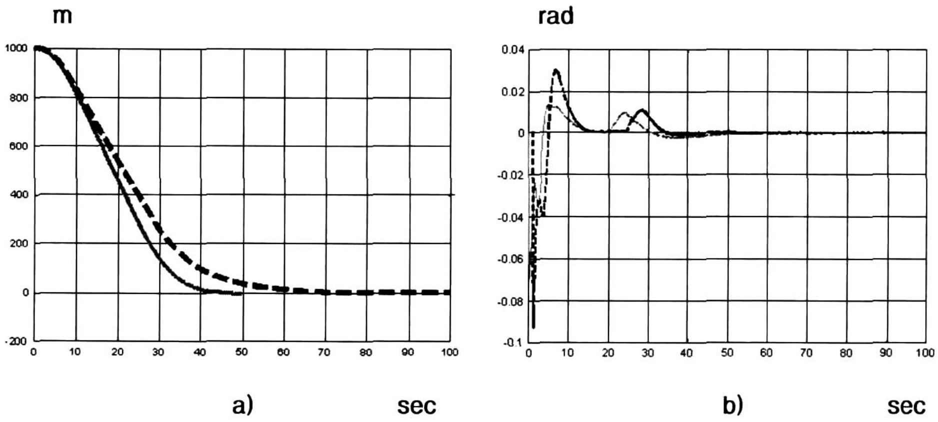

Fig. 5. Transients in the lateral-directional control channel: a) track error(m), b) aileron deflection(rad)

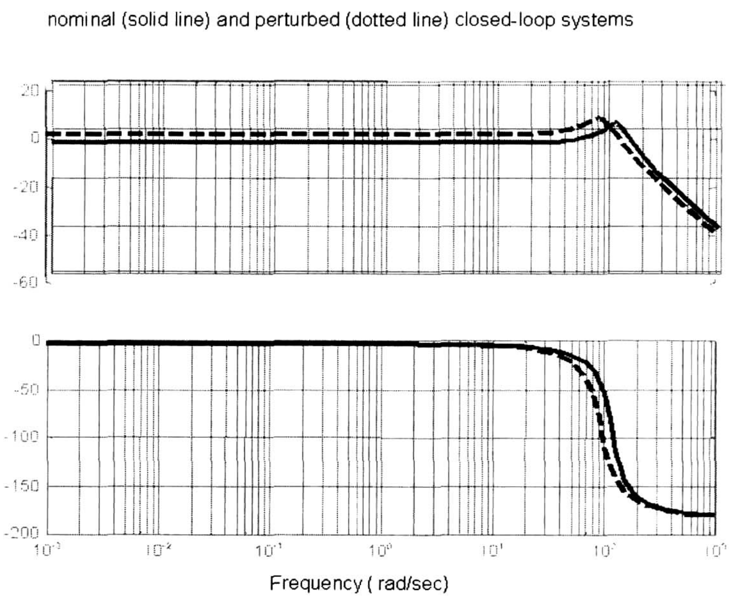

Fig. 6. Bode plots for closed-loop lateral-directional channel

Case Study 2: Robust Optimization of the UAV's Longitudinal Control

Consider the longitudinal FCS for the same UAV with moment of inertia $J_{yy}=111$ Kgm^2, and MAC: $\bar{c}$ = 0.51 m. Likewise the lateral case, longitudinal FCS consists of two loop: inner-loop for attitude control and outer-loop for flight path control. Consider now longitudinal channel of flight control system in Altitude-Hold (Select) mode. Block diagram of this system is depicted in Fig. 7, where elements of outer-loop are shown in the dash-line rectangular. In Fig. 7, h, θ, q stand for sensors of altitude, pitch angle, and pitch rate, respectively, K_h, K_θ, K_q are corresponding gains of control system, "Act(e)" stands for actuator of elevator, "C ϑq" and "Ch" denote pitch and altitude compensators with following transfer functions:

$$W_{Ci}(s)=\frac{T_{ni}s+1}{T_{di}s+1} \qquad (17)$$

where $i=1,2$ - number of compensator ($C\vartheta q$, Ch, respectively). Element "Sat." denotes saturation, limiting the command signal to inner-loop. For diminishing initial deflection of state variables in transient processes command filter "ComF" with transfer function, which is same as in the previous case, is considered.

Vector of adjustable parameters of autopilot $\overrightarrow{C_n}$ which has to be determined by optimization procedure, has the following components:

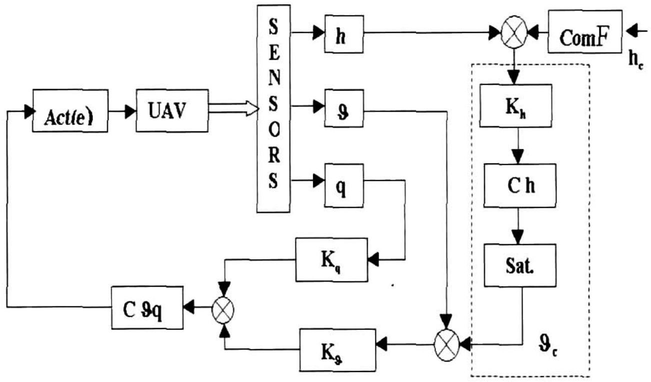

Fig. 7. Block diagram of the longitudinal channel of UAV flight control system

$$\overrightarrow{C_n}=[K_\theta,\ K_q,\ K_h,\ T_{n1},\ T_{d1},\ T_{n2},\ T_{d2}\ ]\ . \qquad (18)$$

UAV longitudinal dynamics state space models corresponding to the nominal and perturbed cases have the following form:

$$A=\begin{bmatrix} -0.0345 & 6 & -9.78 & 0 & 0 \\ -0.0041 & -1.76 & 0 & 0.99 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0.0033 & -25.7 & 0 & -2.19 & 0 \\ 0 & -69.4 & 69.4 & 0 & 0 \end{bmatrix},\quad A_p=\begin{bmatrix} -0.0273 & 6 & -9.78 & 0 & 0 \\ -0.0064 & -1.39 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0.0036 & -16.1 & 0 & -1.73 & 0 \\ 0 & -55.6 & 55.6 & 0 & 0 \end{bmatrix}$$

$$B=[0.36\ \ -0.16\ \ 0\ \ -31.1\ \ 0\ ]^T,\qquad B_p=[0.36\ \ -0.13\ \ 0\ \ -19.9\ \ 0\ ]^T \qquad (19)$$

where matrices for perturbed case are provided with subscript "p".

After few tentative executions of optimization program, made in accordance with aforementioned steps of design procedure, these parameters were chosen as $\lambda_{0d}=\lambda_{pd}=1$; $\lambda_{0s}=\lambda_{ps}=10$, $\lambda_\infty=\lambda_{p\infty}=1.5$. For penalty function the following parameters were used: $K=\tan(\alpha)=4.7$, $\Omega_{\max}=25$ rad/sec, $d_{m1}=0.006$, $d_0=0.0006$ to determine the region of poles location of closed-loop systems. To restrict unnecessary increasing pitch gain in optimization procedure in the penalty function (PF) the following term was added: $P_{fa}=0$, if $K_\theta \geq -20$; $P_{fa}=(K_\theta+20)^2$, if $K_\theta < -20$. Using these PI and PF, optimization procedure determined the following vector of control law's parameters: $\overrightarrow{C_n}=[-20,\ -5.5,\ 0.067,\ 0.1,\ 0.015,\ 0.44,\ 0.04]$. Numerical characteristics of nominal and perturbed systems are summarized in Table 2.

Table 2. Comparison of characteristics for nominal and perturbed plant.

Plant	R.m.s. of state space variables					Stability margin		H_2	H_∞
	α (rad)	ϑ (rad)	q(rad/sec)	h (m)	δe(rad)	Phase (deg)	Ampl. (dB)		
Nominal	0.019	0.019	0.0205	0.28	0.0064	55.5	∞	0.96	1.02
Perturb.	0.038	0.037	0.0213	0.57	0.024	54	∞	0.89	1.23

As it follows from this table r.m.s. of state variables in stochastic case are varying in reasonable limits, whereas such integral characteristics as H_2- and H_∞-norms, defined by expressions (2), (3) and phase stability margins are varying in a very small limits. Amplitude Bode plots of closed-loop systems(see Fig. 8) are very flat, thus proving robustness of system. Transient processes in nominal and perturbed systems are shown in Fig. 9 and 10. From these figures it is obvious that processes in the perturbed systems are slower, than in the nominal. It could be explained by the fact that the airspeed of UAV is not stabilized and changing in the transient process, so UAV reaches given altitude with smaller speed for longer time. In the "Pitch-Hold(Select)" mode the transient processes in both systems are very similar.

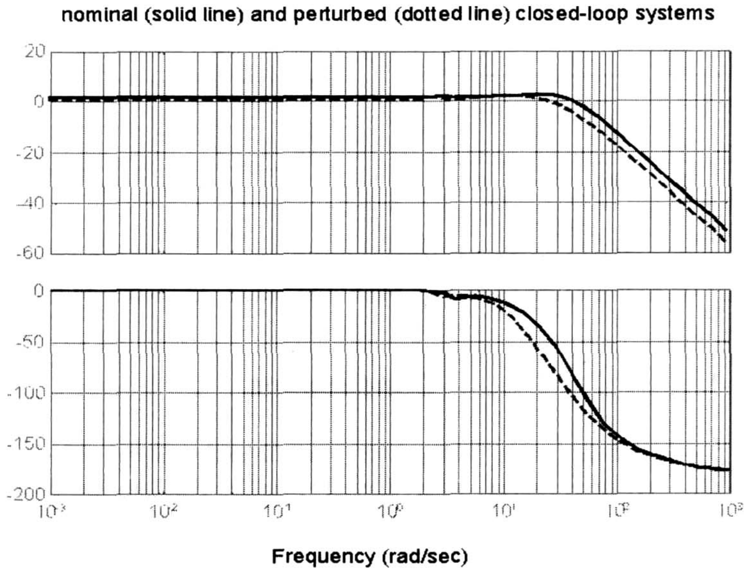

Fig. 8. Bode plots for closed-loop longitudinal channel

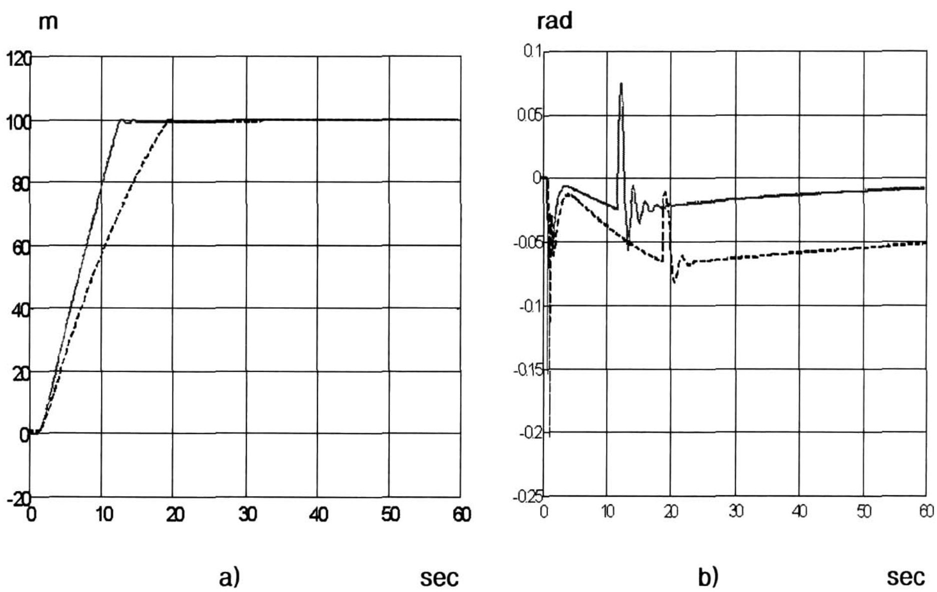

Fig. 9. Transient processes : a) altitude(m), b) elevator deflection angle(rad)

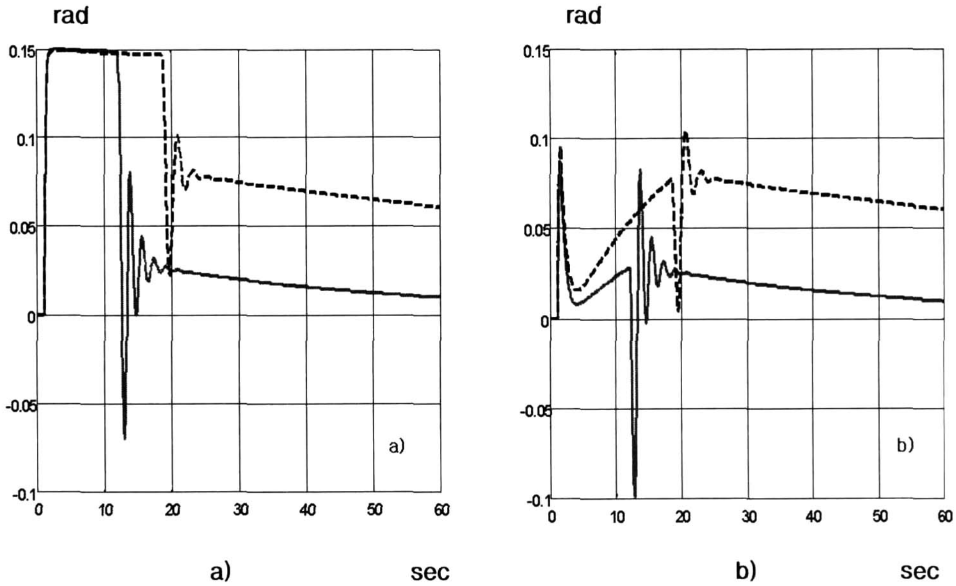

Fig. 10. Transient responses : a) pitch angle(rad), b) angle of attack(rad)

Conclusions

Parametric optimization using composite performance index (PI) based on mixed multi-model H_2/H_∞- criterion along with special penalty function achieves simultaneously robustness and good performance of UAV flight control system, which must follow deterministic command signals in turbulent atmosphere. This composite PI is computed for nominal model of system reflecting the nominal conditions of UAV flight, and for parametrically perturbed model reflecting changes of flight conditions (for example, the change of air speed).

Corresponding weighting coefficients could change contribution of each component of partial PI to the total cost function of optimization. Design procedure consists of optimization program, program for evaluation of optimization results, and final simulation of control system. It is organized like iterative procedure, in which the weighting coefficients in composite PI are corrected after each evaluation of optimization results, and then optimization program is executed again until acceptable values of all components of composite PI would be achieved.

Practical implementation of optimization results requires some adaptation to the nonlinear functions, which are inherent to the actual aircraft control system and include saturation elements, restricting some input and output signals. This adaptation is made at the final stage - simulation of system including all necessary non-linear elements.

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