Design of a Robust Target Tracker for Parameter Variations and Unknown Inputs

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Abstract

This paper describes the procedure to develop a robust estimator design method for a target tracker that accounts for both structured real parameter uncertainties and unknown inputs. Two robust design approaches are combined : the Mini-p-Norm design method to consider real parameter uncertainties and the H_∞ design technique for unknown disturbances and unknown inputs. Constant estimator gains are computed that guarantee the robust performance of the estimator in the presence of parameter variations in the target model and unknown inputs to the target. The new estimator has two design parameters. One design parameter allows the trade off between small estimator error variance and low sensitivity to unknown parameter variations. Another design parameter allows the trade off between the robustness to real parameter variations and the robustness to unknown inputs. This robust estimator design method was applied to the longitudinal motion tracking problem of a T-38 aircraft.

Key Word : Robust, Target Tracker, Parameter Variations, Estimator, Uncertainties

Introduction

In a target tracking problem, several types of uncertainties may be encountered. For the design and analysis of a tracker, mathematical model of a target aircraft is necessary. This mathematical model should be complete enough for an adequate description of the system and also sufficiently simple such that the resulting algorithms are computationally feasible for real time operation. The exact mathematical model for a target aircraft is nonlinear and is very complicated, furthermore it is not known to the tracker. Therefore the mathematical model used in the tracker design will be different from the exact target model. For the simplicity, the high order structural mode of the target aircraft is neglected in the tracker design and the linear model instead of nonlinear model is often used. This uncertainty corresponds to unstructured uncertainty.

The parameters in this mathematical model such as aerodynamic coefficients, mass, and moments of inertia are unknown and time varying. This uncertainty corresponds to parameter uncertainty (or structured uncertainty). The control input applied to the target vehicle by the pilot and the disturbances such as atmospheric turbulence are unknown and time varying. This

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corresponds to the input uncertainty.

The objective of this research is to design a nonadaptive estimator which is robust to parameter uncertainties and input uncertainties or disturbances, and therefore its function performs satisfactorily in the presence of these uncertainties. Mini-p-Norm (MpN) estimator design method was developed by Kim and Andrisani[1][2] to guarantee robust performance against parameter variations. This method computes estimator gains that minimize the new cost function which is directly related to the p-norm of the estimation error variances computed at each parameter values in the range of parameter variations. The MpN estimator has a new design parameter which allows the trade off between small estimator error variance and low sensitivity to unknown parameter variations.

The robust performance of an estimator with unknown inputs can be achieved by minimizing the H_∞-norm of disturbance transfer function. Bernstein and Haddad[3] and Nagpal and Khargonekar[4] applied the H_∞ synthesis to design robust estimators against the uncertainties in the input and initial conditions.

This paper describes a procedure to develop a robust estimator design method that accounts for both structured real parameter uncertainties and unknown inputs. Two robust design approaches are combined: the MpN design method to consider the real parameter uncertainties and the H_∞ design technique for the unknown disturbances and unknown inputs.

Mathematical Formulations

In this paper, continuous, linear, time invariant plant dynamics are assumed for the target vehicle. The state equations modeling the target dynamics can be written in the form:

$$\begin{aligned} \dot{x} &= A(a)x + B(a)w \\ z &= H(a)x \\ y &= C(a)x + v \end{aligned} \tag{1}$$

where x = an n_x dimensional state vector,
a = a vector of unknown system parameters,
z = an n_z dimensional output state vector we want to estimate,
y = an n_y dimensional measurement output vector,
$A(a)$, $B(a)$, $H(a)$ and $C(a)$ = the plant system matrices of proper size,
w = a zero-mean Gaussian white process noise vector with intensity Q_c, and
v = a zero-mean Gaussian white measurement noise vector with intensity R_c.

Typical inputs to the target vehicle are pilot or driver inputs and disturbances such as those caused by atmospheric turbulence. However, since these typical inputs are never known to the tracker, they cannot be estimated unless an adaptation mechanism is implemented. A simple and most common way to deal with these system inputs in the tracker problem is to model them as Gaussian white process noise or colored noise.

In the design of estimators (specifically for the target tracking case), the exact plant matrices $A(a)$, $B(a)$ and $C(a)$ are also unknown to the estimator designer. Instead the estimator designer must use a different, perhaps, simpler state space model of the following form:

$$\begin{aligned} \dot{x} &= A_o x + B_o w \\ z &= H_o x \\ y &= C_o x + v \end{aligned} \tag{2}$$

where x is an n_{x_o} dimensional state vector, and A_o, H_o and C_o are the system matrices of tracker model.

Using the tracker model of Eq.(2), an estimator or observer can be designed in the following form :

$$\begin{aligned} \dot{\hat{x}} &= A_o \hat{x} + K(y - C_o \hat{x}) \\ \hat{z} &= H_o \hat{x} \\ \hat{y} &= C_o \hat{x} + v \end{aligned} \tag{3}$$

where

$\hat{x}$ = an n_{x_o} dimensional estimator state vector,
$\hat{z}$ = an n_z dimensional estimator output vector,
$\hat{y}$ = an n_y dimensional measurement output vector,
K = an $n_{x_o} \times n_y$ gain matrix of the estimator.

In the estimation problem, we are concerned about the estimation error of the states of interest. The estimation error is defined as the difference between the estimated outputs and exact outputs:

$$e(t) \equiv z(t) - \hat{z}(t) = Hx - H_o \hat{x}$$

Robustness to Parameter Variations

Robust performance can be obtained in the presence of parameter variations by introducing the Mini-p-Norm(MpN) estimator with the following cost function which is related to the p-norm of the estimation error variances calculated at a_i, discrete points in the range of parameter variations [1][2].

$$J_p(K) = \sum_{i=1}^{\ell} [P_i \, J_i(K)\,]^p \tag{4}$$

where P_i is the probability of $a = a_i$ and

$$J_i(K) = tr \lim_{t \to \infty} [E(e_i^T W e_i)]_{a = a_i}$$

where W is a weighting matrix for estimation error states.

Note that $(P_i J_i(K))^{p-1}$ has a role of a weight function, in the sense that when $P_i J_i(K)$ is large, $(P_i J_i(K))^{p-1}$ is large and when $P_i J_i(K)$ is small, $(P_i J_i(K))^{p-1}$ is small for all $p > 1$. Therefore, the above cost function gives more weight to the large contributors in the performance index and less weight to the smaller ones. When $p = 1$, the MpN estimator minimizes the average values of all $P_i J_i$ and when $p \to \infty$, it minimizes the maximum value of $P_i J_i$.

Robustness to Unknown Inputs

In Eq.(1) and Eq.(3), let us introduce the following variables

$$w = Q^{1/2} \eta_1 \quad \text{and} \quad v = R^{1/2} \eta_2$$

where η_1 and η_2 are white noises with unit intensities.

Then, Eq.(1) and Eq.(3) can be combined to give:

$$\dot{x}_a = A_a x_a + D_a \eta_a \tag{5}$$

where

$$x_a = \begin{bmatrix} x \\ \hat{x} \end{bmatrix}, \qquad A_a = \begin{bmatrix} A & 0 \\ KC & A_o - KC_o \end{bmatrix},$$

$$D_a = \begin{bmatrix} B(a)Q^{1/2} & 0 \\ 0 & KR^{1/2} \end{bmatrix}, \quad \text{and} \quad \eta_a = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}.$$

The steady state value of the state covariance of the augmented state, x_a is solved by the following Lyapunov equation:

$$A_a X + X A_a^T + D_a D_a^T = 0. \tag{6}$$

If we define W_e as:

$$W_e W_e^T = W \tag{7}$$

where W is the weighting matrix defined in Eq.(4), then the weighted estimation error is written as:

$$W_e e = F x_a \tag{8}$$

where

$$F \equiv W_e [H \quad -H_o]. \tag{9}$$

The disturbance transfer function $\boldsymbol{H}(s)$ from the noise input $\eta_a(s)$ to the estimation error $W_e e(s)$ is:

$$\boldsymbol{H}(s) \equiv F(sI - A_a)^{-1} D_a . \tag{10}$$

The relation of $\boldsymbol{H}(s)$ to the robustness of the estimator will be discussed next. From Eq.(10), we obtain:

$$\begin{aligned} W_e e &= \boldsymbol{H}(s)\eta_a \\ &= F(sI - A_a)^{-1} D_a \eta_a \\ &= W_e [H - H_o] \begin{bmatrix} sI - A & 0 \\ -KC & sI - A_o + KC_o \end{bmatrix}^{-1} \begin{bmatrix} BQ^{\frac{1}{2}} & 0 \\ 0 & KR^{\frac{1}{2}} \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} \\ &= \boldsymbol{H}_1(s)\eta_1 + \boldsymbol{H}_2(s)\eta_2 \end{aligned} \tag{11}$$

where

$$\boldsymbol{H}_1(s) = W_e [H - H_o (sI - A_o + KC_o)^{-1} KC](sI - A)^{-1} BQ^{½},$$

$$\boldsymbol{H}_2(s) = -W_e H_o (sI - A_o + KC_o)^{-1} KR^{½} .$$

In Eq.(11), $\boldsymbol{H}_1(s)$ denotes the transfer function from the system input to the estimation error and $\boldsymbol{H}_2(s)$ denotes the transfer function from the measurement noise to the estimation error.

In order to have robustness to the disturbance inputs or unknown deterministic pilot inputs which are modeled as process noises for the estimator design, $\boldsymbol{H}_1(s)$ must be small; in order to have robustness to the unknown measurement noise, $\boldsymbol{H}_2(s)$ must be small. To achieve as much robustness to the process and measurement noises as possible, we need to minimize the H_∞-norm of $\boldsymbol{H}(s)$, $\| \boldsymbol{H} \|_\infty$ which is defined as:

$$\| \boldsymbol{H} \|_{\infty} \equiv \sup_{w} \sigma_{\max} [\boldsymbol{H}(jw)] \tag{12}$$

where $\sigma_{\max}$ is the maximum singular value. The minimization of $\| \boldsymbol{H} \|_{\infty}$ could cause an increase in the estimation error variance or *MpN* cost function J_p . Therefore, instead of minimizing $\| \boldsymbol{H} \|_{\infty}$, we constrain $\| \boldsymbol{H} \|_{\infty}$ to be less than a specified value, *i.e.*

$$\| \boldsymbol{H} \|_{\infty} < \gamma \tag{13}$$

From the definition of H_{∞}-norm in Eq.(12), it is clear that

$$\| \boldsymbol{H} \|_{\infty} = \max (\| \boldsymbol{H}_1(s) \|_{\infty}, \| \boldsymbol{H}_2(s) \|_{\infty}) \tag{14}$$

where $\max(a_1, a_2)$ represents the larger value of a_1 and a_2. Eq.(14) implies that

$$\| \boldsymbol{H}_1 \|_{\infty} < \gamma \text{ and } \| \boldsymbol{H}_2 \|_{\infty} < \gamma \tag{15}$$

In a practical tracking problem, the sensor dynamics and its noise characteristics are usually well known to the tracker. Thus, in this case, we only need the constraint of

$$\| \boldsymbol{H}_1 \|_{\infty} < \gamma \tag{16}$$

However, the minimization of J_p under the constraint of Eq.(16) is very difficult to solve or may even be impossible. Instead, we have to use the constraint of Eq.(13). If $\| \boldsymbol{H}_1 \|_{\infty} \geq \| \boldsymbol{H}_2 \|_{\infty}$, the constraint of Eq. (13) will be the same as Eq.(16), but if $\| \boldsymbol{H}_1 \|_{\infty} \leq \| \boldsymbol{H}_2 \|_{\infty}$, Eq.(13) will lead to a conservative result. Finally, we will design an estimator which satisfies the following design criteria :

Minimize the *MpN* estimator design criterion

$$J_p(K) = \sum_{i=1}^{\ell} [P_i \, tr(X_i N_i)]^p \tag{17}$$

where

$$N_i \equiv F_i^{\mathrm{T}} F_i \tag{18}$$

under the constraints on the disturbance transfer function

$$\| \boldsymbol{H}_i \|_{\infty} < \gamma \quad \text{for every } i \tag{19}$$

where $\gamma > 0$ is a given constant.

To solve this problem, we use the following Lemma.

Lemma 1. Let A_o, C_o, and K be given such that for every i, there exists a nonnegative-definite matrix $\boldsymbol{X}_i$ satisfying

$$0 = A_{a_i} \boldsymbol{X}_i + \boldsymbol{X}_i A_{a_i}^T + \gamma^{-2} \boldsymbol{X}_i N \boldsymbol{X}_i + D_{a_i} D_{a_i}^T \tag{20}$$

Then, (A_{a_i}, D_{a_i}) is asymptotically stable if and only if $(A_o - KC_o)$ is asymptotically stable. Furthermore, in this case,

$$\| \boldsymbol{H}_i \|_{\infty} < \gamma, \text{ and } \quad X_i \leq \boldsymbol{X}_i \quad \text{for every } i , \tag{21}$$

and

$$J_p(K) \le \sum_{i=1}^{\ell}[P_i tr(\mathbf{X}_i N_i)]^p \tag{22}$$

<u>Proof</u> : *Lemma 1* follows immediately from the *Lemma 3.1* in [3] except Eq.(22) which is proved here. From Eq.(21), the following equation can be obtained

$$P_i tr(X_i N_i) \le P_i tr(\mathbf{X}_i N_i) \quad \text{for every } i. \tag{23}$$

Since $tr(X_i N_i)$ is a positive scalar, Eq.(23) implies

$$[tr(X_i N_i)]^p \le [tr(\mathbf{X}_i N_i)]^p \quad \text{for every } i. \tag{24}$$

Summation of Eq.(24) from i=1 to ℓ leads to Eq.(22). ■

Lemma 1 shows that if a non-negative definite solution to Eq.(20) exists, the H_∞ constraint is automatically satisfied. Furthermore, the *MpN* estimator error criterion is guaranteed to be no worse than the bound $\sum_{i=1}^{\ell}[P_i tr(\mathbf{X}_i N_i)]^p$. The minimization of Eq.(17) under the constraint of Eq.(19) is very difficult. Instead, we can minimize the upper bound $\sum_{i=1}^{\ell}[P_i tr(\mathbf{X}_i N_i)]^p$ under the constraint of Eq.(19). Now, $\mathbf{J}(K) \equiv \sum_{i=1}^{\ell}[P_i tr(\mathbf{X}_i N_i)]^p$ can be interpreted as an auxiliary cost.

The Auxiliary Minimization Problem

The new objective is to minimize

$$\mathbf{J}(K) \equiv \sum_{i=1}^{\ell}[P_i tr(\mathbf{X}_i N_i)]^p$$

under the constraint of

$$0 = A_{a_i} \mathbf{X}_i + \mathbf{X}_i A_{a_i}^T + \gamma^{-2} \mathbf{X}_i N \mathbf{X}_i + D_{a_i} D_{a_i}^T \tag{25}$$

for every i. For this optimization problem, a Hamiltonian function can be expressed as :

$$L = \mathbf{J} + \sum_{i=1}^{\ell} tr\{ \mathbf{G}_i (A_{a_i} \mathbf{X}_i + \mathbf{X}_i A_{a_i}^T + \gamma^{-2} \mathbf{X}_i N \mathbf{X}_i + D_{a_i} D_{a_i}^T) \tag{26}$$

where $\mathbf{G}_i$ is a Lagrangian multiplier matrix. This Hamiltonian function leads to the following necessary conditions :

$$0 = \frac{dL}{d\,\mathbf{G}_i} = A_{a_i} \mathbf{X}_i + \mathbf{X}_i A_{a_i}^T + \gamma^{-2} \mathbf{X}_i N_i \mathbf{X}_i + D_{a_i} D_{a_i}^T \tag{27}$$

$$0 = \frac{dL}{d\,\mathbf{X}_i} = A_{a_i}^T \mathbf{G}_i + \mathbf{G}_i A_{a_i} + \gamma^{-2} \mathbf{G}_i \mathbf{X}_i N_i + \gamma^{-2} N_i^T \mathbf{X}_i \mathbf{G}_i + \frac{\partial [P_i tr(\mathbf{X}_i N_i)]^p}{\partial \mathbf{X}_i} \tag{28}$$

$$0 = \frac{dL}{dK} = \sum_{i=1}^{\ell} \frac{\partial}{\partial K}[tr\{ \mathbf{G}_i (A_{a_i} \mathbf{X}_i + \mathbf{X}_i A_{a_i}^T + \gamma^{-2} \mathbf{X}_i N \mathbf{X}_i + D_{a_i} D_{a_i}^T)\}] \tag{29}$$

Let the matrices $\mathbf{G}_i$ and $\mathbf{X}_i$ be partitioned as follows :

$$\mathbf{G}_i \equiv \begin{bmatrix} \mathbf{G}_{11_i} & \mathbf{G}_{12_i} \\ \mathbf{G}_{12_i}^T & \mathbf{G}_{22_i} \end{bmatrix} \quad \text{and} \quad \mathbf{X}_i \equiv \begin{bmatrix} \mathbf{X}_{11_i} & \mathbf{X}_{12_i} \\ \mathbf{X}_{12_i}^T & \mathbf{X}_{22_i} \end{bmatrix} \tag{30}$$

Substitution of Eq.(30) into Eq.(29) and some mathematical manipulations lead to :

$$\frac{\partial L}{\partial K} = 2\sum_{i=1}^{l}[\, G_{12,i}^{T}(X_{11,i}C_i^T - X_{12,i}C_o^T) + G_{22,i}(X_{12,i}^{T}C_i^T - X_{22,i}C_o^T) + G_{22,i}KR_i].$$

If $\sum_{i=1}^{l} G_{22,i}$ is invertible and the measurement error variance is same for all i (i.e. $R_i = R$ for every i), then K can be expressed explicitly as

$$K = -(\sum_{i=1}^{l} G_{22,i})^{-1}\sum_{i=1}^{l}[\, G_{12,i}^{T}(X_{11,i}C_i^T - X_{12,i}C_o^T) + G_{22,i}(X_{12,i}^{T}C_i^T - X_{22,i}C_o^T)]R^{-1}$$

The matrices G_i and X_i are obtained from Eq.(27) and Eq.(28). The equation (28) shows that G_i is dependent on X_i unless γ is infinitely large. Note that if the H_∞ constraint is relaxed, i.e., $\gamma \rightarrow \infty$, then this estimator becomes the *MpN* estimator introduced in [3]. If p=1, ℓ =1, $A = A_o$, $B = B_o$, $C = C_o$, and $H = H_o$, then this estimator becomes the steady state Kalman filter with an H_∞ bound which is described in ref. [3].

Numerical Computations

The equation (27) has the form of Riccati equation even though the third term has a positive sign instead of a negative sign and the N_i matrix may not be semi-positive definite. This non-standard Riccati equation can be solved using the Schur method [5].

The equation (28) can be written as :

$$0 = (A_{a_i} + \gamma^{-2} X_i N_i)^T G_i + G_i (A_{a_i} + \gamma^{-2} X_i N_i) + \frac{\partial [P_i\, tr(X_i N_i)]^p}{\partial X_i}. \tag{31}$$

This is a Lyapunov type equation. If X_i is solved from Eq.(27), $(A_{a_i} + \gamma^{-2} X_i N_i)$ and $\frac{\partial [P_i\, tr(X_i N_i)]^p}{\partial X_i}$ become the known fixed terms, and G_i can be solved from Eq.(31).

An iterative procedure to compute the optimal values for the estimator gains K is briefly described here:

1. Choose an initial K and an initial γ. (The initial γ should be reasonably large.)
2. Calculate X_{11_i}, X_{12_i}, and X_{21_i} from Eq.(27) for i = 1 to k.
3. Calculate G_{12_i} and G_{22_i} from Eq.(28) for i = 1 to k.
4. Calculate a new value for K and name this K_{new}.
5. Let $K_{n+1} = aK_{new} + (1-a)K_n$ where $0 < a < 1$.
6. Compute $J(K_{n+1})$.
7. Go to (2) unless $\Delta J \equiv | J(K_n) - J(K_{n+1})|$ is small enough.
8. If ΔJ is small enough, then reduce the value of γ
9. Go to (2) and repeat the above process until either a desirable value of γ is achieved or no further decreasing is possible.

Application : Tracking a Fixed Wing Aircraft

The MpN/H_∞ estimator was applied to the longitudinal motion tracking of a T-38 aircraft.

The 5th order state equations that describe the longitudinal motion of a target aircraft was used [6]. The state vector of this linear equation has the following elements : the forward velocity u, the vertical velocity w, the pitch rate q, the pitch angle θ, and the distance along the vertical inertial axis z. The system input is an elevation deflection angle δ_e. In this longitudinal motion tracking example, the vertical position(z) of target aircraft is to be estimated while the available measurement is assumed to be also its vertical position which is corrupted by the measurement noises.

As a tracker model, a simple $\alpha-\beta$ filter was used. An $\alpha-\beta$ filter is a typical tracking filter with only two states, the position and velocity of the target, and the acceleration is considered as a white noise. Normally three $\alpha-\beta$ filters are used in tracking the aircraft, one each direction of an inertial rectangular coordinate system. Only one $\alpha-\beta$ is needed here since the target aircraft is assumed to be solely in the vertical plane in this example. The $\alpha-\beta$ filter is described as follows [7] :

$$\begin{bmatrix} \dot{z} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w$$

and

$$C_o = H_o = [1 \quad 0].$$

The $MpN/H\infty$ estimator was designed to be robust to the variation of C_{m_α} of a target aircraft and the unknown inputs to a target aircraft. The C_{m_α} parameter is a stability derivative that determines the static stability of aircraft longitudinal motion. The nominal value of the stability derivative C_{m_α} for T-38 aircraft is -1.0. Th value of C_{m_α} was varied from -1.3 to -0.1, which is the typical range for military aircraft.

The $MpN/H\infty$ estimators are designed with p=1 and p=20 and with $\gamma = 10^8$ and γ=13 and their estimation error variances are plotted against C_{m_α} in Figure 1. The $MpN/H\infty$ estimator designed with p=1 and $\gamma = 10^8$ is the same as the Minisum Estimator [7] and the performance of the $MpN/H\infty$ estimator designed with p=20 and $\gamma = 10^8$ would be very close to that of the Minimax Estimator [8]. The results with very large γ show the inherent properties of the MpN estimator : as p increases, the maximum estimation error variances decreases and the sensitivity to the C_{m_α} variation reduces, but the average performance degrades. With γ =13, which guarantees the most robustness against the unknown inputs, the effect of p is small and the sensitivity to the C_{m_α} variation is small. However, the estimation error variances are large for every values of C_{m_α}.

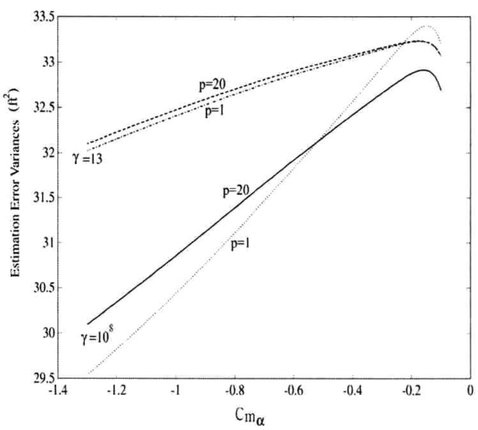

Fig. 1. The Estimation Error Variances for C_{m_α} Variation

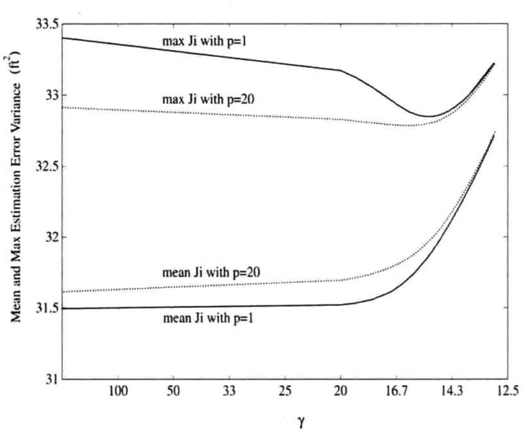

Fig. 2. The Mean and Maximum Estimation Error Variances for γ variation

The mean estimation error variance(mean J_i) and maximum estimation error variance (max J_i) for C_{m_α} variation are plotted against γ in Figure 2. For γ larger than 20, as γ increases the mean estimation error variance increases slightly and maximum estimation error variance decreases slightly. The maximum estimation error variance with p=20 is already small at γ= 10^8, thus, its decrement is negligibly small. The maximum estimation error variance with p=1 decreases noticeably as γ decreases from 20 to 15.5. For γ smaller than 15, both the mean and maximum estimation error variances increase abruptly. Fig. 1 and Fig. 2 show that if we use too small γ to achieve as much as robustness to unknown inputs as possible, the robustness to real parameter variations is sacrificed. Therefore, a trade-off is necessary between the robustness to real parameter variations and the robustness to unknown inputs. The value of γ should be determined by the designer based on the degree of parameter variations and the uncertainty of the unknown inputs that is expected in the tracking scenario.

Conclusions

A new design method for estimators that have the robust performance in the presence of real parameter variations and the unknown inputs was proposed. This method calculates the optimal estimator gains satisfying the H_∞ constraint while minimizing the upper bound of the Mini-p-Norm cost function using an iterative algorithm. It has two design parameters, p and γ. The parameter p has the role of trading off between small estimator error variance and low sensitivity to unknown parameter variations. The parameter γ has the role of trading off between the robustness to real parameter uncertainties and the robustness to unknown inputs. As γ decreases, this new estimator, named as "MpN/H_∞" estimator, becomes less sensitive to the unknown inputs, but the cost function which accounts for parameter variations could increase. If γ approaches infinity, the estimator satisfying the new objective reduces to the *MpN* estimator.

The MpN/H_∞ estimator was applied to the longitudinal motion tracking of a T-38 aircraft. This example shows that if too small γ is used, the robustness to real parameter variation is sacrificed. Therefore a trade-off is required between the robustness to the parameter uncertainties and the robustness to unknown inputs by choosing proper value for γ.

References

1. Kim, E.T. and Andrisani, D., "Design of An Robust Estimator for Target Tracking", AIAA Journal of Guidance, Control and Dynamics, Vol. 16, No. 3, 1992, pp. 595-597.

2. Kim, Eung T., Robust Estimators for Target Tracking, Ph.D. Thesis, Purdue University, Dec., 1991.

3. Bernstein, D.S. and Haddad, W.M., ''Steady-State Kalman Filtering with an H_∞ Error Bound'', System & Control Letters 12, 1989, pp. 9-16.

4. Nagpal, K.N. and Khargonekar, P.P., ''Filtering and Smoothing in an H_∞ Setting'', Proc. of the 28th IEEE Conference on Decision and Control, Tampa, Florida, 1989.

5. Laub, A.J., ''A Schur Method for Solving Algebraic Riccati Equations'', IEEE Trans. Automat. Control, Vol. 24, No. 6, December 1979.

6. McRuer, D., Ashkenas, I. and Graham, D., Aircraft Dynamics and Automatic Control, Princeton University Press, 1972.

7. Singer, R.A., ''Estimating Optimal Tracking Filter Performance for Manned Maneuvering Targets,'' IEEE Trans. on Aerospace and Electronic Systems, Vol. Aes-6, No.4, July 1970.

8. Vinkler, A. and Wood. L.J., ''A Comparison of Several Techniques for Designing Controllers of Uncertain Dynmic Systems, '' Proceedings of the Conference on Decision and Control, 1978, pp. 31-38.

9. Salmon, D.M., ''Minimax Controller Design'', IEEE Trans. on Automatic Control, Vol. AC-13, No.4, August 1968.