Unsteady Temperature Distributions in a Semi-infinite Hollow Circular Cylinder of Functionally Graded Materials

Kui-Seob Kim*
Department of Aeronautical Engineering,
Inha Technical College, 253 Yonghyun-dong,
Nam-ku, Incheon, 402-752, Korea.

Naotake NODA**
Department Mechanical Engineering,
Shizuoka University, 3-5-1 Johoku Hamamatsu, 432-8561, JAPAN.

Abstract

A Green's function approach based on the laminate theory is adopted to obtain the unsteady temperature distributions in a semi-infinite hollow circular cylinder made of functionally graded materials (FGMs). The transient heat conduction equation based on the laminate theory is formulated into an eigenvalue problem for each layer by using the eigenfunction expansion theory and the separation of variables. The eigenvalues and the corresponding eigenfunctions obtained by solving an eigenvalue problem for each layer constitute the Green's function solution for analyzing the unsteady temperature distributions. Numerical calculations are carried out for the semi-infinite hollow circular FGM cylinder subjected to partially heated loads, and the numerical results are shown in figures.

Key Word : Green's function approach, laminate theory, functionally graded materials, unsteady temperature distributions, hollow circular cylinder,

Nomenclature

c_p : Specific heat
G_{ss} : Standard Galerkin-Based Green function
G_{ij} : Green function given in Eq. (22)
k : Thermal conductivity
$P = A\overline{R}(1-\overline{R})$: The porosity
$\overline{R}$, $\overline{z}$: The dimensionless position in the r-direction and in the z-direction
T : Temperature
T_0 : Initial temperature
t : Time
$V_m = (1-\overline{R})^M$: The volumetric ratio of metal
α_m : m-th eigenvalue
ϕ_{im} : Eigenfunction corresponding to m-th eigenvalue
λ_i : Thermal diffusivity of the i-th layer
θ_i : Unsteady temperature of the i-th layer
ρ : Density
τ, λ_m : The dimensionless time and the thermal diffusivity of metal

* Associate professor
E-mail : kuiseob@true.inhatc.ac.kr, TEL : 032-870-2165, FAX : 032-870-2503
** Professor

Introduction

Recently, Functionally Graded Materials (FGMs) have been considered as thermal barrier materials in fields such as the nuclear, the aircraft and the space engineering. A FGM differs from a conventional thermal barrier material like a two-layered cylinder. A FGM is characterized by continuously changing material properties due to a graded composition from one surface to other surface. For such heterogeneous materials such as FGMs, the heat conduction equation is presented in a complex form, and the theoretical treatment for the temperature change is very difficult.

In order to design the effective FGM that can reduce the transient thermal stresses, it is important to know the most effective compositions of FGM and the effect of time on the thermal stresses. But there are few reports [1-5] on the solution of transient problems because it is difficult to analyze the transient temperature fields of FGM. Almost all papers, which treated transient problems, have used the Laplace transformation or Fourier transformation to solve the governing equation.

On the other hand, Diaz and Nomura [6] showed a Green's function approach for two-dimensional homogeneous elastic problems. Nomura and Sheahen [7] used Green's function to analyze the steady thermal stresses of a two-dimensional FGM plate.

In this paper, we discuss the transient problems of FGM. Since compositions of FGM are dependent on the function of one-directional position from a metal surface to a ceramic surface, it is assumed that the thermal properties of FGM are dependent on the one-directional position. The transient temperature solution for a two-dimensional semi-infinite hollow circular FGM cylinder with one-directionally dependent properties is formulated by the Green's function approach based on the laminate theory. An approximate solution of the eigenfunction expansion method for each layer is substituted into the governing equation to yield an eigenvalue problem. The eigenvalues and the corresponding eigenfunctions obtained by solving an eigenvalue problem for each layer constitute the Green's function solution for obtaining the two-dimensional transient temperature distributions. The eigenvalues and the corresponding eigenfunctions for any layer are determined from the homogeneous boundary conditions at outer sides, and the continuous conditions of temperature and heat flux at the interfaces.

As an example, a semi-infinite hollow circular FGM cylinder made of zirconium oxide and titanium alloy under the partially heated loads is selected. The numerical results, such as the temperature distributions and the thermal stress distributions, are shown in figures.

Analysis

We consider the two-dimensional unsteady temperature fields $T(r,z,t)$ in an axisymmetric semi-infinite hollow circular cylinder made of functionally graded material whose thermal properties vary with the radial coordinate r. The thermal properties are dependent on the r-directional position and the temperature is independent on the hoop direction position. The heat conduction equation of the cylindrical coordinate system (r, z) with a heat source $q(r,z,t)$ is expressed by:

$$\frac{\partial}{r\partial r}\left\{rk(r)\frac{\partial T}{\partial r}\right\}+\frac{\partial}{\partial z}\left\{k(r)\frac{\partial T}{\partial z}\right\}+q(r,z,t)=\rho(r)c_p(r)\frac{\partial T}{\partial t} \quad (1)$$

where T is temperature, and t is time. The thermophysical properties ρ, c_p and k are position-dependent density, specific heat and thermal conductivity, respectively.

2.1 Green's function approach for axisymmetric heat conduction equation

For convenience in analysis, we consider the heat conduction equation without a heat source

$q(r,z,t)$, and $T=T(r, z, t)$ can be constructed by the superposition of the simpler problem as:

$$T(r,z,t)=T^s(r,z)+\theta(r,z,t) \tag{2}$$

where T^s and θ are the solutions of the steady-state problem with non-homogeneous boundary conditions and the unsteady-state problem with homogeneous boundary conditions, respectively.

Steady heat conduction equation

The steady solution T^s to Eq. (1) is obtained by the standard Galerkin-Based Green function [8] as follows:

$$T^s(r,z)=T^*(r,z)+\int_V G_{ss}f^* dv \tag{3}$$

where the function $T^*$and the function G_{ss} represent a differentiable temperature function which satisfies the non-homogeneous boundary conditions and the standard Galerkin-based Green function, respectively, and the function f^* is given as follows:

$$f^*(r,z)=\frac{\partial}{r\partial r}\left\{rk(r)\frac{\partial T^*}{\partial r}\right\}+\frac{\partial}{\partial z}\left\{k(r)\frac{\partial T^*}{\partial z}\right\} \tag{4}$$

Unsteady heat conduction equation

We consider a laminated medium consisting of L-layers in the r direction. Assuming that the number of laminae L becomes sufficiently large, and the thermal properties in each layer are constants, the governing equations of Eq. (1) without a heat source $q(r,z,t)$ for each layer and initial conditions are given as

$$\frac{\partial}{r\partial r}\left\{r\frac{\partial \theta_i}{\partial r}\right\}+\frac{\partial^2\theta_i}{\partial z^2}=\frac{1}{\lambda_i}\frac{\partial \theta_i}{\partial t} \tag{5}$$

$$\theta_i(r,z,0)=F_i(r,z)=T_0-T^s(r,z) \text{ at } r_{i-1}\le r\le r_i,\ i=1, 2, \cdots, L \text{ for } t=0 \tag{6}$$

where λ_i and T_0 are the thermal diffusivity of the i-th layer and the initial temperature, respectively.

Using the eigenfunction expansion theory and the separation of variables, we can obtain the solutions of Eq. (5) for an axisymmetric semi-infinite cylinder:

$$\theta_i(r,z,t)=\sum_{m=1}^{\infty}\int_0^{\infty} c_m(\beta)\phi_{im}(r,\alpha_m)\psi(z,\beta)e^{-(\alpha_m^2+\lambda_i\beta^2)t}d\beta \tag{7}$$

at $r_{i-1}\le r\le r_i$, $i=1, 2, \cdots, L$ for $t>0$.

The unknowns c_m are constants to be evaluated, and the function ϕ_{im} and ψ satisfy the eigenvalue problems as follows:

$$\frac{\partial}{r\partial r}\left\{r\frac{\partial \phi_{im}(r)}{\partial r}\right\}+\frac{\alpha_m^2}{\lambda_i}\phi_{im}(r)=0 \text{ at } r_{i-1}\le r\le r_i \tag{8}$$

$$\frac{\partial^2\psi(z)}{\partial z^2}+\beta^2\psi(z)=0 \text{ at } 0\le z\le\infty \tag{9}$$

$i=1, 2, \cdots, L$ for $t>0$

where α_m is m-th eigenvalues, and ϕ_{im} is eigenfunctions corresponding to eigenvalues. Applying the initial conditions of Eq. (6) to Eq. (7), and considering the orthogonality relations of functions ϕ_{im}, we obtain the $c_m(\beta)$:

$$c_m(\beta)=\frac{1}{N(\alpha_m)N(\beta)}\sum_{j=1}^{L}\frac{k_j}{\lambda_j}\int_{z'=0}^{\infty}\int_{r'=r_{j-1}}^{r_j} r'F_j(r',z')\phi_{jm}(r',\alpha_m)\psi(z',\beta)dr'dz' \tag{10}$$

where

$$N(\alpha_m)=\sum_{j=1}^{L}\frac{k_j}{\lambda_j}\int_{r'=r_{j-1}}^{r_j} r'\phi_{jm}^2(r',\alpha_m)dr' \tag{11}$$

and function $\psi(z,\beta)$and $N(\beta)$ are determined from the boundary conditions at z=0.
The substitution of Eq. (10) into Eq. (7) gives the solution for $\theta_i(r,z,t)$:

$$\theta_i(r,z,t)=\sum_{j=1}^{L}\int_{z'=0}^{\infty}\int_{r'=r_{j-1}}^{r_j} r'F_j(r',z')G_{ij}(r,z,t|r',z',t')|_{t'=0}dr'dz' \tag{12}$$

where Green's function $G_{ij}(r,z,t|r',z',t')|_{t'=0}$ is defined as:

$$G_{ij}(r,z,t|r',z',t')|_{t'=0}=\sum_{m=1}^{\infty}\left\{\frac{k_j}{\lambda_j}\frac{1}{N(\alpha_m)}\right\}\phi_{im}(r,\alpha_m)\phi_{jm}(r',\alpha_m)e^{-\alpha_m^2 t}g(z,z',t) \tag{13}$$

$$g(z,z',t)=\int_{\beta=0}^{\infty}\frac{1}{N(\beta)}\psi(z,\beta)\psi(z',\beta)e^{-\lambda_i\beta^2 t}d\beta \tag{14}$$

at $r_{i-1}\le r\le r_i$, i = 1, 2, ⋯, L. for t > 0.
Next, we consider the two-dimensional heat conduction equation with a heat source:

$$\lambda_i\frac{\partial}{r\partial r}\left\{r\frac{\partial\theta_i}{\partial r}\right\}+\lambda_i\frac{\partial^2\theta_i}{\partial z^2}+\frac{\lambda_i}{k_i}q_i(r,z,t)=\frac{\partial\theta_i}{\partial t}\quad \text{at } r_{i-1}\le r\le r_i,\ i=1,2,\cdots,L.\ \text{for } t>0 \tag{15}$$

where k_i and $q_i(r,z,t)$ denotes the thermal conductivity and the internal heat source of i-th layer, respectively.
The general solutions of Eq. (15) are assumed as:

$$\theta_i(r,z,t)=\sum_{m=1}^{\infty}\int_0^{\infty}d_m(\beta,t)\phi_{im}(r,\alpha_m)\psi(z,\beta)e^{-(\alpha_m^2+\lambda_i\beta^2)t}d\beta \tag{16}$$

Substituting Eq. (16) into Eq. (15), and applying the initial conditions in Eq. (6), we can obtain d_m:

$$\begin{aligned}d_m(\beta,t)&=c_m(\beta)+\frac{1}{N(\alpha_m)N(\beta)}\\&\times\sum_{j=1}^{L}\frac{k_j}{\lambda_j}\int_0^t\int_{z'=0}^{\infty}\int_{r'=r_{j-1}}^{r_j} r'\left\{\frac{\lambda_j}{k_j}q_j(r',z',t')\right\}\phi_{jm}(r',\alpha_m)\psi(z',\beta)e^{(\alpha_m^2+\lambda_j\beta^2)t'}dr'dz'dt'\end{aligned} \tag{17}$$

where c_m is given in Eq. (10).
Substituting Eq. (17) into Eq. (16), and introducing the Green's function, the solution of Eq. (15) is obtained

$$\begin{aligned}\theta_i(r,z,t)&=\sum_{j=1}^{L}\int_{z'=0}^{\infty}\int_{r'=r_{j-1}}^{r_j} r'F_j(r',z')G_{ij}(r,z,t|r',z',t')|_{t'=0}dr'dz'\\&+\sum_{j=1}^{L}\int_0^t\int_{z'=0}^{\infty}\int_{r'=r_{j-1}}^{r_j} r'\left\{\frac{\lambda_j}{k_j}q_j(r',z',t')\right\}G_{ij}(r,z,t|r',z',t')dr'dz'dt'\end{aligned} \tag{18}$$

at $r_{i-1} \le r \le r_i$, i = 1, 2, ⋯, L. for t > 0.

where Green's function $G_{ij}(r,z,t|r',z',t')$ is written as

$$G_{ij}(r,z,t|r',z',t') = \sum_{m=1}^{\infty}\left\{\frac{k_j}{\lambda_j}\frac{1}{N(\alpha_m)}\right\}\phi_{im}(r,\alpha_m)\phi_{jm}(r',\alpha_m)e^{-\alpha_m^2(t-t')}g(z,z',t-t') \tag{19}$$

2.2 Determination of eigenfunctions and eigenvalues

The general solution $\phi_{im}(r,\alpha_m)$ of the eigenvalue problems given in Eq. (8) can be written:

$$\phi_{im}(r,\alpha_m) = C_{im}J_0\left(\frac{\alpha_m}{\sqrt{\lambda_i}}r\right) + D_{im}Y_0\left(\frac{\alpha_m}{\sqrt{\lambda_i}}r\right)$$

$$\text{at } r_{i-1} \le r \le r_i,\ i = 1, 2, \cdots, L \text{ for } t > 0 \tag{20}$$

where J_0 and Y_0 denote the Bessel function of the first and second kind of order zero, respectively, and C_{im} and D_{im} are coefficients to be evaluated.

The function $\psi(z,\beta)$ given in Eq. (9) is dependent on the boundary conditions [9], and is given as follows:

$$\psi(z,\beta) = \cos(\beta z),\quad N(\beta) = \pi/2 \text{ for the insulated face at z=0}$$

$$\psi(z,\beta) = \sin(\beta z),\quad N(\beta) = \pi/2 \text{ for the prescribed temperature at z=0}$$

$$\psi(z,\beta) = \beta\cos(\beta z) + h_r/k_r\sin(\beta z),\quad N(\beta) = \pi\{\beta^2 + (h_r/k_r)^2\}/2$$

$$\text{for the face with heat transfer at z=0} \tag{21}$$

where h_r and k_r are the heat transfer coefficient and thermal conductivity at z=0, respectively. Applying the continuous conditions of temperature and heat flux at the interfaces to Eq. (20), the simultaneous equations for C_{im} and D_{im} are written in matrix form as

$$\begin{Bmatrix} C_{im} \\ D_{im} \end{Bmatrix} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}\begin{Bmatrix} C_{i+1,m} \\ D_{i+1,m} \end{Bmatrix} e^{(\lambda_i - \lambda_{i+1})\beta^2 t} \tag{22}$$

at $r = r_i$ in $r_{i-1} \le r \le r_i$, i = 1, 2, ⋯, L for t > 0.

where

$$p_{11} = \frac{1}{DET}\left\{Y_1\left(\frac{\alpha_m}{\sqrt{\lambda_i}}r_i\right)J_0\left(\frac{\alpha_m}{\sqrt{\lambda_{i+1}}}r_i\right) - \frac{k_{i+1}}{k_i}\sqrt{\frac{\lambda_i}{\lambda_{i+1}}}Y_0\left(\frac{\alpha_m}{\sqrt{\lambda_i}}r_i\right)J_1\left(\frac{\alpha_m}{\sqrt{\lambda_{i+1}}}r_i\right)\right\}$$

$$p_{12} = \frac{1}{DET}\left\{Y_1\left(\frac{\alpha_m}{\sqrt{\lambda_i}}r_i\right)Y_0\left(\frac{\alpha_m}{\sqrt{\lambda_{i+1}}}r_i\right) - \frac{k_{i+1}}{k_i}\sqrt{\frac{\lambda_i}{\lambda_{i+1}}}Y_0\left(\frac{\alpha_m}{\sqrt{\lambda_i}}r_i\right)Y_1\left(\frac{\alpha_m}{\sqrt{\lambda_{i+1}}}r_i\right)\right\}$$

$$p_{21} = \frac{1}{DET}\left\{-J_1\left(\frac{\alpha_m}{\sqrt{\lambda_i}}r_i\right)J_0\left(\frac{\alpha_m}{\sqrt{\lambda_{i+1}}}r_i\right) + \frac{k_{i+1}}{k_i}\sqrt{\frac{\lambda_i}{\lambda_{i+1}}}J_0\left(\frac{\alpha_m}{\sqrt{\lambda_i}}r_i\right)J_1\left(\frac{\alpha_m}{\sqrt{\lambda_{i+1}}}r_i\right)\right\}$$

$$p_{22} = \frac{1}{DET}\left\{-J_1\left(\frac{\alpha_m}{\sqrt{\lambda_i}}r_i\right)Y_0\left(\frac{\alpha_m}{\sqrt{\lambda_{i+1}}}r_i\right) + \frac{k_{i+1}}{k_i}\sqrt{\frac{\lambda_i}{\lambda_{i+1}}}J_0\left(\frac{\alpha_m}{\sqrt{\lambda_i}}r_i\right)Y_1\left(\frac{\alpha_m}{\sqrt{\lambda_{i+1}}}r_i\right)\right\}$$

$$DET = \left\{J_0\left(\frac{\alpha_m}{\sqrt{\lambda_i}}r_i\right)Y_1\left(\frac{\alpha_m}{\sqrt{\lambda_{i+1}}}r_i\right) - Y_0\left(\frac{\alpha_m}{\sqrt{\lambda_i}}r_i\right)J_1\left(\frac{\alpha_m}{\sqrt{\lambda_{i+1}}}r_i\right)\right\}$$

Therefore, the eigenvalues α_m are determined from Eq. (22) and the homogeneous boundary conditions at the outer surfaces, and the corresponding eigenfunctions for all layers are determined from Eq. (22).

Numerical Results and Discussion

The numerical calculations are carried out for a hollow circular FGM cylinder made of Zirconium oxide and Titanium alloy, and the position functions of material properties in Ref. [2] are used. For the numerical calculations, we used the dimensionless qualities as follows:

$$\overline{R}=(r-r_a)/(r_b-r_a),\quad \overline{z}=z/(r_b-r_a),\quad \overline{b}=b/(r_b-r_a),\quad \tau=\lambda_m t/(r_b-r_a)^2$$

where $\overline{R}$ and $\overline{z}$ denote the dimensionless position in the r-direction and in the z-direction, b denotes the width of the heating region on $\overline{R}=1$, τ and λ_m are the dimensionless time and the thermal diffusivity of metal, respectively. The volumetric ratio of metal $V_m=(1-\overline{R})^M$ and the porosity $P=A\overline{R}(1-\overline{R})$ as the function of position are used, respectively.

3.1 Semi-infinite hollow circular FGM cylinder

We consider the two-dimensional transient temperature distributions of a semi-infinite hollow circular FGM cylinder subjected to the partially heated loads as follows:

$$\frac{\partial T}{\partial z}=0 \text{ at } z=0,\quad 0\le\overline{R}\le 1$$

$$T=T_0+T_b\cos\left(\frac{\pi}{2b}z\right) \text{ at } \overline{R}=1,\quad 0\le z\le b\ ;$$

$$T=T_0 \text{ at } \overline{R}=1,\quad z>b$$

$$T=T_0 \text{ at } \overline{R}=0,\quad 0\le z\le\infty$$

where b is the width of the heating region on $\overline{R}=1$.

Numerical calculations are carried out for $T_b=5T_0$, $\overline{b}=1$, m=50 in Eq. (19), M=1 and A=0.

Figs. 1-3 present the convergence of dimensionless temperatures (T/T_0) for layer number L at three kinds of dimensionless positions, respectively. The convergence of the solution is very fast at the small dimensionless times ($\tau=0.01$, $\tau=0.05$, $\tau=0.1$). Figs. 4 and 5 show the temperature distributions of a semi-infinite hollow circular FGM cylinder with the dimensionless positions in unsteady and steady state.

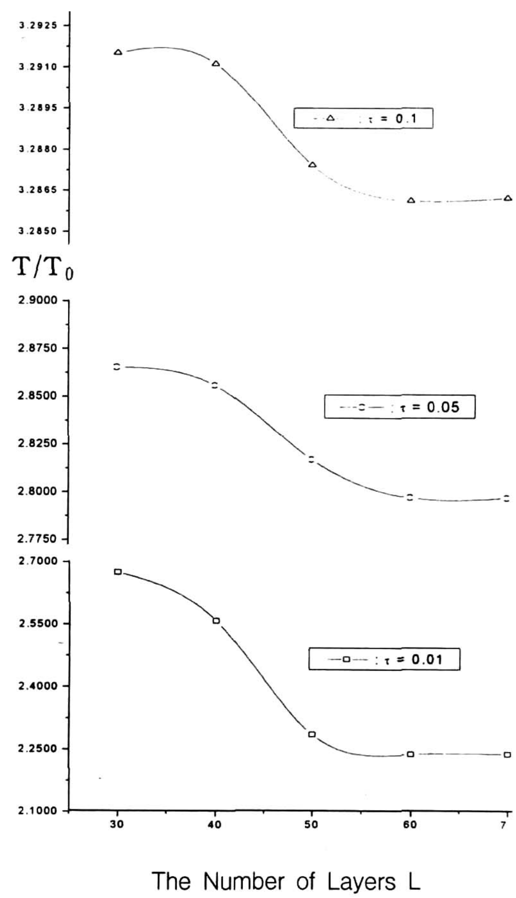

Fig. 1. The dimensionless temperatures vs. the number of layers L at dimensionless position $\overline{R}=0.8$ and $\overline{z}=0.1$.

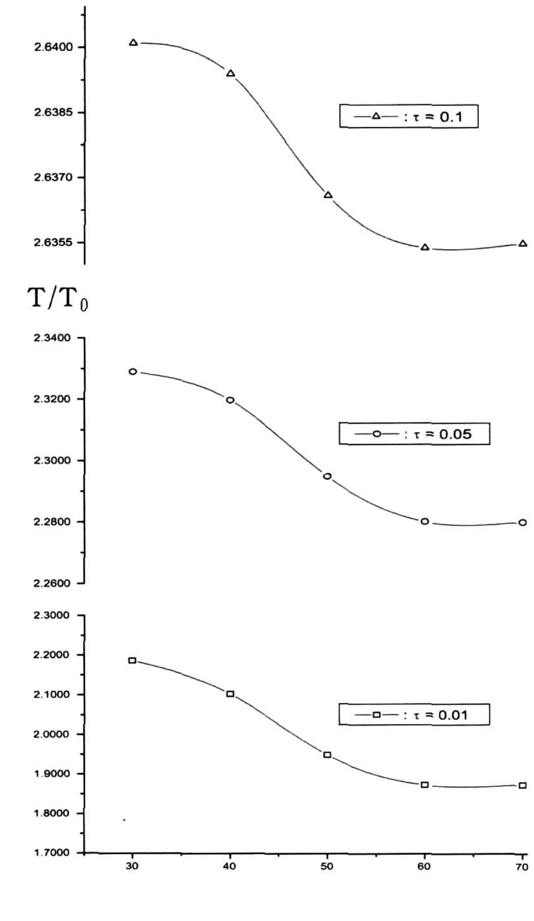

Fig. 2. The dimensionless temperatures vs. the number of layers L at *dimensionless position* $\overline{R}=0.8$ and $\overline{z}=0.5$.

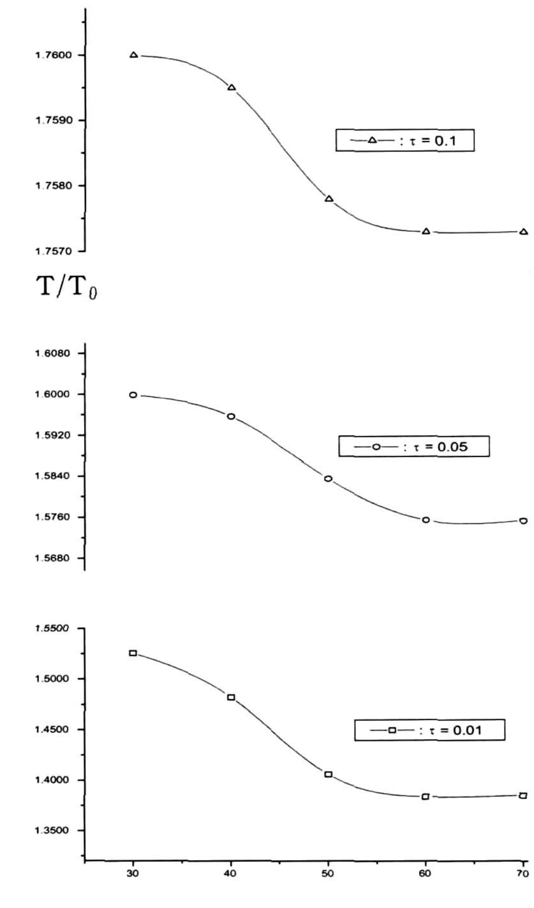

Fig. 3. The dimensionless temperatures vs. the number of layers L at *dimensionless position* $\overline{R}=0.8$ and $\overline{z}=0.8$.

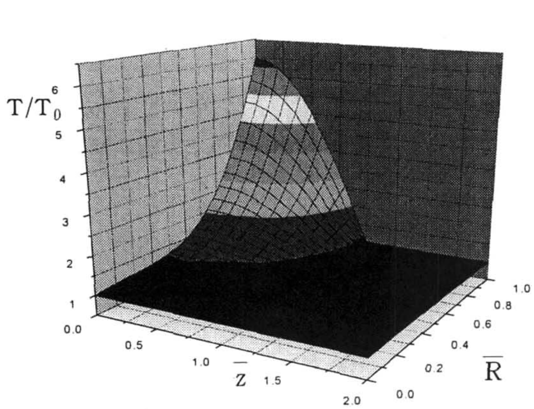

Fig. 4. The two-dimensional dimensionless temperatures vs. the dimensionless positions at $\tau=0.1$.

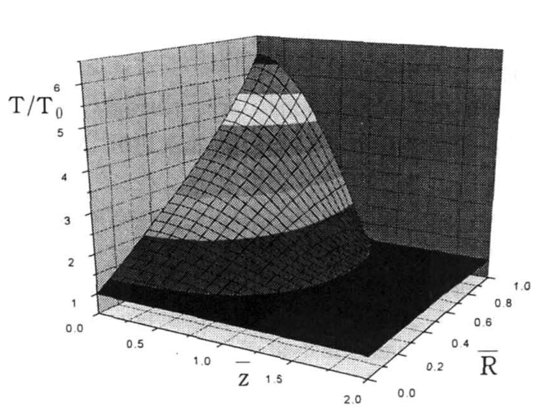

Fig. 5. The two-dimensional dimensionless temperatures vs. the dimensionless position at $\tau=\infty$.

3.2 Applications to thermal stress analysis

It is very difficult to obtain an analytical thermal stress solution of two-dimensional axisymmetric FGM cylinder. Then, we consider thermal stress problem in a one-dimensional hollow circular FGM cylinder with the inner radius r_a and the outer radius r_b by use of the proposed approach. When the material properties are the function of the position, the one-dimensional basic equation for the thermal stress of a hollow circular cylinder is as follows [2]:

$$\frac{d^2u_r}{dr^2}+\frac{1}{r}\frac{du_r}{dr}-\frac{u_r}{r^2}+\delta_1\phi_1(r)\left[\frac{du_r}{dr}+M(r)\frac{u_r}{r}\right]=h_1(r)-h_2(r)\varepsilon_0 \tag{23}$$

where

$$\delta_1\phi_1(r)=\frac{d}{dr}\ln\left\{\frac{1-\nu}{(1+\nu)(1-2\nu)}E\right\}$$

$$M(r)=\frac{d}{dr}\left\{\frac{\nu}{(1+\nu)(1-2\nu)}E\right\}/\left\{\frac{1-\nu}{(1+\nu)(1-2\nu)}E\right\}$$

$$h_1(r)=\frac{(1+\nu)(1-2\nu)}{E(1+\nu)}\frac{d}{dr}\left\{\frac{\alpha E}{1-2\nu}(T-T_0)\right\}$$

$$h_2(r)=\frac{(1+\nu)(1-2\nu)}{E(1-\nu)}\frac{d}{dr}\left\{\frac{E\nu}{(1+\nu)(1-2\nu)}\right\}$$

and u_r, E, ν, α and ε_0 denote the displacement in the radial direction, Young's modulus, Poisson's ratio, the coefficients of linear thermal expansion and the constant strain, respectively.
The solution of Eq. (23) is obtained by the perturbation theory [2], and thermal stresses are carried out under the following conditions:

$$\sigma_{rr}=0 \text{ at } r=r_a$$
$$\sigma_{rr}=0 \text{ at } r=r_b \tag{24}$$

and the both ends of the hollow circular cylinder are free:

$$\int_{r_a}^{r_b}\sigma_{zz}2\pi r dr=0 \tag{25}$$

where σ_{rr} is the thermal stress in radial direction, and σ_{zz} is the thermal stress in axial direction.

We consider the transient temperature distributions and thermal stresses of the one-dimensional FGM hollow circular cylinder under the thermal boundary conditions as follows:

$$T=770(K) \text{ at } \overline{R}=0$$
$$T=1680(K) \text{ at } \overline{R}=1$$

and an initial condition: $T_0=300(K)$ at $\tau=0$.
Figs. 6 and 7 present the temperature distributions with variation of the volumetric ratio of metal and the porosity, respectively. The FGM at M=0.5, M=1.0 and M=1.5 contains about 67%, 50% and 40% of metal, respectively, and the FGM at A=0, A=0.06 and A=0.12 has porosity of 0%, 1% and 2%, respectively. As shown in Figs. 6 and 7, the effects of the volumetric ratio of metal on the temperature distributions are greater than those of the porosity in steady and unsteady state. Figs. 8 and 9 show the effects on axial stress σ_{zz} with variation of the volumetric ratio of metal and the porosity, respectively. As shown in Figs. 8 and 9, the effects of the variation of dimensionless time on the axial stresses are greater than those of the volumetric ratio of metal and the porosity. Figs. 10 and 11 show the effects on hoop stress $\sigma_{\theta\theta}$ with variation of the volumetric ratio of metal and the porosity, respectively. The effects on hoop stress $\sigma_{\theta\theta}$ with variation of the volumetric ratio

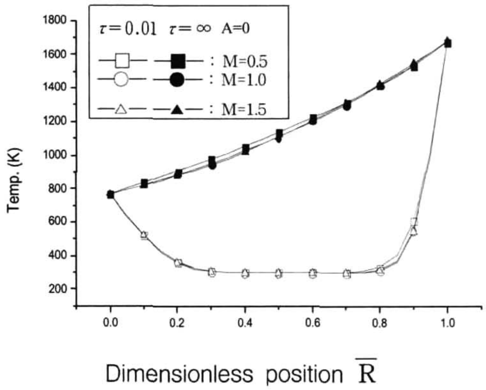

Fig. 6. The temperature distributions vs. the dimensionless position for a one-dimensional FGM cylinder.

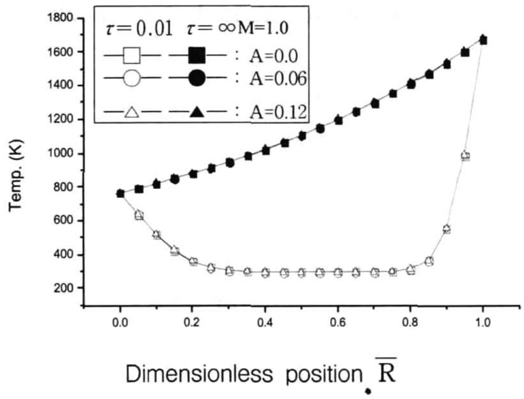

Fig. 7. The temperature distributions vs. the dimensionless position for a one-dimensional FGM cylinder.

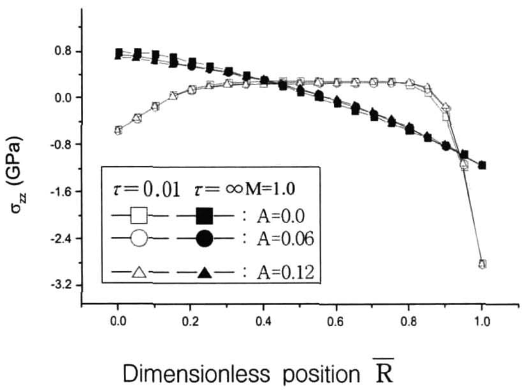

Fig. 8. The axial thermal stresses vs. the dimensionless position for a one-dimensional FGM cylinder.

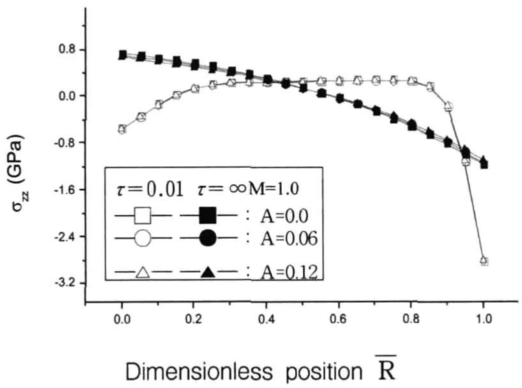

Fig. 9. The axial thermal stresses vs. the dimensionless position for a one-dimensional FGM cylinder.

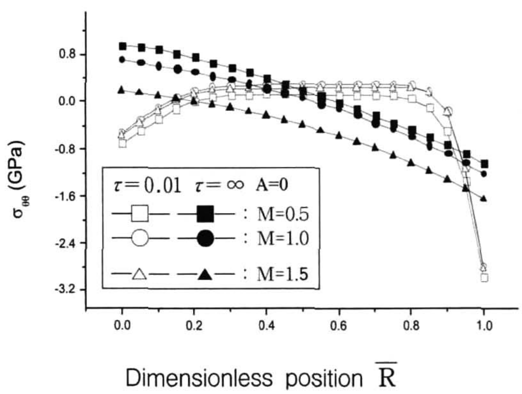

Fig. 10. The hoop thermal stresses vs. the dimensionless position for a one-dimensional FGM cylinder.

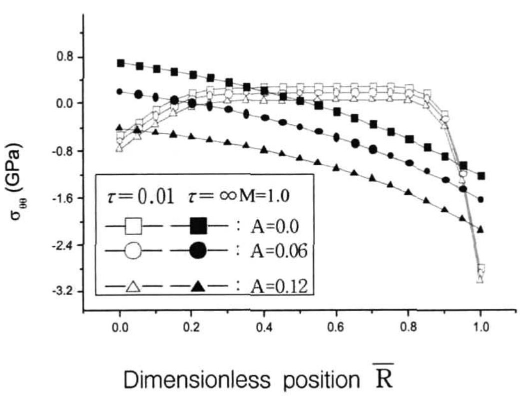

Fig. 11. The hoop thermal stresses vs. the dimensionless position for a one-dimensional FGM cylinder.

of metal and the porosity are greater than those on axial stress σ_{zz} as shown in Figs. 8-11. The maximum compressive axial stress and hoop stress produce on the outer surface at very small time, while the maximum tensile axial stress and hoop stress on the inner surface at steady state.

Conclusions

A Green's function approach for analyzing the unsteady temperature distributions of a semi-infinite hollow circular FGM cylinder with one-directionally dependent properties is proposed. The Green"s function is formulated by using the laminate theory, and is expressed by the eigenvalues and the corresponding eigenfunctions for each layer. The eigenvalues and the corresponding eigenfunctions for each layer satisfy the continuous conditions of temperature and heat flux at each interface, and the homogeneous boundary conditions at outer sides. Therefore, it is shown in figures that the convergence of temperature solution is very fast in steady and unsteady state. It is shown in figures that the application of the proposed method to the analysis of one-dimensional thermal stresses and the calculations of the volumetric ratio of metal and the porosity, which is required for the design of FGM cylinder, are possible.

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