Sliding Mode Attitude Control for Momentum-Biased Spacecraft

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Abstract

In this paper, we present a sliding mode control strategy for the re-orientation maneuver of rigid spacecraft containing rotating wheels. The wheels are considered as internal devices, and external inputs are employed for generation of control commands. The formulation is developed for a general case while particular example is applied to pitch bias momentum spacecraft with a single momentum wheel. The resultant control commands are used to take the gyroscopic effects into account which are caused by the rotating wheels. The controller designed demonstrates that the nutational motion of the pitch bias momentum spacecraft is effectively controlled. It is also assumed that the external control torque device is of on-off nature, and pulse width modulation technique is applied to construct proper control torque history.

Key Word : Spacecraft attitude control, sliding mode control, attitude quaternion, bias momentum stabilization, roll/yaw coupling

Introduction

Sliding mode control or Variable Structure Control(VSC) is an efficient control technique applicable to systems with significant nonlinearity and modelling uncertainty.[1,2] The controller design is based upon the so-called sliding surface on which system states remain while converging to an equilibrium state. The sliding surface is a pre-determined manifold to prescribe the trajectory of state variables by active control actions. The controller is constructed in a manner to drive the system states into the sliding surface from arbitrary initial states. General sliding mode controllers, therefore, consist of terms handling system nonlinearities and feedback terms to secure stability of the closed-loop system under unknown disturbances.

Sliding mode control has been addressed in some previous studies for spacecraft attitude control.[3-9] Most maneuver strategies are expressed in nonlinear feedback control using body angular rate and attitude parameters such as quaternions and modified Rodriguez parameters.[7] The resultant control laws led to three-axis nonlinear attitude maneuvers along the sliding-surface. The global stability is usually guaranteed by Lyapunov stability theory. The control device is assumed to be continuous type handling the continuous control command from the sliding mode control.

The rotating wheel mechanisms inside spacecraft may be either reaction wheels or momentum wheels. For the reaction wheel system, the wheel speed is maintained usually at zero being adjusted continuously to produce required control torque command. The reaction wheel itself can be directly used as an actuator for the sliding mode control since it is a continuous device. The momentum wheel also is a continuous device but with different operating range compared to the reaction wheels. The wheel speed of the momentum wheel is usually set to a non-zero value about which the speed is adjusted to produce desired control torque command.

A single wheel momentum bias system is an attractive choice for three-axis control of spacecraft. In particular, pitch bias momentum spacecraft is provided with a momentum wheel about orbit normal(or pitch) direction. The pitch momentum wheel possesses inherent angular momentum being adjusted within a certain range to control the pitch axis degree of freedom. The roll/yaw axes orthogonal to the pitch axis are controlled by additional control devices such as thrusters and magnetic torquers. The angular momentum of the pitch momentum wheel causes cyclic nutational motion about roll/yaw axes.

In this study, we design and conduct analysis on the sliding mode control for spacecraft model containing wheels as internal devices. By internal it is implied that the primary actuation device are external types such as thrusters and magnetic torquers. The formulation is a slight extension of other previous works on three-axis nonlinear control law. As a special case, a single wheel pitch bias momentum satellite is closely investigated to establish three-axis re-orientation maneuver strategy. The pitch axis is assumed to be controlled by continuous change of the angular momentum of a momentum wheel. For the roll/yaw degrees of freedom, on-off thrusters are employed to produce the desired control command. The continuous control command is pulse width modulated by the on-off thrusters. Simulation results verify three-axis re-orientation is achievable by the proposed controller.

In order to provide extended analysis, linearized roll/yaw dynamics are used to build simultaneous roll/yaw control by the sliding mode control approach. The dynamic coupling effect existing in the roll/yaw dynamics due to the pitch momentum wheel does not exist under the sliding mode control input. This is because gyroscopic terms are cancelled by the control command, so that conventional control strategies need to be modified. A dynamic observer is designed in order to estimate yaw attitude from roll measurement. The estimated yaw attitude is used to build sliding mode control input is yaw axis. The new controller therefore leads to independent roll/yaw control in sliding mode control technique.

Attitude Dynamics and Kinematics

The governing equations of motion for generic rigid spacecraft rotational motion are given by[10]

$$(J - J_w) \dot{\omega} + \omega \times \mathbf{H} + C^{-1} \mathbf{u}_w = \mathbf{T} \tag{1}$$

$$\mathbf{H} = J\omega + C^{-1} J_w \Omega \tag{2}$$

$$\dot{\Omega} = J_w^{-1} \mathbf{u}_w - C^{-1} \dot{\omega} \tag{3}$$

where $J(J_w)$ represents spacecraft body(wheel) moment of inertia, ω spacecraft body angular velocity vector, $\mathbf{H}$ body angular momentum, C^{-1} direction cosine matrix between the body and wheels, Ω wheel angular velocity vector, $\mathbf{u}_w$ control torque produced by the wheels, and $\mathbf{T}$ denotes external torque components. The external torque sources are assumed to be magnetic torquer and/or on-off thrusters.

For spacecraft attitude representation, quaternion parameter is a popular choice. The quaternion is defined first as

$$\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \equiv \begin{bmatrix} \mathbf{q}_{13} \\ q_4 \end{bmatrix} \tag{4}$$

where

$$\mathbf{q}_{13} = \mathbf{1}\sin\frac{\phi}{2}, \qquad q_4 = \cos\frac{\phi}{2} \tag{5}$$

and $\mathbf{1} = [l_1, l_2, l_3]^T$ denotes Euler's principal axis vector, and ϕ is the Euler principal angle. The quaternion satisfies the kinematics

$$\dot{\mathbf{q}} = \frac{1}{2}\Omega(\omega) = \frac{1}{2}\Xi(\mathbf{q})\omega \tag{6}$$

where $\omega = [\omega_1, \omega_2, \omega_3]^T$ is the angular velocity vector about spacecraft body axes, and

$$\Omega(\omega) = \begin{bmatrix} -[\omega\times] & \omega \\ -\omega^T & 0 \end{bmatrix}, \qquad \Xi(\mathbf{q}) = \begin{bmatrix} q_4 I_{3\times 3} + [\mathbf{q}_{13}\times] \\ -\mathbf{q}_{13}^T \end{bmatrix} \tag{7}$$

and

$$[\omega\times] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{8}$$

also similar definition is applied to $[\mathbf{q}_{13}\times]$. The error quaternion is defined as

$$\delta\mathbf{q} = \begin{bmatrix} \delta\mathbf{q}_{13} \\ \delta q_4 \end{bmatrix} = \mathbf{q} \otimes \mathbf{q}_d^{-1} \tag{9}$$

where $\otimes$ denotes the quaternion multiplier. The quaternion inverse can be expressed as

$$\mathbf{q}^{-1} = [-q_1, -q_2, -q_3, q_4]^T \tag{10}$$

Other useful relationships are given by

$$\delta\mathbf{q}_{13} = \Xi^T(\mathbf{q}_d)\mathbf{q}, \qquad \delta q_4 = \mathbf{q}^T\mathbf{q}_d \tag{11}$$

The subscript d denotes the *desired* state. The time derivatives of Eq. (11) can be shown to be

$$\delta\dot{\mathbf{q}}_{13} = \frac{1}{2}\delta q_4\omega + \frac{1}{2}[\delta\mathbf{q}_{13}\times]\omega \tag{12}$$

$$\delta\dot{q}_4 = -\frac{1}{2}\omega^T\Xi^T(\mathbf{q}_d)\mathbf{q} = -\frac{1}{2}\omega^T\delta\mathbf{q}_{13} \tag{13}$$

Attitude dynamics and quaternion kinematics have been introduced. They are now used to derive the sliding mode controller in the following section.

Sliding Mode Controller Design

The sliding mode controller design usually consists of two stages. The first state is to define a sliding surface, and the second stage is to develop a controller which satisfies the sliding condition which dictates the states remain on the sliding surface. On the sliding surface the sates converge to the desired equilibrium state.

The sliding surface employed in this study is similar to that of other previous studies.

Majority of derivations are taken from Ref. [9] and other references, but still presented to explain the final control law. By taking the quaternion attitude parameters and angular velocity vector, a *sliding surface* equation in vector form is proposed as follows.[7]

$$
\begin{aligned}
\sigma &= \omega + \Lambda \Xi^T(\mathbf{q}_d)\,\mathbf{q} \\
&= \omega + \Lambda \delta\,\mathbf{q}_{13}
\end{aligned}
\tag{14}
$$

where Λ is a positive definite matrix. In the ideal case, the sliding surface equation ($\sigma = 0$) should be satisfied for the convergence of state variables. Usually, there exist initial errors in the sliding surface equation, therefore stability condition must be satisfied as follows

$$
\frac{d}{dt}|\sigma|^2 < 0
\tag{15}
$$

The above equation is called *sliding condition*. In order to expand Eq.(14) further, we note that

$$
\begin{aligned}
\dot{\sigma} &= \dot{\omega} + \Lambda \delta\,\dot{\mathbf{q}}_{13} \\
&= -\bar{J}^{-1}\omega \times \mathbf{H} + \bar{J}^{-1}C^{-1}\,\mathbf{u}_w + \bar{J}^{-1}\,\mathbf{T} \\
&\quad + \frac{1}{2}\Lambda(\delta q_4 \omega + [\delta\,\mathbf{q}_{13}\times]\omega)
\end{aligned}
\tag{16}
$$

where $\bar{J} = J - J_w$ represents the spacecraft inertia matrix without wheels. Substitution of the above expression into the stability condition for the sliding surface yields

$$
\begin{aligned}
\sigma\dot{\sigma} = \sigma\big\{ &-\bar{J}^{-1}\omega \times \mathbf{H} - \bar{J}^{-1}C^{-1}\,\mathbf{u}_w + \bar{J}^{-1}\,\mathbf{T} \\
&+ \frac{1}{2}\Lambda(\delta q_4 \omega + [\delta\,\mathbf{q}_{13}\times]\omega)\big\}
\end{aligned}
\tag{17}
$$

The stability condition in Eq. (15) leads us to a feedback sliding mode controller in the form

$$
\mathbf{T} = \mathbf{T}_{eq} - K\sigma - D\,sign(\sigma)
\tag{18}
$$

where $\mathbf{T}_{eq}$ represents control history dictating the sliding surface equation as given by

$$
\mathbf{T}_{eq} = (\omega \times \mathbf{H} + C^{-1}\,\mathbf{u}_w) - \frac{1}{2}\bar{J}(\delta q_4 I_{3\times3} + [\delta q_{13}\times]\omega)
\tag{19}
$$

By applying the closed-loop control, the sliding condition in Eq. (15) becomes

$$
\frac{d}{dt}|\sigma|^2 = -\sigma^T K\sigma - \sigma^T D\,sign(\sigma) < 0
\tag{20}
$$

As it can be shown the final controller includes the internal wheel torque input ($\mathbf{u}_w$) as well as the total angular momentum ($\mathbf{H}$) of the spacecraft including that of wheels. If there is no wheel torque, i.e., constant wheel momentum, then the controller reduces to a simpler form with $\mathbf{u}_w = 0$. Even if the wheel torque is zero constant wheel angular momentum still explicitly affect the controller. For practical implementation of the sliding mode controller, the *sign* function is usually replaced by the $sat(s, \varepsilon)$ function to reduce the chattering effect on the sliding surface.

$$
sat(\sigma, \varepsilon) = \begin{cases} 1, & \text{if } \sigma_i > \varepsilon \\ \dfrac{\sigma_i}{\varepsilon}, & \text{if } |\sigma_i| < |\varepsilon| \\ -1, & \text{if } \sigma_i < -\varepsilon \end{cases}
\tag{21}
$$

On the condition that the sliding surface given by Eq. (15) is satisfied by the proposed controller, the following Lyapunov function candidate is examined to prove the convergence of

attitude variables.

$$U = \delta \mathbf{q}_{13}^T \delta \mathbf{q}_{13} + (1 - \delta q_4)^2 \qquad (22)$$

which can be rewritten as

$$U = 2(1 - \delta q_4) \qquad (23)$$

Applying Eq. (13), the time derivative of the Lyapunov function becomes

$$\dot{U} = -2\delta \dot{q}_4 = \omega^T \delta \mathbf{q}_{13} \qquad (24)$$

Now by substituting the sliding surface equation in Eq. (14), it can be written as

$$\dot{U} = -\delta \mathbf{q}_{13}^T \Lambda \delta \mathbf{q}_{13} < 0 \qquad (25)$$

Thus stability in the Lyapunov sense is guaranteed with the sliding surface condition. The external control torque stabilize the spacecraft system which contains gyroscopic elements. The internal wheel dynamics are accounted for by the stabilizing control law in Eq. (19). The gyroscopic coupling effect needs to be counteracted exactly to guarantee stability on the sliding surface. In general, the gyroscopic coupling effect leads to so-called nutational motion which could be a disturbing source in three-axis stabilization. Thus, such nutational motion should be handled actively to achieve the control objective.

Pitch Bias Momentum Spacecraft

As a special case of the sliding mode controller, the pitch bias momentum spacecraft model is investigated. The pitch bias momentum spacecraft employ a single momentum wheel which is normally pointed along the orbit normal direction. Figure 1 shows the geometric configuration of a pitch bias momentum spacecraft.
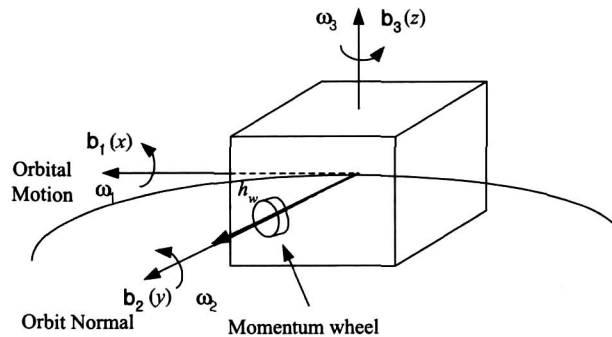

Fig. 1. Geometric configuration of the spacecraft model

For the sake of brevity in notation, the body axes are labeled as 1,2,3 corresponding to *roll, pitch, yaw axes*. The pitch axis is along the orbit normal as explained in the above. The pitch attitude is controlled by adjusting the wheel angular momentum. The roll/yaw control is achieved with the aid of additional actuators such as thrusters and/or magnetic torquers. The combination of pitch momentum wheel and additional actuators provide three-axis attitude control capability. The angular momentum vector of the spacecraft including a single momentum wheel in the pitch axis is defined as

$$\mathbf{H} = H_1 \mathbf{b}_1 + (H_2 + h_w) \mathbf{b}_2 + H_3 \mathbf{b}_3 \qquad (26)$$

where $[H_1, H_2, H_3]$ is a vector of spacecraft body angular momentum, $[\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3]$

represents unit vectors along the spacecraft body axes as shown by Fig. 1, and h_w is angular momentum of the wheel. The generalized governing equations in Eq. (1) can be expanded into set of scalar equations for the pitch bias momentum spacecraft.

$$\begin{aligned}
I_1\,\dot{\omega}_1 + (I_2 - I_3)\omega_2\omega_3 - h_w\omega_3 &= T_1 \\
I_2\,\dot{\omega}_2 + (I_3 - I_1)\omega_1\omega_3 + \dot{h}_w &= T_2 \\
I_3\,\dot{\omega}_3 + (I_1 - I_2)\omega_1\omega_2 + h_w\omega_1 &= T_3
\end{aligned} \tag{27}$$

In the pitch bias momentum spacecraft, the frequency of roll/yaw nutational motion is approximated as[10]

$$\nu = \sqrt{\frac{h_s^2}{I_1 I_2}} \tag{28}$$

where the parameter h_s is defined as

$$h_s = (I_1 - I_2 + I_3)\omega_0$$

and ω_0 is a constant orbital rate. The gyroscopic terms in the controller presented in Eq. (19) are given as

$$\omega \times \mathbf{H} = \begin{bmatrix} \omega_2 H_3 - \omega_3 H_2 - \omega_3 h_w \\ \omega_3 H_1 - \omega_1 H_3 + \dot{h}_w \\ \omega_1 H_2 - \omega_2 H_1 + \omega_1 h_w \end{bmatrix}$$

It should be noted that the gyroscopic coupling effect by the momentum wheel appears in the roll/yaw axes. The pitch axis is independent of the gyroscopic coupling effect due to the wheel. As mentioned above, the pitch momentum wheel is engaged in the pitch axis control, and the time rate of change of the wheel angular momentum can be considered as control torque. In other words,

$$T_2 = 0 \tag{29}$$

Since the momentum wheel is used to generate continuous control command, the pitch axis component of the sliding mode control command from Eq. (18) is directly incorporated into the wheel torque command($\dot{h}_w$).

Next the control commands are distributed to the roll/yaw axes. The roll/yaw control commands correspond to T_1, T_3 in $\mathbf{T}$ in Eq. (18). The roll/yaw controls are assumed to be performed by on-off thrusters. For general pitch bias momentum spacecraft, the roll/yaw axes are controlled by thrusters and or magnetic torquers. Thus control commands need to be handled in a different manner compared to the pitch axis.

Pulse width Modulation

The control signals commanded by the sliding mode controller are continuous type. They can be implemented using actuators such as reaction wheels for continuous control command. By convention, the external torque devices, excluding wheels, are largely on-off actuators. Thus the control signals presented in Eq. (8) needs to be implemented in conjunction with the on-off actuators.

For discrete type actuators, continuous signals can be converted into equivalent discrete signals by PWM(Pulse Width Modulation). The key idea of PWM is to produce pulse trains at equal intervals and each pulse has different pulse width in proportion to the original continuous signal. We assume that the thruster pulse width is modulated so that the thruster on time is manipulated reconstructing the original signal. Figure 2 shows a block diagram for the idea of PWM implementation of the sliding mode controller. The continuous control signal is directly modulated and the final control signal is constant amplitude with varying pulse widths.
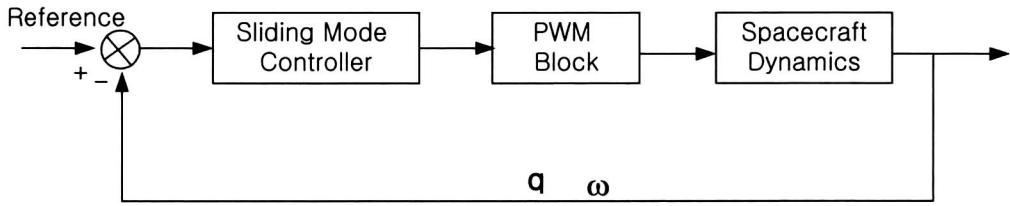
Fig. 2. Feedback Control with PWM

In this study, the pulse width is evaluated as

$$\tau_w = \varDelta T u / N \qquad (30)$$

where τ_w is the pulse width, $\varDelta T$ control signal update interval, N the maximum torque level of thrusters, and u is he continuous control signal.

Simulation

Simulation study by applying the controller designed above has been conducted. The moment of inertia property of the spacecraft itself is assumed to be

$$J = \begin{bmatrix} 9,780 & 0 & 0 \\ 0 & 5,788 & 0 \\ 0 & 0 & 9,966 \end{bmatrix} (\text{in} - lbf - \sec^2)$$

while the nominal angular momentum of the wheel is taken to be $h_w = 275$ (in-lbf-sec). The period of nutational motion is approximately 127 seconds. The control update interval is set to $\varDelta T = 0.5$ seconds and the maximum torque level is $N = 5.0$ (in-lbf). The final desired quaternion is set to be $[0, 0, 0, 1]$. Relatively large initial quaternion errors are prescribed to examine the control law for a large angle maneuver. The control signal is turned on 200 seconds after simulation start. The simulation results are presented in Figs. 3 to 6.

It can be shown that initial attitude errors are eliminated in all three axes. PWM of the continuous sliding mode control signals in roll/yaw axes is effective in achieving the control objective. Another noteworthy point is nutational motion induced by the wheel angular momentum. The initial nutational motion is also controlled quickly after activation of the controller. The sliding mode controller cancelled gyroscopic terms due to the pitch wheel, so that the nutational mode is controlled quickly. The angular velocity history(Fig. 3) and quaternion responses(Fig. 4) imply that the linear sliding surface equation is satisfied upon the control input activation. The pulse width response in Fig. 5 also demonstrates the effectiveness
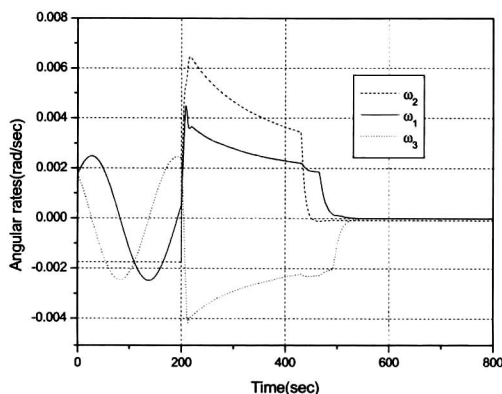

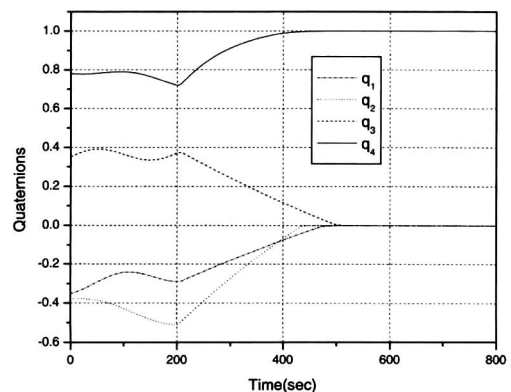Fig. 3 Simulation results of angular velocity histories
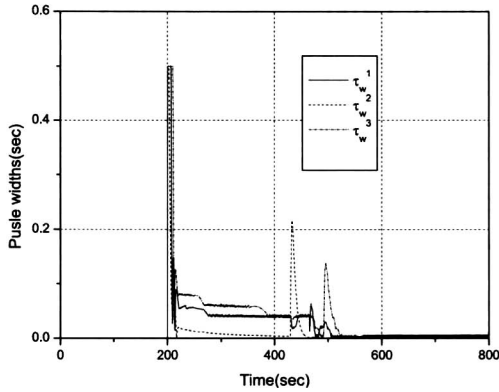

Fig. 4 Simulation results of attitude quaternions

Fig. 5. Response of pulse widths of the
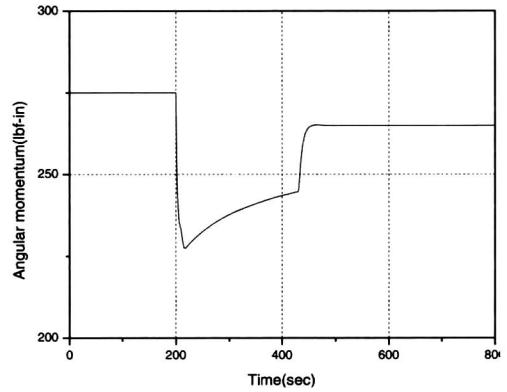control signal

Fig. 6. History of angular momentum
of the wheel

Linearized Roll/yaw Dynamics

So far, the pitch control via a momentum wheel and roll/yaw control using on-off actuators were applied to a nonlinear maneuver of a bias momentum spacecraft. For further analysis, we investigate the controller in terms of linearized governing equations. For pitch wheel bias momentum spacecraft, the roll/yaw axis control is performed independent of pitch axis. Gyroscopic coupling effect by the pitch momentum wheel is exploited for the roll/yaw control. In this section, we try to correlate the sliding mode controller proposed in the previous section to the roll/yaw control over linearized dynamics. This will certainly lead to highlight the characteristic of the original sliding mode controller from different physical viewpoint.

The linearized kinematics between the angular velocity and Euler attitude angles are approximated as

$$\begin{Bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{Bmatrix} \simeq \begin{Bmatrix} \dot{\phi} - \omega_0 \psi \\ \dot{\theta} - \omega_0 \\ \dot{\psi} + \omega_0 \phi \end{Bmatrix} \tag{31}$$

Then the linearized governing equations of motion neglecting gravitational gradient torques are given by

$$I_1 \ddot{\phi} - h_s \omega_0 \dot{\psi} - [(I_3 - I_2)\omega_0 + h_w]\omega_0 \phi = T_1 \tag{32}$$

$$I_2 \ddot{\theta} + \dot{h_w} = T_2 \tag{33}$$

$$I_3 \ddot{\psi} + h_s \omega_0 \dot{\phi} + [(I_2 - I_1)\omega_0 - h_w]\omega_0 \psi = T_3 \tag{34}$$

where $h_s = (I_1 + I_2 - I_3)\omega_0$ is again introduced. The pitch axis degree of freedom is independent of roll/yaw axes. In order to gain some insight about roll/yaw controls, a control law is introduced from Ref. [1]. The bias momentum spacecraft adopts Earth sensors to measure pitch and roll attitude angles. The cyclic nutational and orbital motion leads to alternating roll/yaw dynamics. Hence, the yaw attitude is controlled by the roll/yaw dynamic coupling property. The feedback control laws in roll/yaw axes are proposed as

$$T_1 = -(k_p \phi + k_d \dot{\phi}), \qquad T_2 = a T_1 \tag{35}$$

The control inputs are given using roll information only. The design parameters (k_p, k_d, a) are determined by substituting the control inputs into the governing equations. Then the closed-loop dynamics become a fourth order system, and the design parameters are selected from the stability condition on the closed-loop system.

Meanwhile, the roll/yaw equations of motion can be regrouped as Eqs. (32) and (34). For the sliding mode controller design a sliding surface equation is defined as

$$\sigma = \left\{ \begin{matrix} \sigma_r \\ \sigma_y \end{matrix} \right\} = \left\{ \begin{matrix} \dot{\phi} + k_r \phi \\ \dot{\psi} + k_y \psi \end{matrix} \right\} \tag{36}$$

where subscripts r, y denote $roll(\phi)$ and $yaw(\psi)$ degrees of freedom, respectively. Based upon the final sliding mode controller in Eq.(8), it follows as

$$T_1 = T_{1eq} - K_r \sigma_r - D_r sat(\sigma_r) \tag{37}$$

$$T_2 = T_{2eq} - K_r \sigma_y - D_y sat(\sigma_y) \tag{38}$$

for which T_{1eq}, T_{2eq} correspond to equivalent control inputs in both axes as they are given by

$$T_{1eq} = h_s \omega_0 \dot{\psi} + [(I_3 - I_2)\omega_0 + h_w]\omega_0 \phi \tag{39}$$

$$T_{2eq} = -h_s \omega_0 \dot{\phi} - [(I_2 - I_1)\omega_0 - h_w]\omega_0 \psi \tag{40}$$

The sliding mode control in Eqs. (37) and (38) result in decoupled closed-loop dynamics which are absent from nutational dynamics. This is because the nutational dynamics are cancelled by the control inputs. Therefore, the controller type in Eq. (35) as functions of roll information only may not work with sliding mode control. Independent control action need to be taken for both roll and yaw axes, respectively.

Since both roll and yaw attitudes are involved with the controllers, the yaw angle information need to be provided. In this study, we propose a dynamic observer design to estimate the yaw information by using the roll measurement. The observer design, as it is a conventional approach, takes linear state space form of equations of motion. Hence, roll/yaw dynamics are rewritten

$$\dot{\mathbf{x}} = A \, \mathbf{x} + B \, \mathbf{u} \tag{41}$$

where $\mathbf{x} = [\phi, \psi, \dot{\phi}, \dot{\psi}]^T$ is a state vector, and system matrices are given by

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -p_2/I_1 & 0 & 0 & -p_1/I_1 \\ 0 & -p_4/I_3 & -p_3/I_3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/I_1 & 0 \\ 0 & 1/I_3 \end{bmatrix}$$

and $p_i(i = 1, 2, 3, 4)$ are appropriate parameters from Eqs. (33) and (35). The dynamic observer for the given system dynamics can be written as

$$\dot{\mathbf{x}}_e = A \, \mathbf{x}_e + B \, \mathbf{u} + L(y - C \, \mathbf{x}_e) \tag{42}$$

where $\mathbf{x}_e$ is the estimated state, and L is the observer gain. The measurement equation is given by

$$y = \phi = [1, 0, 0, 0] \, \mathbf{x} \tag{43}$$

Now for the given system dynamics, the observer gain is selected in such a way that

$$\lambda_i(A - LC) < 0, \quad i = 1, 2, 3, 4 \tag{44}$$

for which λ_i is an i-th eigenvalue of the closed-loop system of the error states

$$\dot{\mathbf{e}} = (A - LC) \, \mathbf{e}, \quad \mathbf{e} = \mathbf{x} - \mathbf{x}_e \tag{45}$$

The observer gain(L) is designed by a pole placement technique. The desired closed-loop system dynamics are prescribed first, and the observer gain is computed based upon the given
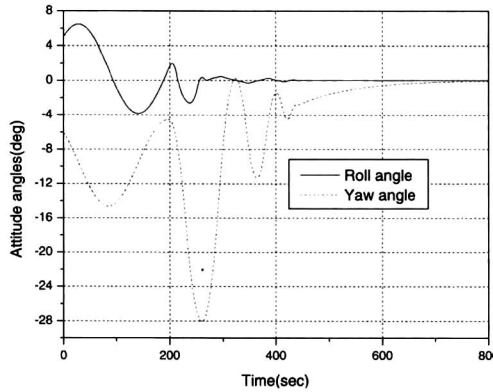
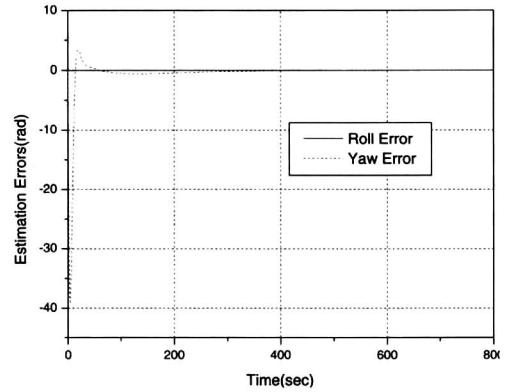Fig. 7. History of angular momentum
of the wheel

Fig. 8. History of angular momentum
of the wheel

dynamics. The estimated yaw state is used to build sliding control commands. The control commands are modified as

$$T_1 = \hat{T}_{1eq} - K_r \sigma_r - D_r sat(\sigma_r) \qquad (46)$$

$$T_2 = \hat{T}_{2eq} - K_y \hat{\sigma}_y - D_r sat(\hat{\sigma}_y) \qquad (47)$$

where the hat sign denotes variables which consist of states estimated by the observer. The stability of the observer now ensures the performance of the roll/yaw controllers without yaw measurement. Simulation has been conducted by combining the controller and observer. The roll/yaw attitude angles are plotted in Fig. 7. The initial attitude errors are shown to be overcome by the proposed control laws in the above. The nutation mode is controlled more quickly than the orbital mode. The steady state error due to orbital mode causes rather sluggish responses.

Figure 8 shows the estimation errors in each axis. The yaw estimation error initially shows excessive overshoot as it is estimated from a large initial uncertainty. There exist certain limitation in achieving yaw estimation performance. This also can be explained by the nature of the controller which decouples the roll/yaw dynamics. The performance of the estimator is essentially dependent upon the dynamic coupling also. Further investigation may be needed in order to achieve satisfactory performance of the observer.

Conclusions

A sliding mode controller, for the attitude maneuver of three-axis spacecraft with internal wheels, has been proposed and verified through simulation study. Special focus was laid on a single axis pitch bias momentum spacecraft model. External actuators producing on-off outputs can be used to implement nonlinear sliding mode control commands in combination with internal reaction wheel elements. Nutational motion caused by the pitch momentum wheel was controlled by on-off thrusters with pulse width modulation of the original control commands. Linearized roll/yaw dynamics led to the potential application of the sliding mode controller with estimated state variables.

Acknowledgement

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