Scalar Adaptive Kalman Filtering for Stellar Inertial Attitude Determination

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Abstract

This paper describes attitude determination algorithm for the low earth orbit(LEO) spacecraft using stellar inertial sensors. The cascaded gyro/star tracker extended Kalman filter is constructed to fuse two sensor data. And then the smoothing of the measurement are proposed for an unreasonable jump of star tracker. The smoothing algorithm for the rejection of star tracker error jumps is designed by scalar adaptive filter. The proposed algorithms operate to process the measurement of gyro/star tracker Kalman filter, therefore, it is comparatively simple to apply these methods to other integration systems. Simulations to gyro/star tracker integrated system show that the proposed method is effective.

Key Word : Scalar adaptive filter, Star tracker, Gyro, Extended Kalman filter, Attitude Determination

Introduction

All spacecrafts require some means of measuring or estimating their attitude relative to given inertial frame either in terms of three-axis knowledge(for three-axis stabilized vehicles) or spin-axis knowledge(for spin stabilized vehicles). To meet the requirements of the attitude determination for spacecrafts, many sensors such as star tracker, horizon scanners, sun sensors, magnetometers, or gyros have been used. In general two or more different attitude determination systems can be combined to improve or stabilize the attitude information. The star tracker and rate gyros are typically utilized as a combined attitude determination sensor package for high-accuracy requirements.[1] The attitude errors of gyroscope only increase with time due to inertial sensor errors, but the star tracker attitude errors are intrinsically bounded. So the star tracker can be combined with gyroscope successfully without error accumulation owing to the compensating properties of the two. Some popular approaches for determining spacecraft attitude and gyro bias are given in [2]. However, since the information of star tracker has a randomly abrupt jump caused by the high frequency jitter, the fast motion of the spacecraft and instrument noise, this approaches make the performance of star tracker/gyro system degraded[3][4].
In this study, extended Kalman filter formulations developed by Thompson and Quasius are utilized to estimate three-axis attitude errors and rate gyro biases from measured star vectors. And then the smoothing of the measurement designed by scalar adaptive filter is applied for an unreason jump of star tracker. An example illustrating the use of filter for spacecraft in an

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elliptical orbit is provided at the end of the paper along with some concluding remarks and recommendations.

Filter Formulation

The basic goal of a Kalman filter formulation applied to spacecraft possessing rate gyros and star tracker is to obtain a filtered three-axis attitude estimate of the vehicle in the presence of gyro and star tracker noise and gyro bias(drift). Historically, the state propagation equations were obtained from kinematics relations between the vehicle rotation rates and the vehicle attitude, and the state update equations were obtained from narrow FOV star tracker measurements of a unit vector to one star. Using modern wide FOV star trackers which can output measurements of three-axis attitude per sample, these state update equations can be modified so as to significantly simplify the filter algorithms. The need for gyros in such a system is to provide relative vehicle attitude between camera measurements and during temporary camera blockages or failures, as well as to provide vehicle angular rates for attitude control. In this section Kalman filter formulations based on [2] are developed for a spacecraft possessing a three-axis attitude (quaternion) sensor.

1. Definition of State Variables

Before one can derive the filter to estimate the attitude errors, one needs to define the desired state variables to be estimated and the noise parameters to be included in the filter. One would select minimum number of states to improve the filter's observability and select states having simple dynamics in order to ease filter's implementation. Therefore, the six component object defined by the vector components of the incremental quaternion and the drift-bias vector will provide a nonredundant representation of the state error[2][5]. The six-dimension body referenced state vector as

$$x=[q^{i} \quad \triangle b]^{T} \tag{1}$$

Tompson and Quasius[2] developed a filter formulation for narrow FOV star cameras in which the attitude was obtained by first rotating the vehicle by a small error relative to inertial space and then rotating the vehicle by an amount equal to the current attitude estimate. Using quaternion notation this process is depicted by

$$Q_b^i = Q_{\hat{i}}^i \otimes \hat{Q}_b^i \tag{2}$$

where Q_b^i is the true quaternion, $Q_{\hat{i}}^i(\equiv[q^i \quad q_{oi}]^T)$ is the error quaternion, $\hat{Q}_b^i$ is the best estimated quaternion, and $\otimes$ represents the quaternion multiplication operator[5].
The inclusion of three gyro bias corrections as state variables is necessary to remove error accumulation of gyro.

2. Dynamic Equations

The gyro bias errors can be modeled as unknown but constant parameters. Hence their dynamics are simply given by:[1]

$$\triangle \dot{b} = \eta_{bias} \tag{3}$$

where, η_{bias} are gyro rate random walk noises.

The dynamics of error quaternion $Q_{\hat{i}}^i$ can be obtained by differentiating Eq. (2) and utilizing the quaternion kinematic relation, $\dot{\hat{Q}}_b^i = \frac{1}{2} \hat{Q}_b^i \otimes (\omega_{ib}^b, 0)$ and the identity $a \otimes (b \otimes c) = (a \otimes b) \otimes c$. The resulting kinematic equations become

$$\dot{Q}_i^i = \frac{1}{2} Q_i^i \otimes \tilde{Q}_b^i \otimes (\omega_{ib}^b - \tilde{\omega}_{ib}^b, 0) \otimes \tilde{Q}_b^{i^{-1}} \tag{4}$$

where ω_{ib}^b represents the true body rate vector in the body frame, $\tilde{\omega}_{ib}^b (\equiv \omega_{ib}^b + \delta\omega_{ib}^b)$ represents the estimated body rate vector in the body frame, and $\tilde{Q}_b^{i^{-1}}$ is the inverse estimated quaternion. Upon assuming that Q_i^i is small, $\delta\omega_{ib}^b$ is small, and utilizing the quaternion transform relation $Q_b^i \otimes (v, 0) \otimes Q_b^{i^{-1}} = C_b^i v$, the final quaternion kinematics can be written in the matrix form[2].

$$\dot{q}^i = \frac{1}{2} \tilde{C}_b^i \delta\omega_{ib}^b = \frac{1}{2} \tilde{C}_b^i (\triangle b - \eta_{gyro}) \tag{5}$$

where $\tilde{C}_b^{i^T}$ represents the direction cosine matrix describing the orientation of the estimated body frame relative to the inertial frame.

3. Measurement Equations

Observations from a star camera are used to correct the estimated quantities at the star camera's update rate. This camera outputs a measured attitude in the form of a quaternion relative to inertial space. The camera output can be written in the form[2].

$$Q_m = n_{sc} \otimes Q_b^i \approx (\boldsymbol{n}_{sc}, 1) \otimes Q_b^i \tag{6}$$

where Q_m is the noisy star camera output quaternion converted to the body frame from the camera frame, and $\boldsymbol{n}_{sc}$ is the star camera random noise vector in the camera frame, modeled as a zero-mean Gaussian process, converted to the body frame from the camera frame.
Combining Eqs. (2) and (6) yields the error quaternion measurement equations

$$Z = [z_1 \ z_2 \ z_3 \ z_4]^T = Q_i^{im} = Q_m \otimes \tilde{Q}_b^{i^{-1}} = n_{sc} \otimes Q_i^i \approx (q^i + \boldsymbol{n}_{sc}, 1) \tag{7}$$

or in matrix form

$$z = [z_1 \ z_2 \ z_3]^T = q^i + C_c^b \boldsymbol{n}_{sc}^c \tag{8}$$

where $\boldsymbol{n}_{sc}^c$ is the star camera noise vector in the camera frame, $C_c^{b^T}$ represents the direction cosine matrix describing the orientation of the camera frame relative to the body frame and Z represents the error quaternion between the measured star camera attitude converted to the body frame and the best estimated attitude.

4. Extended Kalman Filter Formulation

In state-space representation, Eqs. (3), (5) and (8) can be combined and expressed as follows:[2]

$$\dot{x} = Ax + Gw \tag{9}$$

$$z = Hx + L\boldsymbol{n}_{sc}^c \tag{10}$$

where

$$A = \begin{bmatrix} 0_{3\times3} & \frac{1}{2}\tilde{C}_b^i \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix}, \quad G = \begin{bmatrix} -\frac{1}{2}\tilde{C}_b^i & 0_{3\times3} \\ 0_{3\times3} & I_{3\times3} \end{bmatrix}, \quad \omega = [\eta_{gyro} \ \eta_{bias}], \quad H = [I_{3\times3} \ 0], \quad L = C_c^b$$

1. Propagation at gyro sample frequency:

$$\begin{aligned} \dot{\tilde{Q}}_b^i &= \frac{1}{2} \tilde{Q}_b^i \otimes \tilde{\omega}_{ib}^b \\ P &= AP + PA^T + GQG^T \end{aligned} \tag{11}$$

2. Update at star camera measurement frequency:

$$K = P^{-}H^{T}(HP^{-}H^{T} + LRL^{T})^{-1}$$
$$P^{+} = P^{-} - KHP^{-} \tag{12}$$

$$[\tilde{q}^{i+} \ \triangle \hat{b}^{+}]^{T} = Kz \tag{13}$$

$$\hat{Q}_{b}^{i+} = [\tilde{q}^{i+} \ 1]^{T} \otimes \hat{Q}_{b}^{i-}, \qquad \hat{b}^{+} = \hat{b}^{-} - \triangle \hat{b}^{+} \tag{14}$$

Scalar Adaptive Filter

This part considers a adaptive approach to form estimation equation independently for each observable component of state vector. Since the information of star tracker has a randomly abrupt jump caused by various reasons, this adaptive filter is very effective to the performance of gyro/star tracker integration system.

Since the objective of integration system is to estimate the gyro error, the first order pre-filter can be introduced for filtering the measurement without distorting the error of gyro. Therefore the general error signal model can be described as

$$\delta x(k) = \delta x(k-1) + \delta V(k-1)T \tag{15}$$

where δx is measurement error and δV is measurement derivative error in the k'th step.

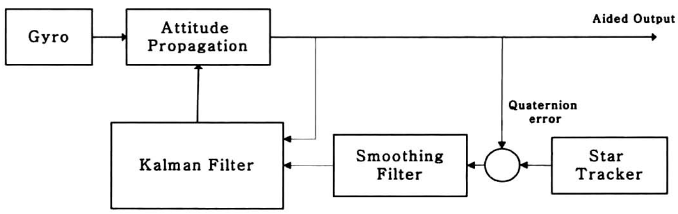

Fig. 1. The block diagram of the preprocessing schemes

A measurement model can be constructed by using attitude error signal

$$z(k) = \delta x(k) + \eta(k) \tag{16}$$

where, $\eta(k)$ is measurement noise

Scalar adaptive filter for the above model can be described by the following equation[3][6].

$$K(k) = \frac{P(k/k-1)}{P(k/k-1) + \hat{R}(k)}$$
$$P(k/k-1) = P(k-1) + \frac{2-K_{st}}{K_{st}} q(k)$$
$$P(k) = [1 - K(k)] \cdot P(k/k-1) \tag{17}$$
$$\delta \hat{x}(k) = \delta \hat{x}(k-1) + K(k)(z(k) - \delta \hat{x}(k-1))$$

where, $P(k)$: the state error variance

$\hat{R}(k)$: the measurement error variance

$P(k/k-1)$: the priori state error variance

K_{st} : the steady state Kalman gain

$q(k)$: the variance of $V(k)$

$\hat{x}(k)$: the estimated state

Hence, we can get an independent estimation Eq. (17) with adaptively adjusted gain $K(k)$ for any observable component of the state vector. The basic idea is to vary (q(k) and $\hat{R}(k)$) to make the filter adaptive. Eq. (17) can be operate to smooth high frequency signals by choosing proper $K(k)$. To implement the filter, suitable K_{st} should be set by using simulations.

To make the filter adaptive, we should suitably change these value in each step. In order to estimate $R(k)$, the innovation sequence ($\nu(k)=z(k)-\delta\hat{x}(k)$) can be used.

$$\hat{R}(k)=\nu(k)^2-P(k/k-1) \tag{18}$$

During the calculation procedure the estimate of $R(k)$ can be negative definite. In order to remove above obstacle, the normalization procedure, such as if $\hat{R}(k)<0$, then $\hat{R}(k)=0$ can be introduced. And the value of $q(k)$ is not a priori known and would depend on the real gyro quality and the time operation. In order to obtain the universal algorithm, q(k) can be calculated within estimation procedure.

$$Z=H\cdot\begin{bmatrix}\delta\hat{x}(0)\\ \delta\hat{V}(0)\end{bmatrix} \tag{19}$$

where,

$$H=\begin{bmatrix}1 & 1 & 1 & \cdots & 1\\ \triangle t(k) & 2\triangle t(k) & 3\triangle t(k) & \cdots & n\triangle t(k)\end{bmatrix}^T,$$

$$Z=[\,\delta\hat{x}(t-n\triangle t(k))\quad \delta\hat{x}(t-(n-1)\triangle t(k))\quad \delta\hat{x}(t-(n-2)\triangle t(k))\quad \cdots\quad \delta\hat{x}(t)]^T$$

The least square estimation about Eq. (19) can be calculated following as

$$\begin{bmatrix}\delta\hat{x}(0)\\ \delta\hat{V}(0)\end{bmatrix}=(H^TH)^{-1}H^TZ \tag{20}$$

Therefore, the estimation of $q(k)$ is obtained by

$$\hat{q}(k)=\delta\hat{V}(0)^2 \tag{21}$$

$n\triangle t(k)$ is time step for the least square estimation. t is current time. So, the least square solution is the velocity error variance of scalar adaptive filter

Simulation

1. Simulation Model

This section describes the computer simulation model that was developed to evaluate the adaptive Kalman filter performance. This study consists of a sun-synchronous spacecraft in an circular Earth orbit possessing a periapsis altitude of 685 km which corresponds to an orbital period of about 98.46 minutes. The three-axis star camera used for this study emulates the Ball CT-633 star tracker with the boresight of the camera aligned along the body fixed unit vector, $e=\sqrt{2}/2\cdot b_x+\sqrt{2}/2\cdot b_y+0.0\cdot b_z$, and with 1-sigma noise errors of 10 arcsec for boresight pointing. The gyro is chosen to emulate the Litton LN200 fiber-optic gyro used on the Clementine spacecraft, with gyro axis aligned along the vehicle axes and with 1-sigma noise errors of 1 deg/hr for each axis. The true bias rates about the x, y, and z axes are chosen to be -1.0 deg/hr, 1.0 deg/hr and 1.0 deg/hr, respectively. The bias error process noise is chosen to have a 1 sigma value 10^{-10} rad/s^2. The sampling rates used for this study are 20 Hz for the gyro-based attitude propagation and 1 Hz for the star camera updates. The intentional jittering noise is included from 900 sec to 940 sec, and the intentional bias noise for fast motion is included from 900 sec to 980 sec

2. Simulation Result

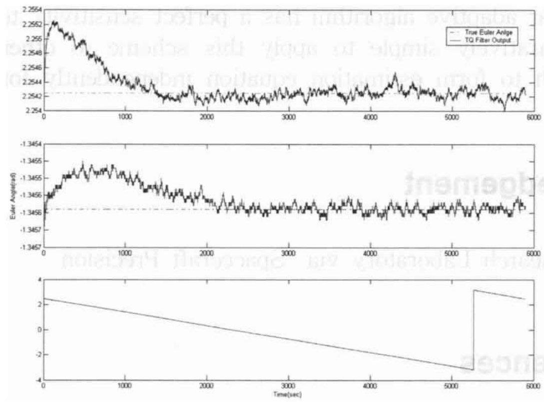

Fig. 2. (a). Estimated Euler Angle (original TQ Filter)

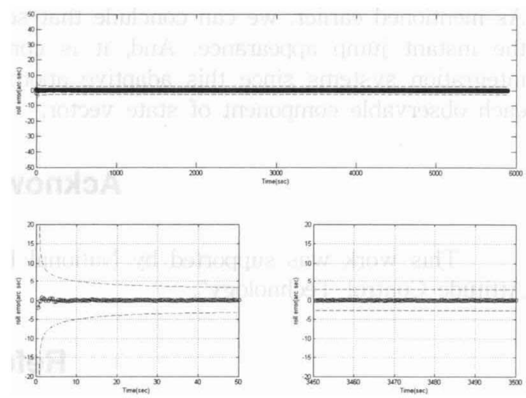

Fig. 2. (b). Attitude Error(original TQ Filter)

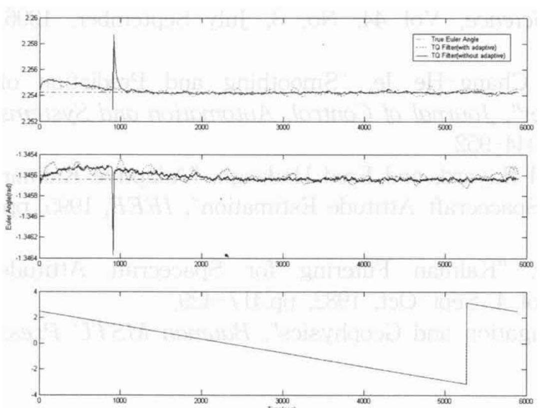

Fig. 3. (a). Estimated Euler Angle(jittering)

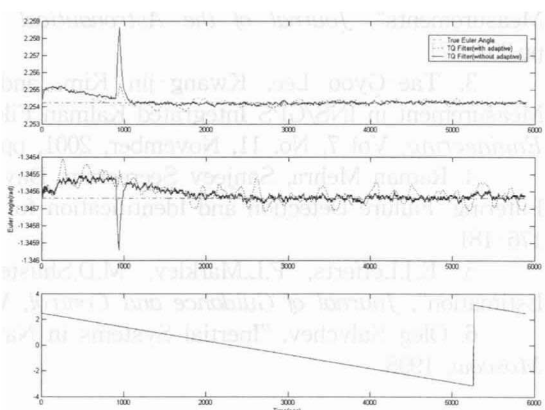

Fig. 3. (b). Estimated Euler Angle(fast motion)

To verify the original TQ(Tompson and Quasius) filter, time histories for estimated attitude and attitude error are presented in Fig. 2(a) and 2(b). In this study, two error jump condition(jittering and fast motion) is considered and the validation of the preprocessing schemes was checked by simulations with estimated attitude. The outputs of Kalman filter with and without the proposed smoothing filter are shown in Fig 3(a) and (b). The interpretation of the algorithm operating has clear physical meaning. When the star tracker measurements have no error jump(jittering or fast motion), the magnitude of innovation is reasonably small and algorithm trusts to the current measurement. In case when the error jump appears the magnitude of innovation instantly increases and gain coefficient is falling down. As a result, the current measurement takes part in estimate of δx with small weight. Consequently, the suggested algorithm actually can be considered to the error jump detector. Comparison of the three plots show that the proposed scalar adaptive filter is very effective to reject a star tracker error jumps in each case.

Conclusion

This study constructs the orignal TQ filter and applies the scalar adaptive filter to stabilize the measurement of gyro/star tracker integration Kalman filter. The effectiveness of the preprocessing scheme is shown by means of simulation in gyro/star tracker integration system. As mentioned earlier, we can conclude that scalar adaptive algorithm has a perfect sensitivity to the instant jump appearance. And, it is comparatively simple to apply this scheme to other integration systems since this adaptive approach to form estimation equation independently for each observable component of state vector.

Acknowledgement

This work was supported by National Research Laboratory via "Spacecraft Precision Attitude Control Technology".

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