Robust Tracker Design Method Based on Multi-Trajectories of Aircraft

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Abstract

This paper presents a robust tracker design method that is specific to the trajectories of target aircraft. This method assumes that representative trajectories of the target aircraft are available. The exact trajectories known to the tracker enables the incorporation of the exact data in the tracker design instead of the measurement data. An estimator is designed to have acceptable performance in tracking a finite number of different target trajectories with a capability to trade off the mean and maximum errors between the exact trajectories and the estimated or predicted trajectories. Constant estimator gains that minimize the cost functions related to the estimation or prediction error are computed off-line from an iterative algorithm. This tracker design method is applied to the longitudinal motion tracking of target aircraft.

Key Word : Tracker, Robust Estimator, Aircraft Trajectory, Prediction, Uncertainty

Introduction

The aircraft tracking problem involves the estimation of current aircraft position and the prediction of the aircraft's trajectory at a future time based on past and current measurement data. When tracking a target aircraft, we are faced with several types of uncertainties, such as real parameter uncertainties, unstructured uncertainties and input uncertainties. Real parameter uncertainties typically arise because of unknown or time varying coefficients in the mathematical model of target plant, such as aerodynamic coefficients, mass and moments of inertia, etc. Unstructured uncertainties arise from the simplification of plant model, that is, the linearization of a nonlinear target model and the ignorance of the higher order modeling effects such as structural dynamics. The unknown control input applied by the pilot and the disturbances such as atmospheric turbulence corresponds to the input uncertainties. When tracking enemy aircraft, the unknown pilot input is the most dominant uncertainty.

All these uncertainties disturb target vehicle's motion and change the trajectory of the target vehicle. Since the ultimate goal of the tracker is to follow the exact trajectory of the target as closely as possible based on the past and current measurement data, it is highly demanded in the practical application of target tracker to design a robust tracker whose performance does not deteriorate seriously when the target trajectory varies by the uncertainties described above.

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An adaptive estimator that updates its parameters based on the latest measurement data, may not be a practical way of tracking a target aircraft since it requires excessive computational load in a real-time tracking scenario. Several attempts to design a robust estimator have been made. Haddad and Berstein [1] developed a state estimator that provides acceptable performance over a range of parametric uncertainty by minimizing estimation error bound. The H_∞ synthesis was applied to an estimator design that would be robust against the uncertainties in the input and initial conditions [2][3]. A robust estimator design method for a target tracker that accounts for both structured real parameter uncertainties and unknown inputs has been suggested in [4].

In this paper, as a new robust tracker design method, the Multi-Trajectory tracker is described, which guarantees the robust performance when tracking different trajectories of different target aircraft or of the same aircraft with different control inputs. In this Multi-Trajectory approach, a finite number of several trajectories selected by the tracker designer are considered and the constant estimator gains are computed off-line that minimize the cost function related to the error between the exact and estimated trajectories.

The cost function of the Mini-p-norm (MpN) estimator introduced in [4][5] is used in this approach for the computation of estimator gains. The MpN estimator provides a design parameter that allows a trade off between small estimator error variance and low sensitivity to unknown parameter variations. Constant Kalman gains that minimize the Mini-p-Norm cost function under the constraint of the Riccati equation are computed off-line from an iterative algorithm.

In [4] and [5], white Gaussian noise input is assumed as a system input, and therefore the estimation error variance is used as a performance metric. However, in this paper, deterministic input is considered and the standard deviation of the errors between the exact and estimated trajectories is used as a performance metric. The tracker design with the consideration of trajectories enables us to handle with a nonlinear target system as well as a linear target.

Mathematical Formulation

In a practical tracking problem, the plant is a nonlinear continuous system, whereas measurement data, which is analyzed on a digital computer, is available only at discrete time points. The plant and measurement system are described by the following nonlinear differential equations:

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t), \omega(t)) \\ z(t) &= h(x(t)) \\ y(k) &= g(x(k), \upsilon(k)) \end{aligned} \tag{1}$$

where x is a plant state vector, z is a output state vector we want to estimate, y is a measurement state vector, f, g and h are nonlinear functions of the plant states, $u(t)$ is a plant input vector, $\omega(t)$ is a zero-mean white Gaussian process noise with spectral density matrix $Q_o(t)$, $\upsilon(k)$ is a zero-mean white Gaussian measurement noise with Covariance matrix R_i, and k is a discrete time index.

The equations with the time index of k denote discrete time equations. It is assumed that $\omega(t)$ and $\upsilon(k)$ are statistically independent for all t and k. For the design of a linear estimator, the nonlinear plant and measurement equations of Eq.(1) are linearized into:

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + \omega(t) \\ z(t) &= H(t)x(t) \\ y(k) &= C(k)x(k) + \upsilon(k) \end{aligned} \tag{2}$$

where the input $u(t)$ is expressed in terms of white noise since the input to the target vehicle is never known in the tracking scenario.

It is further assumed that for some time interval the system matrices defining the state space equations are time invariant. It is very common in the design of tracker that the exact system matrices are unknown to the designer. Instead a different, perhaps, simpler state space models of the following form must be used:

$$\begin{aligned} \dot{x}(t) &= A_o x(t) + w(t) \\ z(t) &= H_o x(t) \\ y(k) &= C_o x(k) + v(k) \end{aligned} \tag{3}$$

The continuous estimator model can be approximately expressed by the following discrete system if the sampling time of measurement is small enough:

$$\begin{aligned} x(k+1) &= \Phi_o x(k) + \Gamma_o \omega(k) \\ z(k) &= H_o x(k) \end{aligned} \tag{4}$$

where $\Phi_o = e^{A_o \Delta t}$, $\Gamma_o = [\int_0^{\Delta t} e^{A_o s} ds]$ and Δt is the sampling time of measurement.

Based on this linearized discrete models, a discrete estimator of the following form can be designed [6]:

$$\begin{aligned} \hat{x}(k+1) &= [\Phi_o - K C_o \Phi_o]\,\hat{x}(k) + K y(k) \\ \hat{z}(k+1) &= H_o \hat{x}(k+1) \end{aligned} \tag{5}$$

where $\hat{x}$ is an estimator state vector, $\hat{z}$ is an estimator output vector, and K is the Kalman gain matrix.

In the tracking problem, the true value of z is to be estimated based on the measurement data y. The objective of this paper is to find the constant values for the estimator gains K that guarantee the robust performance when tracking several different trajectories. The estimation system matrices A_o, H_o, and C_o are assumed to be chosen by the designer by whatever method he prefers.

Multi-Trajectory Tracker Model

When tracking an enemy target vehicle, the enemy vehicle may perform a severe maneuver to escape the bullet, and the trajectory of the target vehicle may be difficult to predict. The number of target trajectories that are expected to be encountered during tracking is infinite. A finite set of trajectories that represent typical flight trajectories are assumed to be known and the design of a robust tracker whose performance does not vary significantly when tracking any of these given trajectories is the main

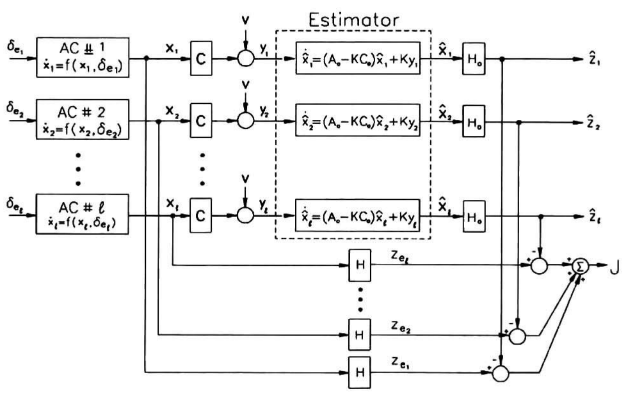

Fig. 1. The block diagram for the Multi-Trajectory tracker mechanism

objective of this paper. The problem of selecting typical flight trajectories is not considered here.

The block diagram in Fig.1 shows the mechanism of the Multi-Trajectory tracker. In this figure, each trajectory belongs to a different nonlinear target model represented by Eq.(1) or the same target model but with different inputs. One linear discrete estimator is used to track all the trajectories. The estimator gains are chosen to minimize the cost function related to the errors between the exact trajectories z, and the estimated trajectories $\hat{z}$.

Multi-Trajectory Estimator

In general tracking approaches, there is no other informations but measurement data. Therefore estimator gains are chosen to minimize the sum of the squares of the deviation $y-\hat{y}$ where $\hat{y}=C_o\hat{x}$. Minimizing this term is related to the fitting of measurement data. In the Multi-Trajectory approach, typical flight trajectories are selected by the tracker designer and those exact trajectories known to the tracker enables the incorporation of the exact data in the tracker design instead of the measurement data. An estimator can be designed which would minimize the difference between the exact states and the estimated states of interest and minimize the sum of the squares of the deviation $z-\hat{z}$, *i.e.* minimize the quantity:

$$J(K)=\sum_{i=1}^{l}\sum_{k=1}^{n_k}P_i\{[z(k)-\hat{z}(k)]^T W[z(k)-\hat{z}(k)]\}_{f=f_i} \tag{6}$$

where W is a symmetrical positive definite weighting matrix, n_k is the number of measurement data for each trajectories, l is the number of trajectories, f_i denotes f of the i-th trajectory, and P_i is the probability of tracking the i-th trajectory.

Multi-Trajectory Predictor

In the last section, a design method for estimating current position is discussed. However, in the practical application tracker performance depends mainly on the prediction of future position rather than the estimation of current position. Prediction refers to estimating the states at a future time. In order to mathematically develop a Multi-Trajectory predictor, the prediction error defined as below is considered.

$$e(k+T_p)=z(k+T_p)-\hat{z}(k+T_p|k)$$

where T_p is the prediction time.

In the predictor design, the following cost function should be minimized.

$$J_p(K)=\sum_{i=1}^{l}\sum_{k=1}^{n_k}P_i\{[z(k+T_p)-\hat{z}(k+T_p|k)]^T W[z(k+T_p)-\hat{z}(k+T_p|k)]\}_{f=f_i} \tag{7}$$

The prediction by the tracker is accomplished on the basis of the current estimation according to the following equations:

$$\dot{\hat{x}}(t)=A_o\hat{x}(t) \tag{8}$$

Prediction from time t_k to time t_k+T_p is done by solving Eq.(8):

$$\hat{x}(k+T_p|k)=e^{A_oT_p}\hat{x}(k) \tag{9}$$

The future output state vector $\hat{z}(k+T_p|k)$ in Eq.(7) is computed from the estimated states,

$\hat{x}(k)$ by using Eq.(9) and the relation of $\hat{z}(k+T_p|k)=H_o\hat{x}(k+T_p|k)$.

Mini-p-Norm Tracker

Robust tracking performance can be obtained by introducing a Mini-p-norm(MpN) estimator with the following cost function which is related to the p-norm of the estimation error variances calculated for each trajectory [4]:

$$J(K)=\sum_{i=1}^{L}(P_i J_i(K))^p \tag{10}$$

where

$$J_i=\sum_{k=1}^{n_k}\{[z-\hat{z}]^T W[z-\hat{z}]\}_{f=f_i}, \tag{11}$$

$z=z(k)$, $\hat{z}=\hat{z}(k)$ for the estimator and $z=z(k+T_p)$, $\hat{z}=\hat{z}(k+T_p|k)$ for the predictor.

Note that $(P_i J_i)^{p-1}$ has a role of a weight function since Eq.(10) can be rewritten as follows:

$$J(K)=\sum_{i=1}^{L}(P_i J_i)^{p-1}(P_i J_i) \tag{12}$$

When $P_i J_i$ is large, $(P_i J_i)^{p-1}$ is large and when $P_i J_i$ is small, $(P_i J_i)^{p-1}$ is small for all $p > 1$. Therefore $(P_i J_i)^{p-1}$ gives more weight to the large contributors in the performance index and less weight to the smaller ones. When $p = 1$, the MpN estimator minimizes the sum of all $P_i J_i$ (or equivalently, the mean value of $P_i J_i$) and when 'p' approaches infinity, it minimizes the maximum value of $P_i J_i$. The choice of the value of 'p' depends on the designer, that is, 'p' acts a new design parameter for the robust tracker.

The following section will discuss the mathematical procedure to find estimator gains K, which minimize $J(K)$ in Eq.(10).

Computation of the Kalman Gains for the MpN Tracker

The computation of the optimal values of estimator gains K minimizing the cost $J(K)$ of Eq.(10) is not practical or may not be possible if the number of measurement states is more than one. Instead, as the suboptimal gains which minimize $J(K)$, we can choose the best estimator gains among the following Kalman gains computed from the steady state discrete Riccati equation [7].

$$K=\Phi_o S H_o^T[H_o S H_o^T+R]^{-1} \tag{13}$$

$$S=\Phi_o(I-SH_o^T[H_o S H_o^T+R]^{-1}H_o)S\Phi_o^T+\Gamma_o Q_o \Gamma_o^T \tag{14}$$

The last term, $\Gamma_o Q_o \Gamma_o^T$, of Eq(14). should be used as a design parameter. Since the number of elements in $\Gamma_o Q_o \Gamma_o^T$ may be too many to compute, it is desirable to reduce the number of unknown elements. It may be assumed that the noise entering each channel is independent of and uncorrelated to each other so that $\Gamma_o=I$ and the non-diagonal terms of Q_o are all zero. Then, the matrix Q_o can be expressed as

$$Q_o=\begin{bmatrix} Q_o(1,1) & 0 & \cdots & 0 \\ 0 & Q_o(2,2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Q_o(n_{x_o}n_{x_o}) \end{bmatrix} \tag{15}$$

where n_{x_o} is the order of the tracker model. The best values for $Q_o(i,i)$ for $i=1,2,\cdots,n_{x_o}$

minimizing $J(K)$ can be obtained by the Newton-Raphson method [8]. Define a design parameter vector ξ as:

$$\xi \equiv [\, Q_o(1,1) \quad Q_o(2,2) \quad \cdots \quad Q_o(n_{x_o}, n_{x_o})\,]^T \tag{16}$$

The Newton-Raphson iteration is:

$$\xi_{n+1} = \xi_n - \alpha(\nabla_\xi^2 J)^{-1} \nabla_\xi J \qquad (0 < \alpha < 1) \tag{17}$$

where subscript $'n'$ is an iteration number and

$$\nabla_\xi J(\xi) \equiv \begin{bmatrix} \frac{\partial J}{\partial \xi_1} \\ \frac{\partial J}{\partial \xi_2} \\ \vdots \\ \frac{\partial J}{\partial \xi_{n_{x_o}}} \end{bmatrix}, \qquad \nabla_\xi^2 J(\xi) \equiv \begin{bmatrix} \frac{\partial^2 J}{\partial \xi_1 \partial \xi_1} & \frac{\partial^2 J}{\partial \xi_1 \partial \xi_2} & \cdots & \frac{\partial^2 J}{\partial \xi_1 \partial \xi_{n_{x_o}}} \\ \frac{\partial^2 J}{\partial \xi_2 \partial \xi_1} & \frac{\partial^2 J}{\partial \xi_2 \partial \xi_2} & \cdots & \frac{\partial^2 J}{\partial \xi_2 \partial \xi_{n_{x_o}}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 J}{\partial \xi_{n_{x_o}} \partial \xi_1} & \frac{\partial^2 J}{\partial \xi_{n_{x_o}} \partial \xi_2} & \cdots & \frac{\partial^2 J}{\partial \xi_{n_{x_o}} \partial \xi_{n_{x_o}}} \end{bmatrix} \tag{18}$$

Using the chain rule, we calculate the gradient and second gradient of the cost function J as:

$$\frac{\partial J}{\partial \xi_j} = 2 \sum_{i=1}^{L} p\, P_i^p\, J_i^{p-1} \sum_{k=1}^{n_k} \left\{ [z - \hat{z}]^T W \left[-\frac{\partial \hat{z}}{\partial \xi_j} \right] \right\}_{f=f_i} \tag{19}$$

$$\begin{aligned} \frac{\partial^2 J}{\partial \xi_j \partial \xi_m} &= 2 \sum_{i=1}^{L} p(p-1)\, P_i^p\, J_i^{p-2} \left(\sum_{k=1}^{n_k} \left\{ [z-\hat{z}]^T W [-\frac{\partial \hat{z}}{\partial \xi_j}] \right\}_{f=f_i} \right)^2 \\ &+ 2 \sum_{i=1}^{L} p\, P_i^p\, J_i^{p-1} \sum_{k=1}^{n_k} \left\{ [-\frac{\partial \hat{z}}{\partial \xi_m}]^T W [-\frac{\partial \hat{z}}{\partial \xi_j}] + \left\{ [z-\hat{z}]^T W [-\frac{\partial^2 \hat{z}}{\partial \xi_j \partial \xi_m}] \right\} \right\}_{f=f_i} \end{aligned} \tag{20}$$

It is advised to neglect the second gradient term $\frac{\partial^2 \hat{z}}{\partial \xi_i \partial \xi_m}$, since this second derivative term approaches its expected value of zero for ξ near the true parameter value and the resulting second derivative matrices may not be positive definite while the computation of this second gradient term is excessive [9].

The gradient of $\hat{z}$ is approximated by

$$\frac{\partial \hat{z}}{\partial \xi_i} = \frac{\hat{z}(\xi_i + \partial \xi_i) - \hat{z}(\xi_i)}{\partial \xi_i} \tag{21}$$

where $\partial \xi_i$ is a small perturbation of the i-th parameter, *i.e.* $Q_o(i,i)$ in our case. The magnitude of the perturbation must be small enough to ensure a linear variation in response, and large enough to avoid round-off errors inherent in the digital computer. The optimal magnitude of the perturbation depends on the machine precision, the units of the states, and the scaling used.

Applications

The Multi-Trajectory MpN estimator introduced in this paper is applied to the longitudinal motion tracking of several aircraft. In this longitudinal motion tracking problem, the vertical position of target aircraft is to be estimated.

The linear equation of longitudinal motion of a target aircraft can be expressed by a 5th order state equation as shown in Eq.(22) where the state vector has the following elements: the forward velocity u, the vertical velocity w, the pitch rate q, the pitch angle θ and the vertical position z. The positive direction of the vertical position (z) is downward. The system input is an elevator deflection angle δ_e.

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g\cos\gamma_o & 0 \\ Z_u & Z_w & Z_q+U & -g\sin\gamma_o & 0 \\ M_u+M_{\dot{w}}Z_u & M_w+M_{\dot{w}}Z_w & M_q+M_{\dot{w}}U & -M_{\dot{w}}\sin\gamma_o & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -U & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \\ z \end{bmatrix} + \begin{bmatrix} X_{\delta_e} \\ Z_{\delta_e} \\ M_{\delta_e}+M_{\dot{w}}Z_{\delta_e} \\ 0 \\ 0 \end{bmatrix} \delta_e \quad (22)$$

If the matrix A_o of the estimator model could be determined to be identically the same as the A matrix of the linear plant model, it will be a perfect model. Unfortunately the parameter values in the A matrix of the plant (target aircraft) are not known and are time-varying. The optimal values for those parameters may be found along with the optimal estimator gain matrix K with much more complicated computing algorithm. To concentrate on the main subject of this paper, the A_o matrix are assumed to be chosen properly by the designer.

24 Different Flight Cases

The example of tracking 8 different aircraft is examined where each aircraft experiences one or more flight conditions. The flight data such as velocity and altitude of 24 different flight cases were obtained from [10][11] and listed in Table 1. The 24 different trajectories were generated from the linear plant model of Eq.(22). The deterministic elevator input was applied to the plant for the simulation of longitudinal motion. The magnitude of this elevator input was adjusted such that the maximum acceleration in the vertical direction becomes $2g$ for each flight case where g is the acceleration of gravity. $2g$ was chosen to represent a moderately large pattern of jinking maneuver. The exact vertical position data of all the simulated trajectories are illustrated in Fig.2.

Table 1. 24 Difference Flight Cases of 8 Different Aircraft

Flight No.	Aircraft	Velocity (ft/sec)	Altitude (ft)	Mass (slug)
FC #1	F-104A	287	0	14126
FC #2	F-104A	893	0	16300
FC #3	F-104A	1228	0	16300
FC #4	A-4D	223	0	22058
FC #5	A-4D	447	0	17578
FC #6	A-4D	950	0	17578
FC #7	A-4D	423	15000	17578
FC #8	A-4D	634	15000	17578
FC #9	F-4C	230	0	33197
FC #10	F-4C	893	0	38925
FC #11	F-4C	1228	0	38925
FC #12	T-38	829	15000	7500
FC #13	T-38	829	15000	9375
FC #14	T-38	829	15000	11250
FC #15	NT-33A	228	0	11800
FC #16	NT-33A	447	0	13700
FC #17	NT-33A	782	0	13700
FC #18	B-52	627	10000	325000
FC #19	B-52	627	10000	406000
FC #20	B-52	627	10000	488000
FC #21	B-1	952	0	227790
FC #22	C-5A	246	0	580756
FC #23	C-5A	502	0	654399
FC #24	C-5A	726	0	654399

It is assumed that the vertical position (z) of the target is measured and the RMS(Root Mean Square) measurement noise is 10 *ft*. To generate measurement data, white noises obtained from a random number generation program were multiplied by the measurement noise intensity R, and added to the exact trajectories. In this example, only the estimation errors are considered, that is, the estimator is designed to minimize $J(K)$ in Eq.(10), where P_i =1/24 and l =24. Mean square estimation errors are used for the performance comparison of the tracker.

Fig.3 illustrates the mean square estimation errors of the Multi-Trajectory MpN estimator with p =1, 5, 10, and 50. This figure shows that the mean square estimation error of the 4-th flight

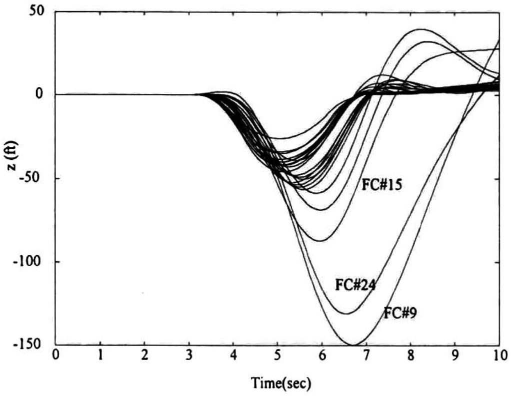

Fig. 2. The vertical position of 24 trajectories generated by a linear simulation

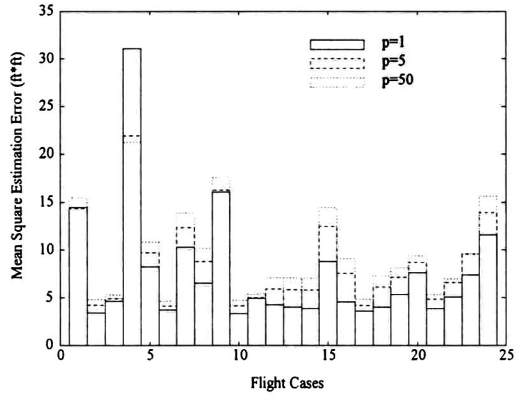

Fig. 3. The effect of p on the performance of the Multi-Trajectory MpN estimator for the 24 different flight cases

condition is the largest. As p increases, the mean square estimation errors at the 4-th flight case decrease; therefore the sensitivity to parameter variations reduces. However, the mean square estimation errors at the other flight cases increase resulting in the increment of the sum of mean square errors.

Note that the maximum mean square error obtained with p =50 is almost the same as the one obtained with p =5, while the sum of mean square errors obtained with p =50 is much larger. Therefore the Multi-Trajectory MpN estimator designed with p =5 may be considered as the most favorable estimator for this example considering its low sensitivity and low sum of mean square error.

This example shows that the Multi-Trajectory MpN estimator guarantees the robust performance when tracking several types of trajectories and with design parameter, p, the tracker designer can deal with the maximum mean square error and the average mean square error simultaneously.

Three Maneuvers of a T-38 Aircraft

In this example, a nonlinear target model is considered and the prediction performance of the tracker is investigated. One level flight and two longitudinal maneuvers (5g pull-up and 2g dive) of a T-38 aircraft were generated by a nonlinear flight simulator program. These maneuvers are primarily in the inertial z-direction. The exact trajectories of these maneuvers are described below:

- Level flight : The aircraft moves in a straight and level flight at the speed of 620 *ft/sec*.
- 5g pull-up : At the start, the aircraft moves in a straight and level flight at the speed of 620 *ft/sec*, the pilot pulls up the elevator stick with full thrust to reach as much as a 5g load factor and then the pilot pushes back the stick to end in a level flight.
- 2g dive : At the start, the aircraft moves in a straight and level flight at the speed of 440 *ft/sec*, the pilot pushes the elevator stick to dive until the aircraft reaches a negative 2g load factor, and then the pilot pulls up to end in a level flight.

Target attitude angle measurement as well as position measurement are incorporated in this example. The RMS noises in the position and pitch angle measurement data are assumed to be 10 *ft* and 5 *deg* respectively. The Multi-Trajectory MpN tracker minimizing in Eq.(10) for 2 second predictions is designed with p =1 and the exact and the predicted trajectories are compared. The Multi-Trajectory MpN tracker with p =1 minimizes the sum of the mean square errors when tracking all three maneuvers at the same time.

Fig.4, Fig.5 and Fig.6 show the 2 second predicted trajectories of the Kalman filter designed

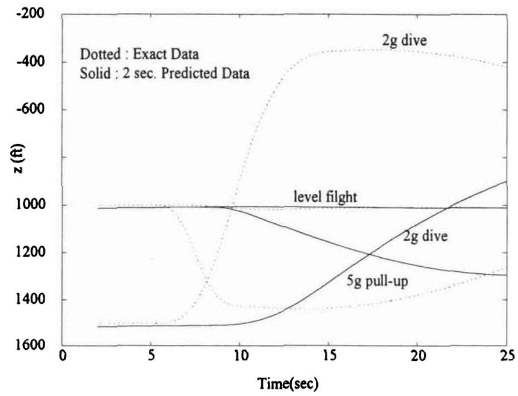

Fig. 4. The 2 second predicted trajectories obtained by the Kalman filter designed for the level flight maneuver of a T-38 aircraft

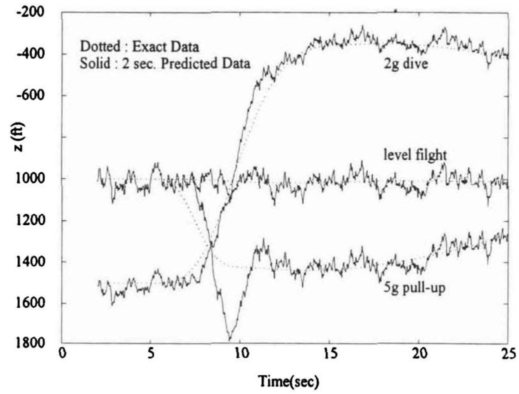

Fig. 5. The 2 second predicted trajectories obtained by the Kalman filter designed for $2g$ dive maneuver of a T-38 aircraft

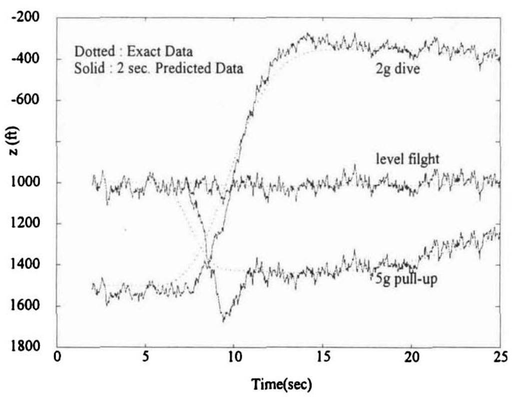

Fig. 6. The 2 second predicted trajectories obtained by the Kalman filter designed for the $5g$ pull-up maneuver of a T-38 aircraft

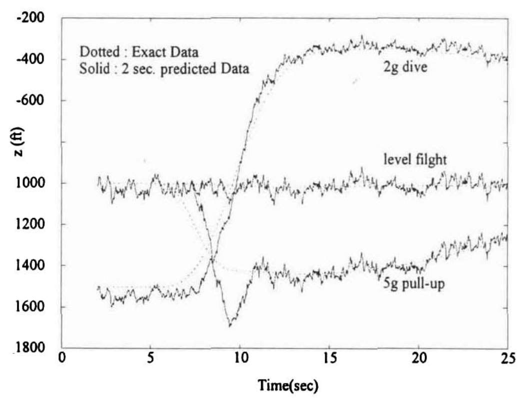

Fig. 7. The 2 second predicted trajectories obtained by the Multi-Trajectory tracker designed for all three maneuvers

by considering only one maneuver at a time. Note that when considering only one maneuver, the Multi-Trajectory MpN tracker with p =1 becomes the standard Kalman Filter. The prediction performance of the Multi-Trajectory tracker considering all three maneuvers is illustrated in Fig.7.

Table 2 represents the standard deviations of the prediction errors of the Multi-Trajectory tracker, which considers all three maneuvers, and three Kalman filters each of which is designed for one maneuver.

The Kalman filter designed for the level flight maneuver shows the excellent prediction performance as shown in Fig.4 when tracking the level flight maneuver as expected. However, this Kalman filter has very larger prediction errors when tracking the $5g$ pull-up and the $2g$ dive maneuver. This is because the estimator designed only for the level flight does not much rely on the measurement data and the estimator gains are very small.

The Kalman filter designed for the $2g$ dive maneuver(Fig.5) has a little better prediction performance when tracking $2g$ dive maneuver than the Multi-Trajectory tracker. However it shows definitely larger overshoot in the $5g$ pull-up maneuver tracking.

The Kalman filter designed for the $5g$ pull-up maneuver(Fig.6) has slightly better prediction performance when tracking the $5g$ pull-up maneuver than the Multi-Trajectory tracker. However, it shows a little bit worse performance in the $2g$ dive maneuver tracking.

Table 2 shows that when tracking all three maneuvers, Multi-Trajectory tracker has the smallest prediction errors. Even though the performance of the Multi-Trajectory tracker is only slightly better than that of the Kalman filter designed for the $5g$ pull-up maneuver in this application, the Multi-Trajectory tracker always guarantees the most robust performance.

Table 2. The standard deviations of the 2 second prediction errors of the Multi-Trajectory MpN estimator with p=1 and three Kalman filters

Tracker Types	Maneuver Types			
	Level Flight	2g dive	5g pull-up	all maneuvers
Kalman filter (Level Flight)	7.1	360.1	147.4	224.7
Kalman filter (2g dive)	37.8	46.5	83.7	59.8
Kalman filter (5g pull-up)	33.0	59.1	65.4	54.4
Multi-Trajectory tracker	31.4	51.2	67.0	51.9

Conclusions

As a trajectory-based approach for the design of a robust tracker, the Multi-Trajectory tracker was introduced. The Multi-Trajectory tracker uses the estimator gains that minimize the p-norm of the estimation or prediction error in tracking several target trajectories. The introduction of the p-norm provides the tracker designer with a capability to trade off the mean and maximum estimation or prediction error between the exact trajectories and the estimated trajectories.

To reduce the computational burden, estimator gains were constrained to be Kalman gains. The diagonal elements of the process noise intensity matrix were used as design parameters. The best values of these parameters that minimize the trajectory based cost function were computed by the Newton-Rapson method, and the steady state Kalman gains were computed by solving the Riccati equation with these parameter values.

The Multi-Trajectory tracker was applied to the longitudinal motion tracking of target aircraft. In two examples, the Multi-Trajectory tracker shows the robust performance in tracking different target maneuvers generated by linear as well as nonlinear simulation models.

The Multi-Trajectory tracker is suitable for the practical application since the estimator gains are computed off-line and the real estimation errors are considered instead of the estimation error variance, which is computed under the assumption of the Gaussian white noise input to the system. This approach would be reliable when the type of trajectory encountered during tracking is similar to one of the typical trajectories considered in the design. The proper choice of the typical trajectories, which is beyond the scope of this paper, is important in this approach. During the initial tracking stage of target aircraft, a set of expected trajectories which is stored as database for the tracker system could be chosen and then the tracking performance of the Multi-Trajectory tracker based on these trajectories can be considerably enhanced.

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