Adaptive Kalman Filter Design for an Alignment System with Unknown Sway Disturbance

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Abstract

The initial alignment of inertial platform for navigation system was considered. An adaptive filtering technique is developed for the system with unknown and varying sway disturbance. It is assumed that the random sway motion is the second order ARMA(Auto Regressive Moving Average) model and performed parameter identification for unknown parameters. Designed adaptive filter contain both a Kalman filter and a self-tuning filter. This filtering system can automatically adapt to varying environmental conditions. To verify the robustness of the filtering system, the computer simulation was performed with unknown and varying sway disturbance.

Key Word : Initial alignment, sway disturbance, ARMA(Auto Regressive Moving Average) Model, Kalman filter, self-tuning filter

Introduction

The initial alignment of the inertial navigation system has a great deal of importance. For vehicle navigation systems, the navigation accuracy mainly depends on the accuracy of the initial alignment [1][2][3]. It is often noted that the conventional control and estimation methods does not have good results in the presence of random sway motions during the initial alignment [4]. Since, the accuracy of alignment depends on the sensor noise and acceleration disturbance, it is reasonable to consider the stochastic uncertainties and to construct a robust controller for varying disturbance. Since the accuracy of an alignment system varies with the environmental disturbance which is actually unknown, the corresponding filter gain must be switched for different operation conditions. Extended Kalman filter and parameter identification algorithms were applied to reduce the effect of random disturbance [5]. Also, a self-tuning filter algorithm is tried for multivariable process and dynamic ship positioning problem [6]. This type of multivariable adaptive filtering algorithms is not tried for the navigation system of vehicles such as airplanes and missiles.

In this paper, an adaptive filtering technique is developed for an alignment system with unknown and varying sway disturbance. The proposed filtering system can automatically adapt to varying environmental conditions. This filtering system contains a Kalman filter, a self-tuning filter and a optimal feedback controller. The random sway motion was assumed as the unknown second order model and parameter identification was performed for the unknown parameters. These estimated parameters and sway state values are used to estimate the alignment system states. Therefore, for designed filtering system, the Kalman filter can calculate state values in the presence of varying environmental disturbance. The robustness of the proposed adaptive filtering system was verified by the computer simulation with the variation of sway disturbance parameters.

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Initial Alignment System

Initial Alignment

The alignment problem can be described as following state space equation.

$$\dot{x} = Fx + Bu + Gw \tag{1}$$

Where x is the state vector representing misalignment angles, F, B, $G$are constant matrixes, w is the gyro-drift, u is the control input. This equation illustrates the relation among the alignment error dynamic motion, the gyro-drift, the misalignment angles and the gyro torque control inputs.

The measurement equation is represented as follows.

$$z = Hx + v \tag{2}$$

Where z is the measurement vector, H is the constant matrix and v is the accelerometer uncertainty error. And the signal w and v are white gaussian noises.

The Alignment Error Model

All of the known major sources of error for an alignment system may be considered as followings.

The gyro drift rate errors, the gyro torque errors, the accelerometer errors, the accelerometer alignment errors, the gyro alignment errors, the system alignment errors, the altitude errors, and so on. The error state vector for all mechanization is composed of the system's attitude and position errors.

$$x = \{\varepsilon_N, \varepsilon_E, \varepsilon_D, \delta L, \delta l, \delta h_i\} \tag{3}$$

Where $\{\varepsilon_N, \varepsilon_E, \varepsilon_D\}$ is the north, the east and the vertical down components of the attitude error, respectively, δL is the latitude error, δl is the terrestrial longitude error and δh_i is the altitude error.

The attitude error will be defined as the orthogonal transformation error between platform and geographic axes. A rigorous, detailed error model of the real-world inertial alignment system contains 50 or more error states. However, H. Winter reported that the error sensitivity studies, 13 states are really significant as far as ground alignment of inertial platforms [2]. We are interested in the case, however, that sway acceleration is severe, the alignment system can be assumed third order model for simplicity. For simplicity, 3 misalignment angles (North, East and Vertical Down misalignment angles) were treated. Fig. 1, represents the diagram of error dynamics and measurement model. The error model is described as following form.

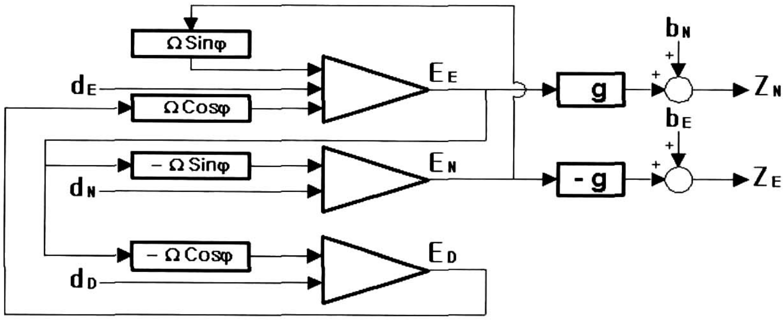


Fig. 1. Error Dynamics and Measurement Model

$$\begin{bmatrix} \dot{E}_N \\ \dot{E}_E \\ \dot{E}_D \end{bmatrix} = \begin{bmatrix} 0 & -\Omega \sin\varphi & 0 \\ \Omega \sin\varphi & 0 & \Omega \cos\varphi \\ 0 & -\Omega \cos\varphi & 0 \end{bmatrix} \begin{bmatrix} E_N \\ E_E \\ E_D \end{bmatrix} + \begin{bmatrix} dN \\ dE \\ dD \end{bmatrix} \quad (4)$$

where Ω is the earth rate, φ is the latitude of alignment, $[dN \;\; dE \;\; dD]^T$ are gyro drift rates and $[E_N \;\; E_E \;\; E_D]^T$ are three misalignment angles.
The measurement equation is as following form.

$$\begin{bmatrix} Z_E \\ Z_N \end{bmatrix} = \begin{bmatrix} -g & 0 & 0 \\ 0 & g & 0 \end{bmatrix} \begin{bmatrix} E_N \\ E_E \\ E_D \end{bmatrix} + \begin{bmatrix} b_E \\ b_N \end{bmatrix} \quad (5)$$

where b_E, b_N are white gaussian accelerometer uncertainty errors.

Structure of Alignment System

In this paper, it can be assumed that the alignment system model is composed of platform error dynamics and sway disturbance model. The former represents system dynamic characteristic and the latter represents a subsystem excited by external disturbance. These two motions can be determined separately and the total motion is the sum of each of them. Since the main disturbance frequency is vary with environmental conditions, the corresponding Kalman filter gain must be tuned. The alignment problem is to control the platform motion with the output which contains both the system output y and the disturbance y_d.

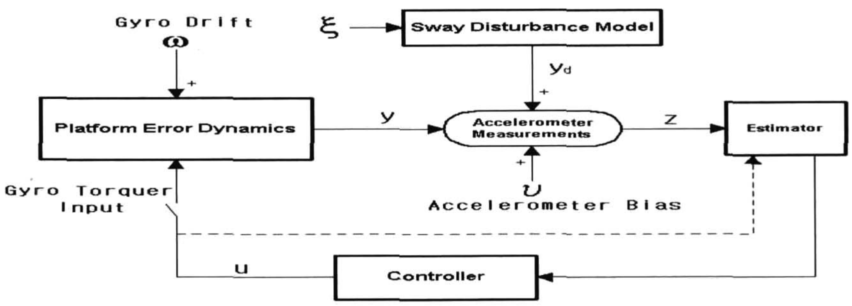


Fig. 2. Alignment System with Platform Error Dynamics and Disturbance model

The structure of the system is represented in Fig. 2. The system can be represented by the following state-space equations.

$$x(t+1) = Ax(t) + Bu(t) + w(t) \quad (6)$$

$$\begin{aligned} y(t) &= Cx(t) \\ z(t) &= y(t) + v(t) + y_d(t) \end{aligned} \quad (7)$$

where the signal w and v are a white gaussian process noise and a measurement noise having diagonal covariance, respectively. The plant measurement equation contains the disturbance signal $y_d(t)$ which is a colored noise.

It is assumed for the moment that the disturbance signal y_d can be measured, and hence z can be calculated. The state x can be estimated using a conventional Kalman filter. It is assumed that a time invariant model for the system motion is known and detectable and that the noise is stationary. The Kalman gain matrix is computed as following equations.

$$Predictor: \;\; \hat{x}(t|t-1) = A\hat{x}(t-1|t-1) + Bu(t-1) \quad (8)$$

$$\hat{y}(t|t-1) = C\hat{x}(t|t-1) \quad (9)$$

$$P(t|t-1) = AP(t-1|t-1)A^T + DQD^T \tag{10}$$

$$Corrector: \ \hat{x}(t|t) = \hat{x}(t|t-1) + K(t)\varepsilon(t) \tag{11}$$

$$\hat{y}(t|t) = C\hat{x}(t|t) \tag{12}$$

$$P(t|t) = P(t|t-1) - K(t)CP(t|t-1) \tag{13}$$

$$K(t) = P(t|t-1)C^T [CP(t|t-1)C^T + R]^{-1} \tag{14}$$

where hat(^) notation represents the estimation values. The matrix K is the Kalman filter gain, P is the error covariance matrix. The signal $\varepsilon(t)$ is represented by following equations.

$$\varepsilon(t) = z(t) - \hat{y}(t|t-1) - y_d(t) \tag{15}$$

Actually, in above equations the disturbance signal $y_d(t)$ is not measured from $z(t)$ separately, thus the signal $\varepsilon(t)$ can not be calculated.

Sway Disturbance

Modeling of Random Sway Motion

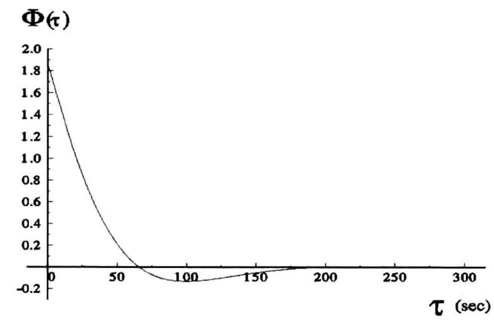


Fig. 3. Autocorrelation Function of Random Sway Motion

The sway motion is the environmental disturbance caused by the random external wind or the wave forces. It has energy at low frequency so that it is the main cause of accuracy decay and time consuming problems for alignment system. Therefore, it is desired that the stochastic consideration of this uncertainties for design of an alignment system which is operated under sway disturbance.

The modeling of random sway is beset with practical difficulties, therefore sway have been measured experimentally. From Fig. 3, autocorrelation function of random sway motion, it is clear that the random sway disturbance represents periodic behavior. The autocorrelation function model of the random variable with periodic behavior has the form.

$$\phi_{x_1,x_1} = \sigma^2 e^{-\beta|\tau|} \cos w|\tau| \tag{16}$$

where σ^2, β and w are chosen on the basis of the physics of the situation or to fit the empirical autocorrelation data. Two state variables are necessary to represent the random variables with above autocorrelation function. For the test aircraft specification HFB 320, the sway has been measured exponentially [4]. The following parameters have been worked out.

$$\phi(\tau) = \sigma^2 e^{-\xi w_n|\tau|} (\cos|\tau|) \tag{17}$$

$$w_n = 2\pi f = 3.33\times 10^{-2}, \ \beta = \xi w_n = 2.33\times 10^{-2} \tag{18}$$

$$w = w_n(1-\xi^2)^{1/2} = 2.378\times 10^{-2} \tag{19}$$

ARMA(Auto Regressive Moving Average) Model of the Random Sway Motion

The external disturbance is represented by the colored noise model. The random sway disturbance can be represented with varying mean frequency, ARMA model. It is assumed that the order of the polynomial matrices, $A_d(z^{-1})$ and $C_d(z^{-1})$, is the second and the first order, respectively. The simulations were performed using two coloring filters driven by white noise of the following forms.

$$\begin{aligned} \dot{x}_d &= A_d x_d + D_d \xi \\ y_d &= C_d x_d \end{aligned} \tag{20}$$

where $A_d = \begin{bmatrix} A_d^s & 0 \\ 0 & A_d^y \end{bmatrix}$ $D_d = \begin{bmatrix} D_d^s & 0 \\ 0 & D_d^y \end{bmatrix}$ and $\xi(t)$ represents an independent white gaussian which has a diagonal covariance matrix Σ_ξ. the matrices $A_d(z^{-1})$and $C_d(z^{-1})$are assumed to be square as follows.

$$A_d(z^{-1}) = I_r + A_1 z^{-1} + A_2 z^{-2} + \cdots + A_{n_a} z^{-n_a} \tag{21}$$

$$C_d(z^{-1}) = C_1 z^{-1} + C_2 z^{-2} + \cdots + C_{n_c} z^{-n_c} \tag{22}$$

where $A_d(z^{-1})$is regular. The order of the polynomial matrices is known, but the coefficient matrices are treated as unknown, since in practice the wave disturbance spectrum varies slowly with environmental condition.

Adaptive Kalman Filter Algorithms

Disturbance Estimator

The disturbance estimator can be designed from the equations (20) to (22). It is assumed that the disturbance motion is the second order ARMA model. Define the new variable $m_d(t)$ as the form.

$$m_d(t) = \varepsilon(t) + y_d(t) \tag{23}$$

where m_d is assumed the output of sway disturbance model with white measurement noise ε having covariance matrix Σ_ε. The innovation signal model is represented by following equation.

$$A_d(z^{-1}) m_d(t) = D_d(z^{-1}) e(t) \tag{24}$$

where e is a random noise having covariance matrix Σ_e. The optimal estimation value of $y_d(t)$ can be obtained by following equation [5][6].

$$\hat{y}_d(t|t) = m_d(t) - A_{n_a}^{-1} D_{n_d} e(t) \tag{25}$$

State Estimator

For the initial alignment system, both system dynamic motion signal and the environmental disturbance signal are measured with two accelerometers simultaneously. Therefore, the disturbance signal $y_d(t)$ is not separately measurable and must be replaced in the above Kalman filter by $\hat{y}_d(t|t)$. The adaptive filter algorithms have both a Kalman and a self-tuning filter and can produce the motion estimates by the following equations [7].

$$\textit{predictor}: \ \hat{x}(t|t-1) = A\hat{x}(t-1|t-1) + Bu(t-1) \tag{26}$$

$$\hat{y}(t|t-1) = C\hat{x}(t|t-1) \tag{27}$$

$$corrector: \hat{x}(t|t) = \hat{x}(t|t-1) + K(t)\bar{e}(t) \tag{28}$$

$$\hat{y}(t|t) = C\hat{x}(t|t) \tag{29}$$

And if the signal $n_d(t)$ is defined as the difference between $y_d(t)$ and its estimation value $\hat{y}_d(t|t)$, i.e. $n_d(t) = y_d(t) - \hat{y}_d(t|t)$ the following equations are derived.

$$\hat{x}_l(t|t) = A_l\,\hat{x}_l(t-1|t-1) + K_l(t)\,n_d(t) \tag{30}$$

$$\begin{aligned}\tilde{y}(t|t-1) &= C(\hat{x}(t|t-1) - \hat{\hat{x}}(t|t-1)) \\ &= CA\,\tilde{\hat{x}}(t-1|t-1)\end{aligned} \tag{31}$$

where, $\hat{\hat{x}}(t|t)$ denotes the change brought about by replacing $y_d(t)$ as $\hat{y}_d(t|t)$. And $\tilde{y}(t|t-1)$ is the difference between $\hat{y}(t|t-1)$ and $\hat{\hat{y}}(t|t-1)$, i.e. $\tilde{y}(t|t-1) = \hat{y}(t|t-1) - \hat{\hat{y}}(t|t-1)$. The state estimates are corrected by using the estimated value $\tilde{y}(t|t-1)$. Thus, the corrected estimate is given the form.

$$\hat{\hat{y}}(t|t) = \hat{y}(t|t) - \tilde{y}(t|t-1) \tag{32}$$

Parameter Identification

If the estimation value of m_d is defined as $\overline{m}_d$, i.e. $\overline{m}_d(t) = m_d(t) - \tilde{y}(t|t-1)$ the following equation is satisfied [7].

$$A_d(z^{-1})\,\overline{m}_d(t) = D_d(z^{-1})e(t) - A_d(z^{-1})\,\tilde{y}(t|t-1) \tag{33}$$

$\tilde{y}_l$ can be treated as a constant during the small calculation period. If the new parameter $s(t)$ is defined as the form, $s(t) = A_d(z^{-1})\tilde{y}(t|t-1)$ then the equation. (33) becomes the following form.

$$A_d(z^{-1})\,\overline{m}_d(t) = D_d(z^{-1})e(t) - s(t) \tag{34}$$

This equation can be represented in the conventional parameter identification method [7].

$$\overline{m}_d(t) = \phi(t)\theta + e(t) \tag{35}$$

For the second order model, the following parameters must be identified for each *i*th channel.

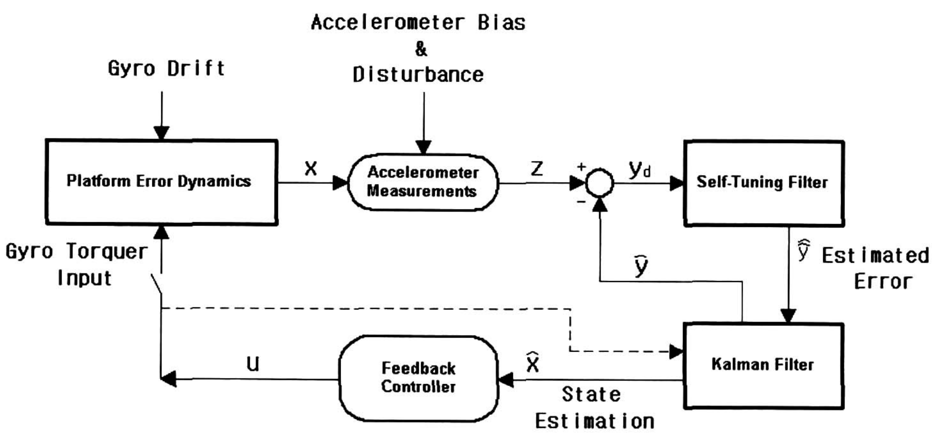


Fig. 4. The Structure of Designed Adaptive Kalman Filter

$$\psi_i(t) = [\ -\overline{m}_{d_i}(t-1),\ -\overline{m}_{d_i}(t-2)\ ;\ e_i(t-1),\ e_i(t-2)\ ;\ 1] \tag{36}$$

$$\theta_i^T = [a_{i1},\ a_{i2};\ d_{i1},\ d_{i2}\ ;\ s_i] \tag{37}$$

The structure of designed adaptive filter system having a Kalman filter, self-tuning filter and a feedback controller is represented in Fig. 4

Simulations

The simulation block was constructed with Matlab and Simulink. Fig. 5 shows the closed loop simulation block diagram having a self-tuning filter, a fixed gain Kalman filter and an optimal feedback controller. The purpose of the initial alignment is to control the three axes(North, East, Vertical Down) misalignment angles within small values. For the computer simulations, the initial 3 misalignment angles are set as 2*mrad*, 2*mrad* and 100*mrad,* in order. The optimal controller gain K_c is calculated by the steady state Riccati equation. The performance index function has the form.

$$J = \lim_{T\to\infty} \frac{1}{2T} E\{ \int_{-T}^{T} (x^T Qx + u^T Ru) dt \} \tag{38}$$

$$u(t) = -K_c \hat{x}(t) \tag{39}$$

It is not considered that the problem of actuator torque limits when the linear optimal controller is designed. But for the simulation of designed control system, the nonlinearity of actuators was considered by introducing saturation elements between controller and actuators, i. e.,

$$\begin{aligned} u_{ai} &= u_i \quad &;\ u_i \le u_{i\max} \\ &= u_{imax} \quad &;\ u_i > u_{i\max} \end{aligned} \tag{40}$$

Fig. 6 represents a typical form of sway disturbance used in this simulations. The simulation parameters are summarized in the following Table 1. Fig.7 represents the estimated parameter *a*, *d* and *s* for one channel. Three misalignment angles with time are shown in Fig. 8. The filtering system can align the two horizontal(North and East axes) misalignment angles within less than 0.1*mrad* and for the vertical down(D axis) misalignment angle can be aligned the angle less than 1*mrad* within 3 minutes.

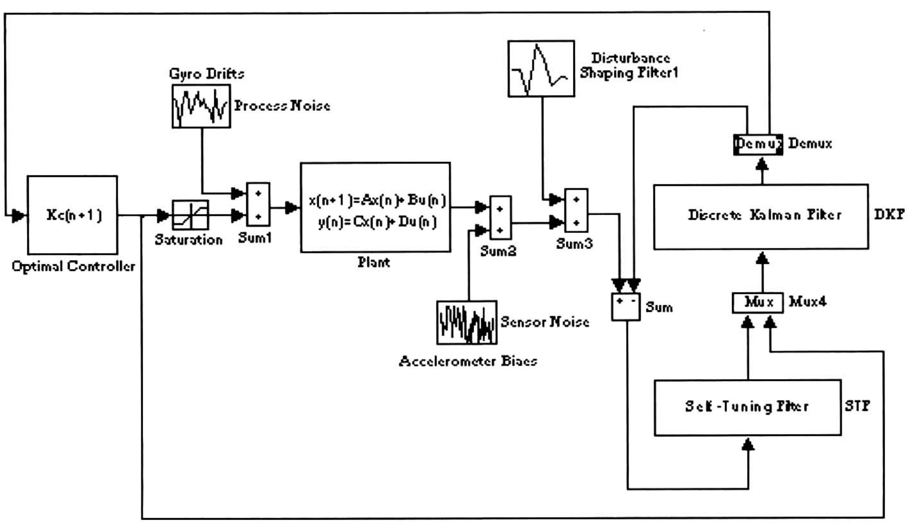


Fig. 5. Simulation Block Diagram

Table 1. The Simulation Parameter Values.

Initial values of misalignment angle	N-axes	2 *mrad*
	E-axes	2 *mrad*
	D-axes	100 *mrad*
Variance of gyro drift rates	4.25e-15 rad/sec^2	
Variance of accelerometer bias	9.6e-8 m^2/sec^4	
Variance of Random Sway acceleration	1.86e-6 m^2/sec^4	
Center frequency of sway acceleration	0.0053 ~ 0.053 *Hz*	

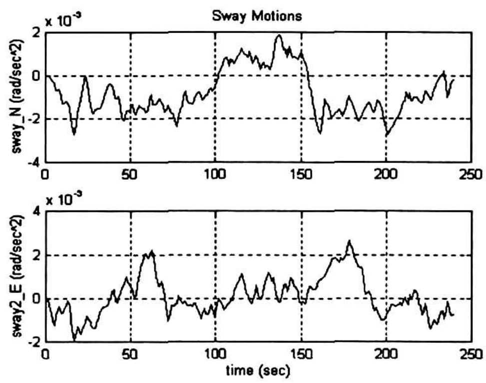
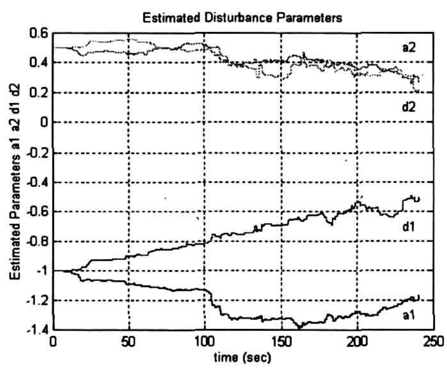
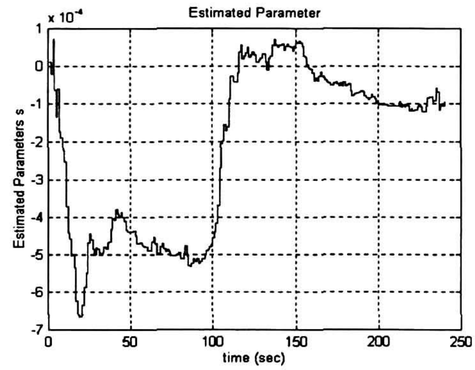


Fig. 6. Sway Disturbance

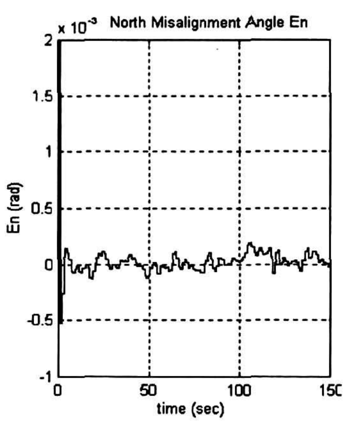


(a) a1, a2, d1, d2

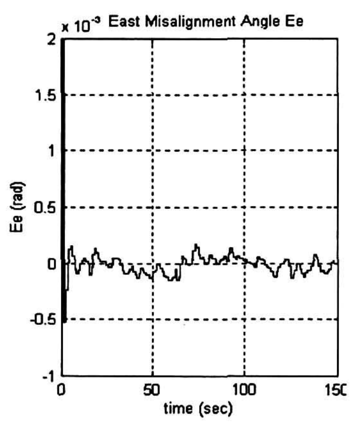


(b) s

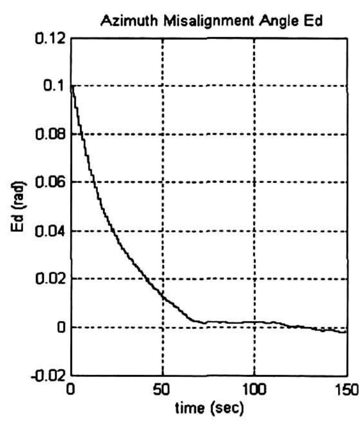
Fig. 7. Estimated Parameters



(a) Misalignment Angle En



(b) Misalignment Angle Ee



(c) Misalignment Angle Ed

Fig. 8. North, East and Vertical Down Axes Misalignment Angles

Conclusions

An adaptive filtering technique is developed for the intial alignment of inertial platform with varying sway disturbance. This filter can replace conventional fixed gain Kalman filter. Also, this adaptive filtering algorithm can be easily implemented to a actual vehicle navigation system owing to its low computation burden. The implementation of this type of self-tuning filter is fine solution to an aerial vehicle having unknown and varying environments. The designed filter can satisfy the desired accuracy performance in the presence of gyro drifts and acceleration uncertainties. Especially, for the case that the random acceleration is severe and varying, the filtering system can automatically adapt to the varying environmental conditions. The robustness is verified with unknown and varying disturbance with the large variation of its parameters. The result of this studies can be applied to the systems which have severe environmental conditions, but do not have automatic adaptation to the varying disturbance.

References

1. Cannon, R. H., 1961, "Alignment of Inertial guidance Systems by Gyro-compassing Linear Theory", *Journal of the Aerospace Sciences*, Vol.28, pp. 885-895

2. Winter, H., 1962, "The Modelling Error Sensitivity of Digital Filter for the Alignment of Inertial Platforms", AGARD Conference Proceeding, No. 116, papers 21C-1-21C-15

3. Parvin, R. H., 1962, "Inertial Navigation Systems; Prelaunch Alignment", *IRE Trans. on Aerospace Navigational Electronics*, pp. 141-145

4. Vathsal, S., 1986, "Design and Simulation of Closed-Loop Ground Alignment of Inertial Platforms with Sway Motion", *Journal of Guidance*, Vol. 9 No3, pp. 332-338

5. Panuska, V., 1980, "A new form of the extended Kalman filter for parameter estimation in linear systems with correlated noises', *IEEE Trans. Automatic Control*, Vol. AC-25, pp. 229-235

6. Hagander, P and Wittenmark, B., 1977, "A Self-tuning filter for fixed-lag smoothing", *IEEE Trans. Inform. Theory*, Vol. IT-23, pp. 377-384

7. Fung, P. T. and Grimble, M. J., 1983, "Dynamic Ship Positioning Using a Self-Tuning Kalman Filter", *IEEE Trans. on Automatic Control*, Vol. AC-28, No.3, pp. 339-349

8. Brown, R. G., 1991, *Random Signal Analysis and Kalman Filtering*, John Wiley & Sons, New York, pp. 296-326