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Research Advances on Tension Buckling Behaviour of Aerospace Structures: A Review

Prosun Kumar Datta* and Sauvik Biswas**

Department of Aerospace Engineering, Indian Institute of Technology, Kharagpur, 721302, India

Abstract

This paper reviews most of the research done in the field of tensile buckling characteristics pertaining to aerospace structural elements with special attention to local buckling and parametric excitation due to periodic loading on plate and shell elements. The concepts of buckling in aerospace structures appear as the result of the application of a global compressive applied load or shear load. A less usual situation is the case, in which a global tensile stress creates buckling instability and the formation of complex spatial buckling pattern. In contrast to the case of a pure compression or shear load, here the applied macroscopic load has no compressive component and is thus globally stabilizing. The instability stems from a local compressive stress induced by the presence of a defect, such as a crack or a hole, due to partial or non-uniform applied load at the far end. This is referred to as tensile buckling. This paper discusses all aspects of tensile buckling, theoretical and experimental. Its far reaching applications causing local instability in aerospace structural components are discussed. The important effects on dynamic stability behaviour under locally induced periodic compression have been identified and influences of various parameters are discussed. Experimental results on simple and combination resonance characteristics on plate structures due to tensile buckling effects are elaborated.

Key words: Parametric instability, Vibration, Curved panels, Nonlinear behaviour, Tensile buckling

1. Introduction

Literally hundreds of papers dealing with the buckling of plates may be found in published literature. The majority of these assumed that the loading is uniform compressive stress, either uniaxial or biaxial, and various types of edge constraints or other features (e.g., orthotropy) are considered. In all these cases, buckling appears as the result of the application of global compressive applied load. A significant number of papers also treat uniform shear stress or linearly varying normal stress (e.g., pure in-plane bending moment). Here buckling occurs since one of the principal stresses is compressive in nature. The difficult problem of buckling of a rectangular plate subjected to in plane concentrated compressive forces has also been addressed, although significant disagreement in buckling loads is found. However, plates can also buckle when subjected to forces that are only tensile. Perhaps the simplest example is what happens when one takes a sheet of paper (i.e., plate having very small bending stiffness) in both hands holding each end between thumb and forefinger, and pulls on it. One observes the sheet wrinkling in the direction transverse to the loading. If the applied tensile forces are more distributed, but still non-uniform, the wrinkling (i.e., buckling) load will be higher. Indeed, such wrinkling can occur whenever the solution to the plane elasticity problem yields internal stresses that are compressive in any direction at any point. Induced wrinkling may be important in certain practical applications such as stressed panels in aircraft or satellite structures. In the above discussions, the overall effect is globally stabilizing with respect to buckling instability. The instability stems from local compressive stresses induced. The buckling instability may also appear from local compressive stresses induced in the presence of a defect, such as hole or crack. It is known

* Professor, Corresponding author
Email: prosundatta@gmail.com
** Graduate student

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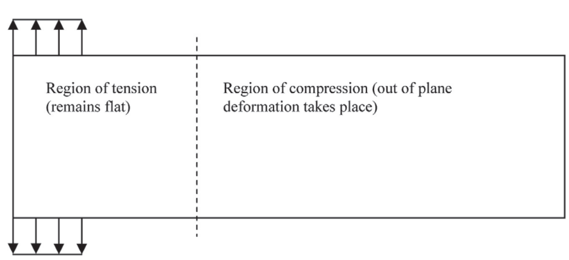
(Dixon, 1960; Dixon and Strannigan, 1969; Inglis, 1913) but not well recognized that, in pure uniaxial tensile applied stress, the second principal stress component (tangential or hoop stress) is negative in four domains around hole or crack. Therefore buckling undulations can appear if this stress is sufficiently compressive. Compression effects due to tension are shown in Figs. 1a and b.

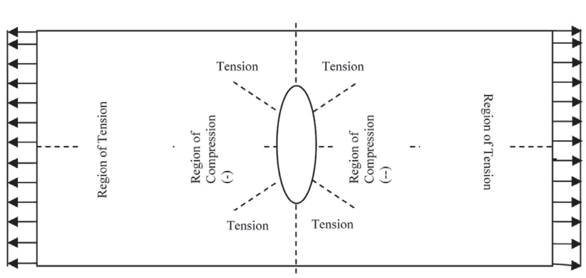
The early solutions for the stresses in tensile sheets with holes obtained by Kirsch (1898) clearly exhibited the region of compressive stresses. It is the existence of region of compressive stresses that can cause out-of plane deflection or local buckling, in the vicinity of cracks or holes. Later, Dixon (1960) gave a solution for the elastic stresses around a crack in the infinite plate under tension. Cherepanov (1963) studied the buckling behaviour of a membrane containing holes under uniaxial tension. Brock et al. (1968) made an investigation of the buckling of tensioned panel containing a circular hole. Carlson et al. (1970) made an experimental study on the buckling of thin cracked sheets under tension and suggested an empirical formula for tensile buckling stress depending on the plate parameters. The empirical formula for the buckling stress is expressed as

$$\sigma_b = \kappa E(t/L)^2 \quad (1)$$

where σ_b is the applied traction necessary to cause buckling, E is the Young's modulus, t is the sheet thickness and L is the crack length. The value of k appears to depend on material properties, the modal geometry (specimen width, aspect ratio, etc.) and very likely in the manner in which critical stress is defined. This empirical formula is useful for estimating buckling stresses for thin cracked sheets in practice for design purpose. Zielsdorff and Carlson (1972a, b) presented results on the buckling in cracked thin sheets and out of plane deflection characteristics around the crack. Later, some other investigators (Larsson, 1989; Markstrom and Stoakers, 1980; Sih and Lee, 1986) studied the buckling behaviour of cracked plates.



(a)



(b)

Fig. 1. Combination of tension-compression zone (a) non-uniform edge load, (b) presence of hole.

Clarkson (1965) studied the propagation of fatigue cracks in tensioned plate subjected to acoustic loads. Petyt (1968) made an analytical solution on the vibration characteristics of a tensioned plate containing fatigue crack using finite element approach and compared his results with experimental study. Datta and Carlson (1973) presented the results of an analytical and experimental study of both buckling and vibration behaviour of a thin sheet with an elliptical hole in a tensile field. The analytical methods used in stress distribution around the opening were based on the elasticity solution presented in Timoshenko and Goodier (1951). Datta (1976) made an investigation of the study of the static and dynamic behaviour of tensioned sheet with rectangular opening. The experimental results have been verified with the empirical formula (Eq. 1) for the limiting case. Datta (1982) studied static stability characteristics of a thin plate with eccentric loading.

Datta and White (1979, 1980) performed extensive experimental study on the acoustic excitation behaviour of aluminium alloy panel having a central opening under biaxial tension, using the Acoustic tunnel facility of Institute of Sound and Vibration Research, University of Southampton. It was observed that the response phenomenon is highly nonlinear. Datta (1980, 1981) made an extensive study on the nonlinear vibration behaviour of fuselage panel with a central opening, subjected to tension loading under acoustic excitation. It was observed that the response behaviour was governed by a combination of hardening-softening spring type nonlinear characteristics near the tensile buckling stress.

In the investigations mentioned above, only the buckling of the 'lips' of crack was demonstrated, because of the relatively large thickness (1 mm above) of the plate used. Very recently Gilabert et al. (1992) presented an experimental work in which an applied tensile stress creates a complex spatial buckling pattern resembling a "Maltese cross" around a defect such as a hole or a crack in a thin plate. The orientations of the folds of buckling pattern were shown to follow closely the isostatics calculated for the unbuckled state. A beautiful buckling pattern, reminiscent of a maltese cross along four branches

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around the crack, have been presented by the authors in their paper. The wave vector of the buckling undulation is parallel to the direction of compressive stresses and thus perpendicular to the applied tensile stress since the induced compressive stress is approximately perpendicular to the global applied tensile stress. Shimizu and his co-authors (Shimizu and Yoshida, 1991; Shimizu et al., 1991) have investigated the strength of a plate with hole under tensile loading using finite element approach. The above works mainly concentrated on the overall strength characteristics of the plate rather than the local instability phenomenon.

1.1 Vibration and dynamic instability

The vibration behaviour for tension buckling problems is very interesting. It depends on the in-plane stress distribution of the panel and it is a combined effect of tension-compression zone. Initially, with the application of tensile load, the tensile effect is dominant over the compression and the frequency of excitation increases with the applied load and attains a maximum value. As the load is further increased, the compression effect starts overshooting the tension zone and the frequency of the panel starts decreasing reaching upto a minimum value. This is the prebuckling zone. When the frequency attains the minimum value, buckling occurs locally. This is the concept of local buckling, termed as tension buckling, an important concept of aerospace structural analysis. As the applied tensile load is further increased, frequency increases which correspond to the postbuckling phenomenon in the light of structural instability. The above mentioned phenomenon has been discussed in the experimental works by Datta (Datta, 1976; Datta and Carlson, 1973).

The concept of parametric instability under in-plane pulsating compressive loading has been discussed by Bolotin (1964), leading to the subject of extensive research investigations. Structural elements subjected to in-plane periodic forces may undergo unstable transverse vibrations, leading to parametric resonance, due to certain combinations of the values of load parameters and natural frequency of transverse vibration. Since the excitations when they are time dependent appear as parameters in the governing equations, these excitations are called parametric excitations. This instability may occur below the critical load of the structure under compressive loads over a range or ranges of excitation frequencies. Several means of combating parametric resonance, such as damping and vibration isolation, may be inadequate and sometimes dangerous with reverse results. A number of catastrophic incidents can be traced to parametric resonance. In contrast to the principal resonance, the parametric instability may arise not merely at single excitation frequency, but even for small excitation amplitudes and combinations of frequencies. Thus the dynamic stability characteristics are of great technical importance for understanding the dynamic systems under periodic loads. In structural mechanics dynamic stability has received considerable attention over the years and encompasses many classes of problems. The distinction between "good" and "bad" vibration regimes of a structure subjected to in-plane periodic loading can be distinguished through a simple analysis of dynamic instability region (DIR) spectra.

Oscillating tensile in-plane load at the far end causing parametric instability effects around the free edge of the cutout in a plate/panel or in a partially loaded plate/panel under tension is an interesting phenomenon in structural instability. The study of dynamic instability behaviour of plates with opening is relatively new and the available works are mostly experimental in nature. Backer (1972) has studied the instability behaviour of a plate with circular hole subjected to pulsating load and the simple resonance zones were presented. Carlson (1974) has given experimental study of the parametric excitation of a tensioned sheet with crack like opening. Datta (1978) investigated experimentally the dynamic instability phenomena of tensioned sheet with a central opening. Datta (1983) later studied the parametric instability of tensioned plates with central opening and edge slot. Tani and Nakamura (1978) studied theoretically the dynamic stability of clamped multiply connected annular plates using Galerkin procedure. It was found that principle resonance was of most practical importance, but that of combination resonance cannot be neglected for the application of in-plane loading.

There was a gap in the research activities on tension buckling for sometimes, though the concept was well introduced long back, demonstrating its importance. The

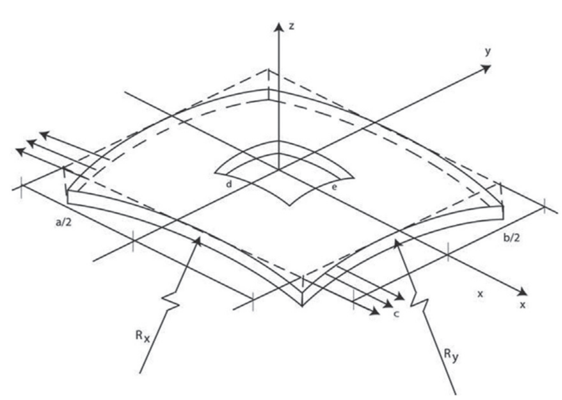

Fig. 2. Geometry of laminated composite doubly curved panel with cutout under periodic load.

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present author of this review paper along with his co-workers later made a very systematic approach to study the concept of tension buckling associated with vibration and dynamic instability phenomenon, both theoretical and experimental. The results have appeared in several papers which will be reviewed here with brief theory and results.

2. General Theories Involving Tension Buckling, Vibration and Dynamic Instability

The basic configuration of the problem to discuss 'tension buckling' is a laminated composite, doubly curved panel with cutout subjected to in-plane partial edge loading as shown in Fig. 2. In the figure, R_x, R_y and R_{xy} identify the radii of curvatures in the X and Y directions and the radius of twist respectively. a/b is the plate aspect ratio and d/e is the general representation of hole ratio. Any hole geometry–circular, elliptic, rectangular can be considered. Removing the hole provides the uniform panel. c/b is the load width parameter. Different non-uniform tensile loading is shown in Fig. 3.

The choice of the laminated doubly curved panel geometry has been made as a basic configuration so that, depending on the value of curvature parameter, plate, cylindrical panels and different doubly curved panels including twist, can be considered as special cases. Specific problems can be explained from the general theory by proper choice of geometry, load, material and other parameters. Isotropic problem as a special case is considered.

2.1 Governing equations

The governing differential equations, given by Bert and Birman (1988) for dynamic stability of orthotropic cylindrical shells, modified for the static, free vibration and parametric excitation of laminated composite, shear deformable, doubly curved panels, can be expressed as

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}-\frac{1}{2}C_2\left(\frac{1}{R_y}-\frac{1}{R_x}\right)\frac{\partial M_{xy}}{\partial y}+C_1\frac{Q_x}{R_x}+C_1\frac{Q_y}{R_{xy}} = \rho h\frac{\partial^2 u}{\partial t^2}$$

$$\frac{N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}+-\frac{1}{2}C_2\left(\frac{1}{R_y}-\frac{1}{R_x}\right)\frac{\partial M_{xy}}{\partial x}+C_1\frac{Q_y}{R_y}+C_1\frac{Q_x}{R_{xy}} = \rho h\frac{\partial^2 v}{\partial t^2} \quad (2)$$

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}-\frac{N_x}{R_x}-\frac{N_y}{R_y}-2\frac{N_{xy}}{R_{xy}}+N_x^0\frac{\partial^2 w}{\partial x^2}+N_y^0\frac{\partial^2 w}{\partial y^2} = \rho h\frac{\partial^2 w}{\partial t^2}$$

$$\frac{\partial M_x}{\partial x}+\frac{\partial M_{xy}}{\partial y}-Q_x = \frac{\rho h^3}{12}\frac{\partial^2\theta_x}{\partial t^2}$$

$$\frac{\partial M_{xy}}{\partial x}+\frac{\partial M_y}{\partial y}-Q_y = \frac{\rho h^3}{12}\frac{\partial^2\theta_y}{\partial t^2}$$

where N_x^0 and N_y^0 are the external loading in X and Y directions respectively. C_1 and C_2 are tracers by which the analysis can be reduced to that of Sanders', Love's and Donnel's theories. If the tracer coefficients C_1=C_2=1, the equation corresponds to the generalization of Sanders' first approximation theory. The case C_1=1, C_2=0 correspond to Love's theories of thin shells generalized to include shear deformations. Finally, if C_1=C_2=0, the equation is reduced

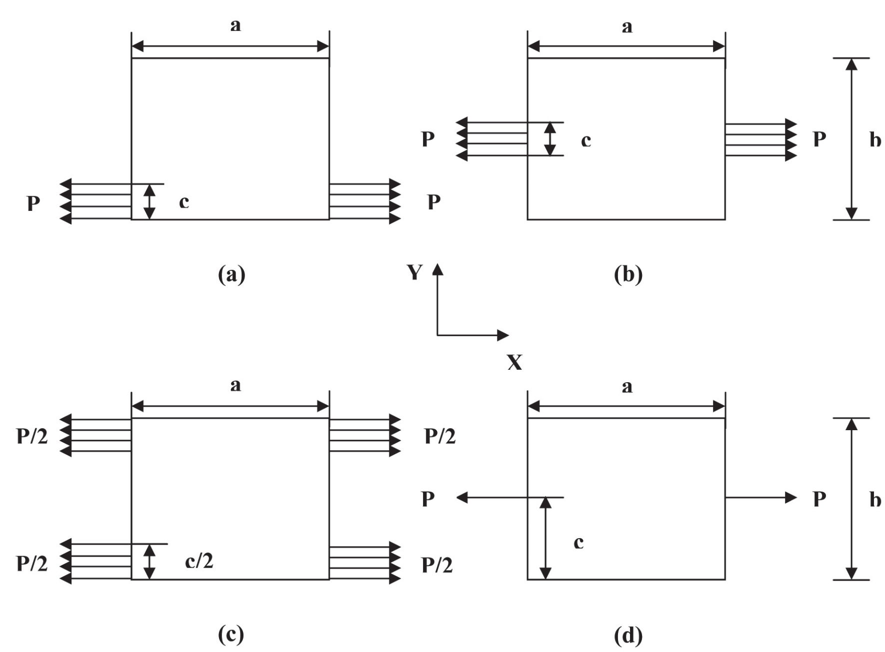

Fig. 3. Description of the problem: (a) partial edge loading at one end (b) partial edge loading at the centre (c) partial edge loading at both ends (d) concentrated edge loading.

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to a shear deformable version of Donnell's theories. The equation of motion can be expressed in matrix form as

$$[M]\{\ddot{q}\}+[[K_e]-P(t)[K_g]]\{q\}=0 \tag{3}$$

The in-plane load $P(t)$ is periodic and can be expressed in the form:

$$P(t)=P_s+P_t\cos\Omega t \tag{4}$$

where P_s is the static portion of load. P_t is the amplitude and Ω is the frequency of the dynamic portion of P_t. The static buckling load P_{cr} is the measure of the magnitude of P_s and P_t.

$$P_s=\alpha P_{cr},\ P_t=\beta P_{cr} \tag{5}$$

where α and β are termed as static and dyamic load factors respectively. Using Eqs. (3-5) the equation of motion is obtained as:

$$[M]\{\ddot{q}\}+[[K_e]-\alpha P_{cr}[K_g]-\beta P_{cr}[K_g]\cos\Omega t]\{q\}=0 \tag{6}$$

Eq. (6) represents a system of second order differential equations with periodic coefficients of the Mathieu-Hill type.

The boundaries of the DIRs are formed by the periodic solutions of period T and $2T$, where $T=\frac{2\pi}{\omega}$. The boundaries of the primary instability regions with period $2T$ are of practical importance (Bolotin, 1964) and the solution can be achieved in the form of the trigonometric series

$$q(t)=\sum_{k=1,3,5,..}^{\infty}\left[\{a_k\}\sin\left(\frac{k\Omega t}{2}\right)+\{b_k\}\cos\left(\frac{k\Omega t}{2}\right)\right] \tag{7}$$

Putting this in Eq. (6) and if only first term of the series is considered, equating coefficients of $\sin\left(\frac{\Omega t}{2}\right)$ and $\cos\left(\frac{\Omega t}{2}\right)$, the Eq. (6) reduces to

$$[[K_e]-\alpha P_{cr}[K_g]\pm\frac{1}{2}\beta P_{cr}[K_g]-\frac{\Omega^2}{4}[M]]\{q\}=0 \tag{8}$$

Eq. (8) represents an eigenvalue problem for known values of α, β and P_{cr}. The two conditions under a plus and minus sign correspond to two boundaries of the DIR. The eigenvalues are Ω, which give the boundary frequencies of the instability regions for given values of α and β.

The theory presented so far are general formulation for instability phenomena and is applied for plate, curved panels depicting tensile buckling effects. The eight-node curved isoparametric quadratic element is employed in the analysis with five degrees of freedom - u, v, w, θ_x and θ_y per node. First order shear deformation theory is used and the shear correction coefficient has been employed to account for the non-linear distribution of the shear strains through the thickness. The displacement field assumes that mid plane normal remains straight but not necessarily normal after deformation, so that

$$\begin{aligned}\bar{u}(x,y,z)&=u(x,y)+z\theta_x(x,y)\\ \bar{v}(x,y,z)&=v(x,y)+z\theta_y(x,y)\\ \bar{w}(x,y,z)&=w(x,y)\end{aligned} \tag{9}$$

where θ_x and θ_y are the rotations of the mid-surface. Also $\bar{u}$, $\bar{v}$, $\bar{w}$ and u, v, w are the displacement components in the x, y, z directions at any section and at mid-surface respectively. The constitutive relationships for the shell are given by

$$\{F\}=[D]\{\varepsilon\} \tag{10}$$

where

$$F=\left[N_x,N_y,N_{xy},M_x,M_y,M_{xy},Q_x,Q_y\right] \tag{11}$$

Expressions for $[D]$ for isotropic materials can be found in Bert and Birman (1988) and Reddy and Phan (1985).

For composite material, the basic doubly curved laminated shell is considered to be composed of composite material laminate (typically thin layers). The material of each lamina consists of parallel continuous fibres embedded in a matrix material. Each layer may be regarded as on a microscopic scale as being homogenous and orthotropic. The constitutive relation Eq. (10) for a laminated shell element is expressed as

$$\begin{Bmatrix}N_i\\M_i\\Q_i\end{Bmatrix}=\begin{bmatrix}A_{ij}....B_{ij}....0\\B_{ij}....D_{ij}....0\\0......0....S_{ij}\end{bmatrix}\begin{Bmatrix}\varepsilon_j\\k_j\\\gamma_m\end{Bmatrix} \tag{12}$$

The extensional, bending-stretching coupling and bending stiffnesses are expressed as

$$\left(A_{ij},B_{ij},D_{ij}\right)=\sum_{k=1}^{n}\int_{Z_{k-1}}^{Z_k}\left(\overline{Q_{ij}}\right)_k\left(1,z,z^2\right)\mathrm{d}z \qquad i,j=1,2,6 \tag{13}$$

The transverse shear stiffness is expressed as

$$\left(S_{ij}\right)=\sum_{k=1}^{n}\int_{Z_{k-1}}^{Z_k}\kappa\left(\overline{Q_{ij}}\right)_k dz \qquad i,j=1,2,6 \tag{14}$$

Where k is the transverse shear correction factor and $\overline{Q_{ij}}$ terms are the conventional off axis stiffness values, which depends on the material constants, and ply orientations.

Green Lagrange's strain displacement relations are presented in general throughout the structural analysis. The linear part of the strain is used to derive the elastic stiffness matrix and the nonlinear part of the strain is used to derive

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the geometric stiffness matrix. The total strain is given by

$$\{\varepsilon\} = \{\varepsilon_{l}\} + \{\varepsilon_{nl}\} \tag{15}$$

The linear strain displacement relations are

$$\begin{aligned} \varepsilon_{xl} &= \frac{\partial u}{\partial x} + \frac{w}{R_x} + zk_x \\ \varepsilon_{yl} &= \frac{\partial v}{\partial y} + \frac{w}{R_y} + zk_y \\ \gamma_{xyl} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{2w}{R_{xy}} + zk_{xy} \\ \gamma_{xzl} &= \frac{\partial w}{\partial x} + \theta_x \\ \gamma_{yzl} &= \frac{\partial w}{\partial y} + \theta_y \end{aligned} \tag{16}$$

where the bending strains are expressed as,

$$k_x = \frac{\partial \theta_x}{\partial x}, \quad k_y = \frac{\partial \theta_y}{\partial y}, \quad k_{xy} = \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \tag{17}$$

The element geometric stiffness matrix for doubly curved panel is derived using the non-linear strain component as

$$\begin{aligned} \varepsilon_{xnl} &= \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial x} - \frac{u}{R_x}\right)^2 + \frac{1}{2}z^2\left[\left(\frac{\partial \theta_x}{\partial x}\right)^2 + \left(\frac{\partial \theta_y}{\partial x}\right)^2\right] \\ \varepsilon_{ynl} &= \frac{1}{2}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{v}{R_y}\right)^2 + \frac{1}{2}z^2\left[\left(\frac{\partial \theta_x}{\partial y}\right)^2 + \left(\frac{\partial \theta_y}{\partial y}\right)^2\right] \\ \gamma_{xynl} &= \frac{\partial u}{\partial x}\left(\frac{\partial u}{\partial y}\right) + \frac{\partial v}{\partial x}\left(\frac{\partial v}{\partial y}\right) + \left(\frac{\partial w}{\partial x} - \frac{u}{R_x}\right)\left(\frac{\partial w}{\partial y} - \frac{v}{R_y}\right) \\ &\quad + z^2\left[\left(\frac{\partial \theta_x}{\partial x}\right)\left(\frac{\partial \theta_x}{\partial y}\right) + \left(\frac{\partial \theta_y}{\partial x}\right)\left(\frac{\partial \theta_y}{\partial y}\right)\right] \end{aligned} \tag{18}$$

The element matrices are derived as

Elastic stiffness matrix –

$$[k_e]_e = \int [B]^T [D][B] \, dx \, dy \tag{19}$$

Geometric stiffness matrix –

$$[k_g]_e = \int [G]^T [S][G] \, dx \, dy \tag{20}$$

Consistent mass matrix –

$$[m]_e = \int [N]^T [I][N] \, dx \, dy \tag{21}$$

The overall matrices, $[K_e]$, $[K_g]$ and $[M]$ are obtained by assembling the corresponding element matrices.

The panels of various geometries can be considered as follows: spherical (R_y/R_x=1), cylindrical (R_y/R_x=0) and hyperbolic paraboloid (R_y/R_x=−1). With ratios a/R_x and b/R_y as (0.2, 0.2), (0.0, 0.2) and (-0.2, 0.2) respectively for shallow panel, b/h=100 for thin shell analysis.

Eq. (3) can be reduced to the governing equations for buckling and vibration problems as follows:

Buckling problem: $\{\ddot{q}\}$=0

$$[K_e]\{q\} \pm P_{cr}[K_g]\{q\} = 0 \tag{22}$$

Vibration problem: Assuming that the panel with applied load vibrates harmonically with angular frequency, ω

$$-\omega^2[M]\{q\} + [[K_e] \pm P[K_g]]\{q\} = 0 \tag{23}$$

Both Eqs. (22) and (23) represent the eigenvalue problems. The eigenvalues are the critical buckling loads, P_{cr}, and the square of the natural frequency, ω^2, for a known load, P, respectively. Eigenvectors, $\{q\}$, correspond to the mode shapes of buckling or vibration.

Dynamic Instability: This is being analyzed when all the terms in Eq. (3) are present and follows the analysis as per the Eqs. (4) to (8). The onset of instability and DIRs are found for different panel and load parameters. The dynamic stability behaviour that are usually discussed involves determination of simple resonance zones, where driving frequency for resonance is related to only one natural frequency. The boundaries between stable and unstable regions are formed by periodic solutions of period T and $2T$ of the Mathieu-Hill equation describing its solution (Bolotin, 1964), where $T=\frac{2\pi}{\Omega}$, Ω being the frequency of excitation. The onset of principal instability regions is given by $\Omega=\frac{2\omega}{k}$, where ω is the frequency of free vibration and k=1, 2, 3, …

In many practical cases involving tension buckling instability, the driving frequency of excitation may be related to two or more natural frequencies, known as combination resonance, expressed as $\Omega \approx \omega_m \pm \omega_n$; sum and difference type (Nayfeh and Mook, 1979). The existence of the combination resonance phenomenon is well established in dynamic instability studies. Bolotin's method (1964) is very common for obtaining primary instability regions (simple resonance zones), but it cannot be used for solving combination resonance problems. The boundaries of the simple and combination parametric resonance zones are obtained by using the method of multiple scales (MMS) given by Nayfeh and Mook (1979) on the modified modal equation,

$$\Omega \approx \omega_m \pm \omega_n \tag{24}$$

where, terminologies are explained in Udar and Datta (2007).

When the frequency of the excitation is close to the sum or the difference of two natural frequencies of the system, a combination resonance of the summed type or the difference

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type exists between various modes. The nearness of Ω to $\omega_m \pm \omega_n$ is expressed by introducing a detuning parameter, σ, such that

$$\{\ddot{\xi}\} + [\hat{C}]\{\dot{\xi}\} + [\Lambda]\{\xi\} + 2\varepsilon\cos\Omega t[\hat{K}]\{\xi\} = 0 \quad (25)$$

where σ is obtained from the solution of a quadratic equation

$$A\sigma^2 + B\sigma + C = 0 \quad (26)$$

The constant coefficients A, B and C lained in Udar and Datta (2007).

The two roots of Eq. (26) correspond to two boundaries of the DIR. The case $m=n$ gives the simple resonance zone and the case $m \neq n$ gives the combination resonance zone of the summed or difference type as explained by Choo and Kim (2000). The critical dynamic load factor, β^*, corresponds to the value of β for which the expression $B^2-4AC=0$. For values of β below β^*, Eq. (26) gives complex roots, which means dynamic instability cannot occur.

2.2 Computer program

A computer program has been developed using FORTRAN 77 to perform all the necessary computations. The element elastic stiffness matrices and mass matrices are obtained using standard procedure of assembly. The geometric stiffness matrix, $[K_g]$, is essentially a function of in-plane stress distribution in the element due to applied edge loadings. Since the stress field is non-uniform, plane stress analysis is carried out using finite element method to determine the stresses and these are used to formulate the geometric stiffness matrix. Element matrices are assembled to obtain the global matrices, using skyline technique. The subspace iteration method is adopted throughout to solve the eigenvalue problems.

3. Buckling, Vibration and Dynamic Stability Behaviour of Plate and Doubly Curved Panels Under Non-Uniform Tensile Loading

Datta and Deolasi (1993) studied on the buckling and vibration behaviour of rectangular plates, subjected to band loading at the centre. The study showed that tension buckling occurs due to compressive stresses produced in the middle region of the plate in transverse direction due to band tensile loading. The value of tensile buckling load depends on the strength and area of compressive zones. It is lowest under concentrated loading, increases rapidly as load bandwidth increases. For tensile loading of small bandwidth, natural frequencies initially rise with the load, but begin to fall down beyond certain value of load and become zero at buckling loads.

Deolasi et al. (1995) presented results on the static and dynamic behaviour of rectangular plates subjected to wide varieties of partial edge loading. The results show that tensile buckling occurs due to compressive stresses produced somewhere in the plate due to localized compression zones. Initial increase of natural frequencies and then its decreasing trend on the onset of tensile buckling could be observed. Stress singularities and stress concentration due to narrow patch loadings do not seem to have serious effects on the behaviour of the plates. It is seen from convergence studies that the use of finer mesh, which models the singularities better, does not show drastic changes in the results. A typical frequency plot is shown in Fig. 4 for partial edge loading at one end.

Prabhakara and Datta (1996a, b) investigated static and dynamic elastic behaviour of damaged plates subjected to non-uniform edge loading. The effect of a region of damage is introduced by the use of an idealized model having a reduction in the elastic property at the zone of damage. The study shows the variation in tensile buckling and vibration characteristics due to the presence of damage. The vibration behaviour shows that, for certain combination of load and damage parameters, the plate with lesser flexural rigidity yields higher frequencies compared to an undamaged plate.

Tension buckling and vibration behaviour of curved panels subjected to non-uniform in-plane edge loading were investigated by Ravi Kumar et al. (2002b). The theory

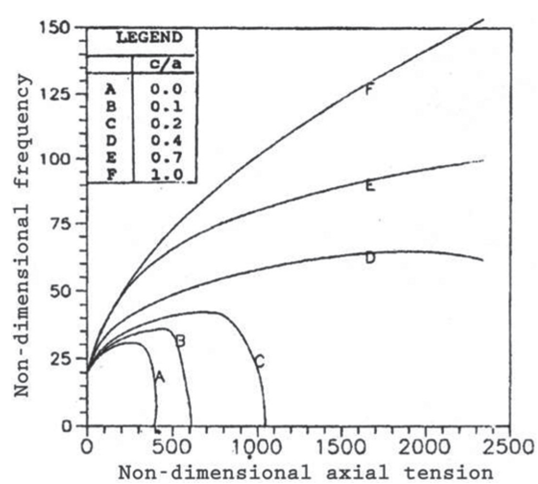

Fig. 4. Non-dimensional frequencies versus non-dimensional load for different values of *c/a* for partial edge loading at one end, for *a/b*=1.

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used is extended from the dynamic, shear deformable theory based on the Sandars' first approximation for doubly curved shells. It was observed that tensile buckling load, generally, increases with the increase in curvature of the panel. However, the buckling behaviour for certain curved panels is similar to that of the flat panels, depending on the edge conditions and aspect ratio of the panel. Initial increase and then decreasing behaviour of the frequency of the panel with applied edge tension could be observed as discussed earlier, to determine the onset of tension buckling. Effect of curvatures on the vibration behaviour was also shown. Ravi Kumar et al. (2002a) studied the tensile buckling and vibration behaviour of curved square panels subjected to combination of tension-tension, tension-compression and compression-compression biaxial edge loadings. It was seen that the value of tensile buckling load depends on the stiffness and area of compressive zones. It is the lowest under compression-compression with the load ratio being unity for all types of panels. Vibration and buckling behaviour of composite curved panels under biaxial non-uniform edge loading was studied by Ravi Kumar et al. (2003a). Vibration behaviour shows that natural frequency is varying with applied load and becomes zero at the respective values of buckling loads. The effects of ply orientation and variation of transverse load (P_r) significantly affect the buckling load.

Datta and Deolasi (1995) have studied the parametric resonance characteristics of rectangular plates under partial tensile loading at the edges, considering the effects of damping. Simple and combination resonance zones were obtained. It was observed that under certain conditions, combination resonance zones are very significant. The destabilizing effect of damping could be identified for certain load parameters in tensile buckling environment.

Deolasi and Datta (1995a) have studied the effects of damping on the dynamic instability behaviour of a plate subjected to localized tensile periodic edge loading. The effect of damping on the instability regions show that there is a critical value of the dynamic load factor beyond which damping makes the system unstable. The effect is more pronounced for a localized loading near the ends of the edge.

Parametric instability behaviour of rectangular plates under tensile periodic edge loading was investigated by Deolasi and Datta (1995b). The results show that the instability regions appear early for positions of the load near the corner of the edge. Simple and combination parametric resonance zone characteristics of rectangular plates subjected to non-uniform edge loading were studied by Deolasi and Datta (1997b). The results show that the damping exhibit destabilizing effect on the combination resonance phenomenon.

Ravi Kumar et al. (2003c) studied the tensile buckling, vibration and dynamic stability behaviour of multilaminated curved panels subjected to uniaxial in-plane point and patch tensile edge loadings. The results in general show that the introduction of curvature improves the instability behaviour of the panels. The tensile buckling value for angle-ply laminates is less than cross-ply laminates because of the effect of coupling in angle-ply panels.

Effect of aspect ratio and boundary conditions on the tensile buckling and vibration behaviour of curved panels was studied Ravi Kumar et al. (2004b, 2006). The results show that the tensile buckling characteristics are significantly influenced by the aspect ratio and boundary conditions, because of extensive non-uniform stress field developed.

Udar and Datta (2007) studied the dynamic analysis of parametrically excited composite curved panels under non-uniform edge loading. It was observed that the combination resonance zones of sum type contributes a considerable amount of instability region and the effects of damping are destabilizing for higher values of dynamic load factor.

Ravi Kumar et al. (2003b, 2005) has studied the dynamic instability behaviour of laminated composite plates subjected to partially distributed non-conservative follower forces. The in-plane stress distribution is a combination of tensile and compression zones. In certain domain, when the tensile zone dominates, it gives rise to stiffening effect. On the other hand, domination of compressive zone leads to de-stiffening behaviour. Their effects on divergence and flutter characteristics are discussed.

3.1 Conclusions

The results from the study of tensile buckling, vibration and dynamic stability behaviour of curved panels subjected to non-uniform in-plane stress distribution can be summarised as follows:

1) The value of tensile buckling load depends on the strength and area of compressive zones. It is the lowest under concentrated loading, increases significantly as the width of loading increases, and approaches infinity as the load approaches the full load over the edges.

2) Tensile buckling load, generally, increases with the increase of the curvature of the panel. However, the buckling behaviour for certain curved panels is similar to that of flat panels, depending on the edge conditions and aspect ratio of the panel.

3) The vibration behaviour due to tensile edge loading shows that for small bandwidths of loading, the natural frequency initially rises with the load, but begins to fall beyond certain value of load. The frequencies reduce to

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zero at the respective values of buckling loads. Also there is a significant change in the natural frequencies with width of loading and its position over the edges.

4) Stress singularity and stress concentration due to concentrated loads do not seem to have serious effects on the behaviour of the panels because these are present over a small area near the point of application of the load. It is seen from the convergence studies that the use of finer mesh, which models the singularities better, does not show drastic changes in the results.

5) Parametric stability study shows that under certain in-plane load conditions, the combination resonance zones are very significant. The destabilizing effects of damping on instability regions have been exhibited.

6) Dynamic instability study under partially distributed follower forces show the stiffening–de-stiffening behaviour affecting the divergence and flutter characteristics.

4. Tension Buckling, Vibration and Dynamic Instability of Plates and Panels with a Hole under Tension

The concept of tension buckling in a panel having a crack or hole has been briefly discussed in the Section 1.

Plate and panels with different type of cutouts are extensively used in transport vehicles (e.g. automotive and aircraft). Cutouts are made to lighten the structure, for venting, to provide accessibility to other parts of the structure and altering the resonant frequency. The behaviour of a thin plate with an opening has been the subject of a number of investigations (Joga Rao and Picket, 1961; Picket, 1964; Ritchie and Rhodes, 1975; Sabir and Chow, 1983; Yettram and Brown, 1985). In this type of problem a skin carries all or some of the in-plane load as well as lateral pressure load. A variety of modes of static and dynamic behaviour is possible and failures often result from the development of fatigue cracks which propagate from a stress concentration at the cutout. Openings considered by different investigations have ranged in shape from a crack to a circular hole, and both static and dynamic behaviour have been investigated. The static problem of interest is associated with out-of-plane deflections, which accompany buckling and post-buckling. The dynamic behaviour, which has been studied for this class of problems, involves bending vibration in the presence of an initial, in-plane stress (Lee and Lim, 1992).

Two features of the problems under consideration are of special interest. First, the pre-buckle stress state for the cutout is non-uniform. A review of standard texts on elastic stability (Timoshenko and Gere, 1961) reveals that most solved problems involve simple, uniform pre-buckle stress state. The second feature is both more fundamental and has more important repercussions. The sheet with an opening can exhibit buckling both when tensioned and when compressed. Under compressive buckling the out-of-plane deflection is global in character. When the sheet containing central opening is subjected to unidirectional tensile load, a region adjacent to the internal free edge perpendicular to the direction of loading is in a state of compression. The presence of this compressive zone can create the necessity of considering local buckling with out-of-plane deflection as being confined primarily to regions adjacent to the hole. The tensile buckling, which is predominantly localized, is of considerable interest.

Few relevant works pertaining to tensioned plate having a crack, opening have been discussed in the Section 1. Few other works on this topic are reviewed here. Zielsdorff (1971) investigated out-of-plane deflection behaviour of thin sheets with opening in a tensile field.

Prabhakara and Datta (1993) studied buckling and vibration of a plate with a central opening under uniformly distributed compressive and tensile loading. The vibration behaviour and buckling loads with slot geometry as parameter were discussed. The vibration, buckling and parametric instability behaviour of a plate with internal opening subjected to in-plane compressive and tensile periodic edge loading were studied by Prabhakara and Datta (1997). The dynamic instability and buckling characteristics with cutout size, load parameters are discussed.

The dynamic instability of doubly curved panels with a centrally located circular cutout, subjected to non-uniform harmonic edge loading was studied by Udar and Datta (2008). The occurrence of combination resonance in contrast to single (mode) resonances was observed. The MMS was discussed for analysis.

Ravi Kumar et al. (2004a) studied the vibration and dynamic instability behaviour of laminated composite plates with cutout, subjected to non-uniform follower edge loading with damping. It was observed that in most cases the damping effect gives destabilizing behaviour, making the plate prone to flutter.

4.1 Conclusions

The results from a study of the static anti dynamic analysis of plates with internal opening subjected to uniaxial uniform periodic loading, compressive or tensile can be summarized as follows:

1) The fundamental frequency of the plate is affected by the size and shape of the cutout. Natural frequencies decrease as the compressive load increases and frequency

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becomes zero at the buckling load of the plate. Natural frequencies under tensile loading initially rise with the load and gradually approach zero as the free edge of the opening buckles locally.

2) Static stability behaviour shows that the compressive buckling load of the plate increases with increase in the slenderness of the slot for a given size of the cutout. On the other hand, the tensile buckling load of the plate decreases with increase in the slenderness of the slot for a given size of the opening.

3) Dynamic instability results under compressive periodic loading show that as the static load factor increases, the width of the instability regions move inward on the frequency ratio axis, making the plate more susceptible to instability. Under tensile periodic loading, as the static load factor increases, the width of instability regions move outward on the frequency ratio axis, thus indicating less susceptibility to instability. The instability regions are in general affected by the size and shape of the cutout.

4) The destabilizing effect of damping due to non-uniform follower force, leading to flutter behaviour of the panel has been observed.

5. Experimental Program

5.1 An experimental study of the static and dynamic behaviour of a tensioned sheet with a rectangular opening

Datta (1976) performed an experimental study of the buckling and vibration behaviour of a thin sheet with rectangular hole in a tensile field. Few salient features of the experiment are presented here.

The choice of specimen with its geometry and the hole dimensions is such that it corresponds to an infinite sheet. This eliminates boundary effects. A typical specimen is illustrated in Fig. 5, following Carlson et al. (1970) and Datta and Carlson (1973). The test facility used to conduct the static and dynamic experiments is shown in Fig. 6.

The loading assembly is capable of applying a tensile static load up to 2,500 kg to the specimen and can be varied in a very controlled manner. The same test setup is used for parametric excitation experiment, the result for which will be discussed later. The loading system provides an axial dynamic load of the form, $P(t)=P_0+P_1 \sin \Omega t$. The oscillatory component of the dynamic load is applied through shaker and spring mechanism. Three separate experiments, a) static (no oscillation is provided), b) vibration with axial load (keeping the amplitude of the oscillatory force very small) and c) dynamic instability (with axial static and oscillatory load) can be performed using this test setup.

To analyze the experimental results, some measure of the lateral deflection of the free edge of the opening was required. In the present case, the bending strain, which provides a local measurement of the deflection, was measured using strain gauges mounted back-to-back at the slot edge. The strain sensitive direction was mounted perpendicular to the direction of the loading. The static tensile buckling loads were estimated using the modified Southwell method on the applied load versus bending strain data. The applicability of the Southwell technique to the present problem involving non-uniform pre-buckle stress state has been discussed in Datta and Carlson (1973).

The dynamic characteristics of uniaxially tensioned

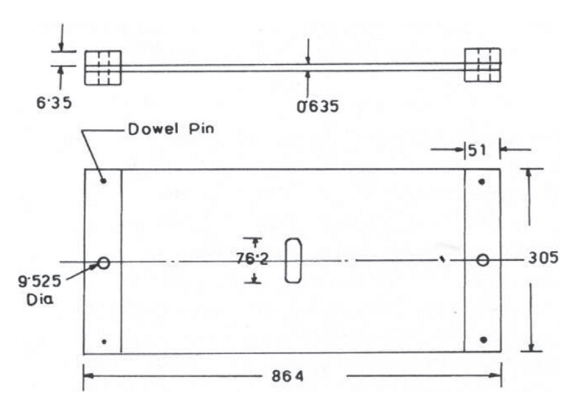

Fig. 5. Plate specimen. All dimensions are in mm.

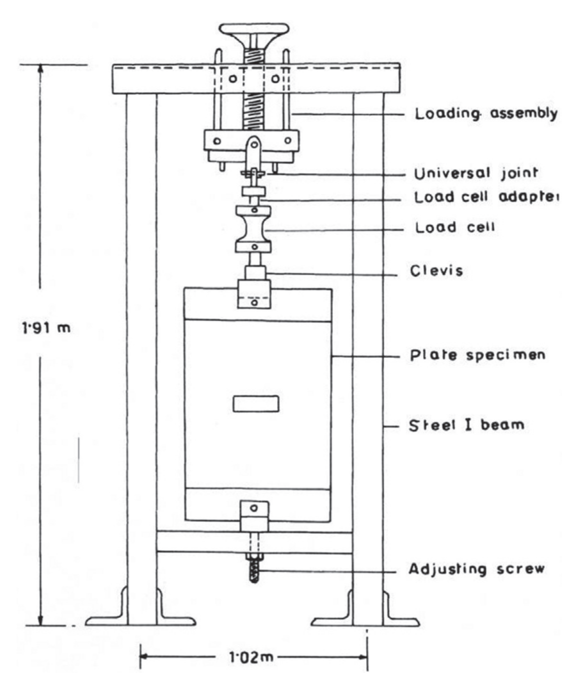

Fig. 6. Schematic diagram of the test facility.

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sheet with rectangular cutouts have been studied. The experimental frequency versus applied tensile stress plots for two hole parameters (*a/b*) are shown in Fig. 7.

It is seen that for a narrow opening (*b/a*=0.1), the load versus frequency plot has three distinct regions:

1) The frequency increases with load (A to B) because of the dominance of tension effect.

2) In the region B to C, the frequency decreases with load. Here the compression effect dominates over tension. This is the pre-buckle zone, till the minimum occurs at C and the local buckling occurs.

3) In the region C to D, the frequency increases with load. This is the post buckling zone.

For wider slots, the minimum point C is replaced by an inflexion point at buckling.

5.1.1 Conclusions

The local buckling phenomena of the free edge of the opening are seen to be dependent on cut-out parameters like aspect ratio, ratio of cut-out width to plate width, sheet thickness, cut-out corner radius and edge supports. It is observed that critical buckling load decreases as the length of the slot is increased and the critical load increases as the end radius is increased.

The vibration experiments indicate the frequency versus applied tension behaviour is markedly affected by the hole shape. For narrow hole, the plot of frequency versus applied tension shows a well-defined minimum, corresponding to the local buckling of the free edge of the hole. For wide hole, the plots of frequency versus applied tension show kinks, corresponding to the buckling load. The pre-buckling, onset of buckling and post buckling phenomenon of the free edge of the opening are clearly demonstrated.

The stress distribution in rectangular slotted plates under tensile loading also reveals some of the trends described. From this study it is possible to arrive at an optimum configuration of the cut-out.

5.2 Experiments on parametric vibration response of plates under tensile loading

Experiments on parametric instability behaviour of structural elements are very few in the literature because of the complexities involved. Somerset and Evan-Iwanoski (1966) and Evan-Iwanoski (1976) have demonstrated some experimental works on rectangular plates. Ostiguy et al. (1993) have studied the occurrence of simultaneous resonances in parametrically excited rectangular plates under compressive loads. As mentioned earlier, Carlson (1974) and Datta (1978) presented some experimental results on tensioned sheet with crack-like opening and slot respectively. A very comprehensive and controlled experiment work by Deolasi and Datta (1997a) on parametric resonance of plates under tensile loading is briefly presented here.

In this work, parametric instability characteristics of rectangular plates with nearly concentrated eccentric periodic tensile loading at various locations on the two opposite edges are studied. Test specimens are shown in Fig. 8.

The test facility, shown in Fig. 6, applying a dynamic load $P(t)=P_0+P_1 \sin\Omega t$, is used. A piezoelectric accelerometer located on the plate measures the plate response. A schematic diagram for the measurement of vibration response is shown in Fig. 9.

The details of the measurement procedure and experimental techniques are described in Deolasi and Datta (1997a). The measurement method involves the determination of the lower

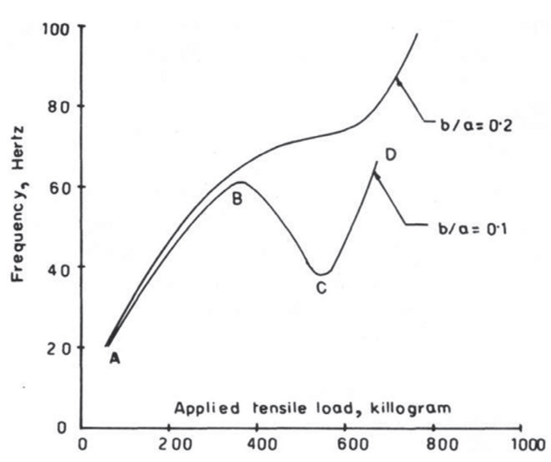

Fig. 7. Experimental frequency versus load.

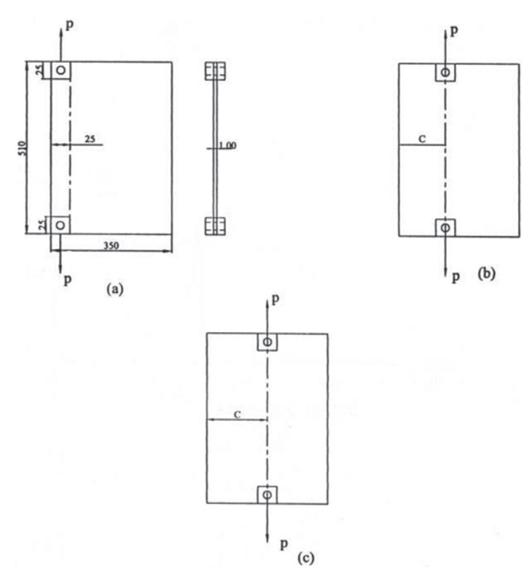

Fig. 8. Test specimens: (a) c = 15 mm, (b) c = 75 mm and c = 175 mm.

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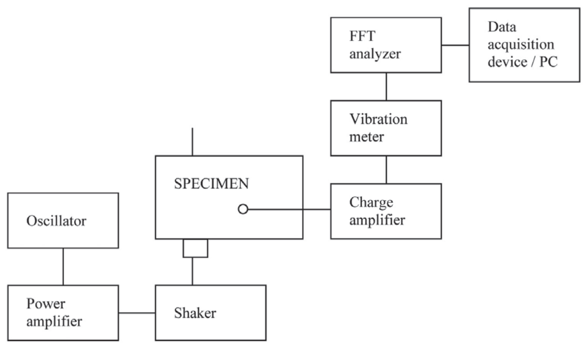
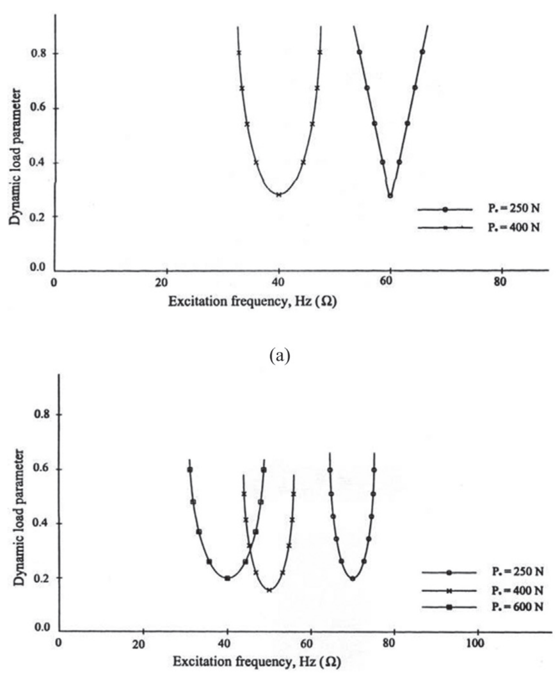


Fig. 9. Schematic setup diagram for parametric resonance experiments.



(a)

(b)

Fig. 10. Parametric resonance zones for a plate with tensile load at corner. (a) Simple resonance zones. (b) Combination resonance zones.

and upper boundaries of instability regions by observing the response signal. The excitation frequency is swept both in increasing and decreasing directions, for different static and dynamic load parameters of the parametric loading. Fig. 10 shows the simple and combination resonance instability for parametric tensile load at the corner of the plate.

5.2.1 Conclusions

The results of the present experimental work indicate that large amplitude vibrations may be developed in the plate due to parametric excitations and over a wide range of excitation frequency, mainly due to principal parametric resonances. Due to the presence of overhang there may be two upper and two lower boundaries of the principal parametric resonance zone depending on the direction of frequency sweep. However, it has been demonstrated that the boundaries which give the maximum width of instability region are of practical interest.

Results also indicate that principal resonances are not possible below certain amplitude of the dynamic load. This is probably due to the essence of inherent damping in the system. Secondary parametric resonances were found to be insignificant in comparison to principal resonances. However, the data from the secondary resonances can be used to determine the natural frequencies of the plate without separate tests for natural frequencies. The plots of natural frequencies versus static tensile load show decreasing nature of frequencies as the load increases, indicating the onset of buckling under tension.

The location of loading on the edge is found to have a considerable influence on the natural frequencies and parametric instability behaviour of plates. This indicates that the free vibration and parametric instability behaviour of the plate are considerably affected by the nature of the in-plane loading.

6. Concluding Remarks

This paper has surveyed the tension buckling behaviour of aerospace structural elements such as plates and curved panels. Apart from local static buckling phenomenon, emphasis is on the DIRs due to periodic parametric excitation loading. Interesting problems related to flat and curved panels subjected to eccentric patch loading have been extensively studied, showing undulations due to load buckling effects. Plates and curved panels having discontinuities such as damage, narrow slit and different openings cannot be avoided in aerospace structure. The present review paper has discussed the local instability problems due to the presence of opening in a highly non-uniform stress zone. The occurrence of flutter due to non-uniform tensile follower edge loading is an important phenomenon and is discussed.

Dynamic instability phenomenon which are mostly studied as global effects, have shown to have significant influence on locally induced tension buckling environment. On the experimental side, nicely controlled experiments have demonstrated local buckling phenomenon and its influence on vibration behaviour and parametric instability through developments of simple and combination resonance zones.

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The experiments which have been reviewed here may inspire and guide a sound refinement of tensile buckling concept.

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