# Reconfigurable Flight Control System Design Using Sliding Mode Based Model Following Control Scheme

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## Abstract

In this paper, a reconfigurable flight control system is designed by applying the sliding mode control scheme. The sliding mode control method is a nonlinear control method which has been widely used because of its merits such as robustness and flexibility. In the sliding mode controller design, the signum function is usually included, but it causes the undesirable chattering problem. The chattering phenomenon can be avoided by using the saturation function instead of signum function. However, the boundary layer of the sliding surface should be carefully treated because of the use of the saturation function. In contrast to the conventional approaches, the thickness of the boundary layer of our approach does not need to be small. The reachability to the boundary layer is guaranteed by the sliding mode controller. The fault detection and isolation process is operated based on a sliding mode observer. To evaluate the reconfiguration performance, a numerical simulation using six degree-of-freedom aircraft dynamics is performed.

**Key Word** : Reconfiguration, Sliding mode control, Sliding mode observer, Fault detection

## Introduction

As the missions of flight control systems are more complicated, the reconfigurability of the flight control systems becomes more important. The purpose of the reconfigurable flight control system (RFCS) is to compensate faults during the flight system operation.[1-3] For pre-determined fault modes, the RFCS has been usually implemented using the fault detection and isolation (FDI) process. This kind of RFCS must have the fast adaptivity to the trim condition.

In this paper, a new RFCS using the sliding mode controller and observer is proposed. The proposed method is classified into the FDI based RFCS. The proposed method utilizes the dynamic characteristics of the equations of aircraft motion that the equations of aircraft motion can be divided into two parts: the kinematic equation part and dynamic equation part. The kinematic equation part is used for the design of the sliding surface, and the model following technique is applied to the sliding mode controller. The control input in dynamic equation part should be determined such that the system states be remained on the designed sliding surface in spite of the occurrence of the fault. The sliding mode observer is introduced to estimate the unmeasurable value of the time derivative of sliding mode variables, and the FDI process is performed using the sliding mode observer. The sliding mode observer can enlarge the thickness of the boundary layer as well as detect the fault. To verify the effectiveness of the

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proposed RFCS design method, a numerical simulation is performed using a nonlinear 6-DOF aircraft equation and a elevator fault model.

## Aircraft Equations of Motion

Consider the linearized aircraft equations of motion as follows:

$$\dot{x} = Ax + Bu \tag{1}$$

where $x$ denotes the state vector, and $u$ denotes the control input vector. Generally, the aircraft equations of motion can be separated into two parts: the kinematic part equation and the dynamic part equation. The kinematic part equation is not explicitly related with the control input $u$, and the kinematic equation part includes explicit control input terms. By separating the kinematic and dynamic equations, Eq.(1) can be rewritten as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \tag{2}$$

where $x_1$ is the kinematic part state vector, and $x_2$ is the dynamic part state vector. To divide $x$ into $x_1$ and $x_2$, the condition $B_1 = 0$ should be satisfied. In this case, Eq. (2) represents the system whose relative degree is one. However, the separation of the equations of motion into two parts may not be possible in real aircraft systems because the value of $B_1$ is small but non-zero. Thus, the control method compensating the weak coupling effect between $\dot{x}_1$ and $u$ is required. Rewriting the kinematic equations and the dynamic equations independently, the following expressions can be obtained.

*Kinematic part:*

$$\dot{x}_1 = A_{11} x_1 + A_{12} x_2 + d_1 \tag{3}$$

*Dynamic part:*

$$\dot{x}_2 = A_{21} x_1 + A_{22} x_2 + B_2 u \tag{4}$$

In Eq.(3), the disturbance-like term of $d_1$ is introduced to accommodate the effects of non-zero of $B_1$. The fault considered in this study is the addictive fault in the dynamic part equation, which can be expressed as follows:

$$\dot{x}_2 = A_{21} x_1 + A_{22} x_2 + B_2 u + Ef \tag{5}$$

where $f$ denotes a fault and $E$ denotes a fault matrix. In most cases, the fault matrix $E$ is not known, and therefore replacing $Ef$ by $F$ gives

$$\dot{x}_2 = A_{21} x_1 + A_{22} x_2 + B_2 u + F \tag{6}$$

Note that the considered fault $F$ only affects the dynamic part equation. This kind of fault modeling includes the faults such as the actuator mal-function and control surface damages. Eqs. (3) and (6) are jointly used to design the RFCS. The sliding surface is designed using Eq. (3) while the FDI process is designed using Eq. (6).

## Sliding Mode Based Reconfigurable Flight Control System Design

### Sliding surface design using the kinematic part equations

The conventional sliding mode control method consists of two phases. One is the reaching phase to the sliding surface and the other is the maintaining phase on the sliding surface. To apply the sliding mode controller, a sliding surface should be designed. Although the design procedure of the sliding mode controller is well-constructed, the sliding mode control theory does not prescribe how the sliding surface should be designed. Therefore, the effective procedure for the design of the sliding surface is required. In this study, the model following controller presented in Ref. [1] is used for the sliding surface design. In the kinematic equation Eq.(3), the state vector $x_2$ can be considered a pseudo-input, thus, the controller can be determined and the resulting control law becomes the desired sliding surface. Other control methods can be also an alternative approach for the sliding surface design. To apply the model following controller in Ref. [1], the reference model for the model following controller is chosen as

$$\dot{x}_m = A_m x_m + B_m r \tag{7}$$

where $x_m$ denotes state variables of reference model, and $r$ denotes the reference command. It is assumed that the system matrix $A_m$ is stable, and the input matrix $B_m$ is invertible. The sliding surface is represented as

$$x_{2,d} = C_0 r + C_0 G_0 x_1 + C_0 v + C_0 K_0 x_m \tag{8}$$

where $v$ is a control vector, and $C_0$, $G_0$, and $K_0$ are control gains. Subscript $d$ is introduced because Eq. (8) is not a actual control law, but the desired relation. To guarantee the asymptotic stability, matrix gains in Eq.(8) should be designed to satisfy the following relations.

$$G_0 = B_m^{-1}(A_e - A_{11}) \tag{9}$$

$$C_0 = A_{12}^{-1} B_m \tag{10}$$

$$K_0 = B_m^{-1}(A_m - A_e) \tag{11}$$

where $A_e$ is an arbitrary stable matrix and $A_{12}$ is an invertible matrix. The vector $v$ in Eq.(8) is designed as

$$v = -B_m^{-1} d_1 \tag{12}$$

Let us define the sliding surface variable $s$ as

$$s = x_2 - x_{2,d} \tag{13}$$

The time derivative of Eq.(13) is given by

$$\dot{s} = A_{21} x_1 + A_{22} x_2 + B_2 u + F - \dot{x}_{2,d} \tag{14}$$

## Sliding mode controller design

Consider the following Lyapunov candidate function for the dynamic equation part.

$$V = \frac{1}{2} s^T s \tag{15}$$

Time derivative of Eq.(15) is given by

$$\dot{V} = s^T \dot{s} = s^T \left[ A_{21} x_1 + A_{22} x_2 + B_2 u + F - \dot{x}_{2,d} \right] \tag{16}$$

By the sliding mode control theory, the control input is designed as follows:

$$u = B_2^{-1} \left[ -A_{21} x_1 - A_{22} x_2 + \dot{x}_{2,d} - K\, sgn\left(\frac{s}{\delta}\right) \right] \tag{17}$$

where $K = diag[k_1, \cdots, k_n]$ is a control gain matrix and $sgn(s)$ denotes $[sgn(s_1), \cdots, sgn(s_n)]^T$. Substituting Eq.(17) into Eq.(16) yields

$$\dot{V} = -s^T K sgn(s) + s^T F \tag{18}$$

If the following conditions are satisfied, then $\dot{V} \leq 0$ is obtained.

$$k_i > |F_i|, \quad (i = 1, \cdot \cdot \cdot, n) \tag{19}$$

In a practical point of view, the use of signum function $sgn(\cdot)$ in Eq.(17) may cause the chattering problem which is the control input vibration phenomenon with high frequency. Since the chattering is undesirable, the signum function is usually replaced by the first-order or third-order saturation function. In our study, the third-order saturation function is adopted. The third-order saturation function can be expressed as

$$sat(\frac{s}{\delta}) = \begin{bmatrix} -1, & when \ s < -\delta \\ \frac{-1}{2\delta^3}(s^3 - 3\delta^2 s), & when \ -\delta < s < \delta \\ 1, & when \ s > \delta \end{bmatrix} \tag{20}$$

where $\delta$ represents the boundary layer thickness of the sliding surface. Using Eq.(20), Eq.(17) is rewritten as follows under the assumption that $B_2$ is nonsingular.

$$u = B_2^{-1}\left[ -A_{21}x_1 - A_{22}x_2 + \dot{x}_{2,d} - K sat(\frac{s}{\delta}) \right] \tag{21}$$

The thickness of the boundary layer $\delta$ should be determined considering the performance of the controller and the chattering regulation. The large value of $\delta$ could effectively prevent the chattering phenomenon, however, the small value of $\delta$ is more desirable in a point of the performance of the sliding mode controller.[4]

## FDI with the sliding mode observer

In the presence of the fault, the condition of Eq.(19) would not be always satisfied. Within the boundary layer, $\dot{V} \leq 0$ is not guaranteed even if Eq.(19) is satisfied. The boundary layer is resulted from the use of saturation function instead of the signum function.

If the value of fault is properly estimated, then the performance of the controller can be improved. In this section, the FDI method based on the sliding mode observer is discussed. Substituting Eq.(21) into Eq.(14) yields

$$\dot{s} = -K sat[\frac{s}{\delta}] + F \tag{22}$$

Note from Eq.(22) that the fault can be estimated by

$$\hat{F} = \dot{s} + K sat[\frac{s}{\delta}] \tag{23}$$

To implement Eq.(23), the values for $s$ and $\dot{s}$ must be known. The sliding variable $s$ can be easily calculated from the state measurements and the relation of Eq.(8). However, the variable $\dot{s}$ cannot be measured, and therefore the estimator for $\dot{s}$ is required. The sliding mode observer[5-6] contributes to estimate $\dot{s}$ in our approach. The sliding mode observer for estimating $\dot{s}$ can be constructed as

$$\hat{s}_1 = s \tag{24}$$

$$\hat{s}_2 = \dot{s} \tag{25}$$

$$\dot{\hat{s}}_1 = -L_1 sgn[\hat{s}_1 - s] \tag{26}$$

$$\tau \dot{w}_1 = -w_1 - L_1 sgn[\hat{s}_1 - s] \tag{27}$$

$$\dot{\hat{s}}_2 = -L_2 sgn[\hat{s}_2 - w_1] \quad (28)$$

where $L_i$'s are gains, $\tau$ is a time constant for adjusting the bandwidth of the low pass filter, and $w_i$ is parameter for smooth estimation. If the value of fault vector is available, the fault effect can be compensated by simply adding the estimated fault to the real control input. Thus, the final form of the proposed control law is described by

$$u = B_2^{-1}\left[ -A_{21}x_1 - A_{22}x_2 + \dot{x}_{2,d} - K sat(\frac{s}{\delta}) + \hat{F} \right] \quad (29)$$

Substituting Eq.(29) into Eq.(16) yields

$$\dot{V} = -s^T K sat(s) + s^T(F - \hat{F}) \quad (30)$$

The stability condition $\dot{V} \le 0$ is guaranteed if $F = \hat{F}$. Because the saturation function is used in Eq. (30), $\dot{V} \le 0$ holds regardless of the boundary layer. This observation implies that the thickness of the boundary layer can be arbitrarily expanded as long as the FDI process is properly operated.

## Numerical Simulation

In the numerical simulation, it is investigated how the aircraft lateral dynamics is affected by the elevator control surface damage. Consider the linearized aircraft lateral motion equation with elevator damage.

$$\dot{x} = Ax + Bu + Ef \quad (31)$$

where $x = [\beta, \phi, p, r]^T$, $u = [\delta_a, \delta_r]^T$ and $f = \delta_{e,f}$ denotes a elevator deflection angle with damage. Note that the states are divided into $x_1 = [\phi, \beta]^T$, and $x_2 = [p, r]^T$. The system matrix $A$ and input matrix $B$ are expressed as follows:[7]

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{V_T} & \frac{g\cos\theta}{V_T} & \frac{\cos\gamma}{\cos\theta} & \frac{\sin\gamma}{\cos\theta} \\ 0 & 0 & \frac{Y_p}{V_T} & \frac{Y_r}{V_T} \\ \mu L_\beta + \sigma_1 N_\beta & 0 & \mu L_p + \sigma_1 N_p & \mu L_r + \sigma_1 N_r \\ \mu N_\beta + \sigma_2 L_\beta & 0 & \mu N_p + \sigma_2 L_p & \mu N_r + \sigma_2 L_r \end{bmatrix} \quad (32)$$

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \frac{Y_{\delta_a}}{V_T}(\approx 0) & \frac{Y_{\delta_r}}{V_T}(\approx 0) \\ \mu L_{\delta_a} + \sigma_1 N_{\delta_a} & \mu L_{\delta_r} + \sigma_1 N_{\delta_r} \\ \mu L_\beta + \sigma_2 N_\beta & \mu L_{\delta_r} + \sigma_2 N_{\delta_r} \end{bmatrix} \quad (33)$$

The coefficients of $\sigma_i$ and $\mu$ are the constants related with the conversion of the moment of inertia from the body axis to the stability axis. The detail definitions and explanations are presented in Ref. [7]. The fault matrix E is given by

$$Ef_{elev} = d_a(1-\zeta_e)\begin{bmatrix} 0 \\ 0 \\ \mu L_{\delta_a} + \sigma_1 N_{\delta_a} \\ \mu N_{\delta_a} + \sigma_2 L_{\delta_a} \end{bmatrix}\delta_e \tag{34}$$

where $d_a$ is the ratio of the lateral elevator location to the aileron location from the center of mass, $\zeta_e$ is the ratio of effective elevator area. In the numerical simulation, $d_a = 2.0$, and $\zeta_e = 0.7$, are chosen.

The considered dynamics are F-16 nonlinear equations of motions.[8] Through the numerical linearizaion with respect to the level flight condition, the following system matrix $A$ and input matrix $B$ are can be obtained.

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} -0.1295 & 0.0698 & 0.1308 & -0.9882 \\ 0 & 0 & 1.0000 & 0.1317 \\ -34.6234 & 0 & -2.7237 & 0.8917 \\ 7.8114 & 0 & -0.0478 & -0.3850 \end{bmatrix}$$

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0.0068 & 0.0186 \\ 0 & 0 \\ -29.3421 & 4.5153 \\ -1.2046 & -2.4006 \end{bmatrix}$$

From the above matrices, the following statements can be made.

*Remarks 1.* $A_{12}$ and $B_2$ are invertible.

*Remarks 2.* Elements of $B_1$ are nearly zero.

Remarks 1 and 2 jointly support the validity of the assumptions in the controller design. The model dynamics for the reference system is chosen as

$$\begin{bmatrix} \dot{\beta}_m \\ \dot{\phi}_m \end{bmatrix} = \begin{bmatrix} -1.5 & 0 \\ 0 & -1.5 \end{bmatrix}\begin{bmatrix} \beta_m \\ \phi_m \end{bmatrix} + \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}\begin{bmatrix} \beta_{com} \\ \phi_{com} \end{bmatrix} \tag{35}$$

The roll maneuver is considered. The reference command for the roll maneuver is chosen as

$$\beta_{com} = 0^o$$

$$\phi_{com} = \begin{cases} 0^o & 0 \le t \le 5 \\ 5^o & 5 \le t \le 10 \\ 0^o & 10 < t < 15 \end{cases}$$

Actuator dynamics is modeled as the following first-order filter.

$$H(s) = \frac{20}{s+20} \tag{36}$$

Fig. 1 shows the considered elevator fault, $F = Ef = [F_1, F_2]^T$. The fault is assumed to occurs at 8 seconds, which is the elevator stuck with angle of 5 degrees. Fig. 2 shows the estimated fault by FDI process based on the sliding mode observer. As shown in Fig. 2, the fault is estimated with accuracy by the proposed FDI process. A little deviation at 5 and

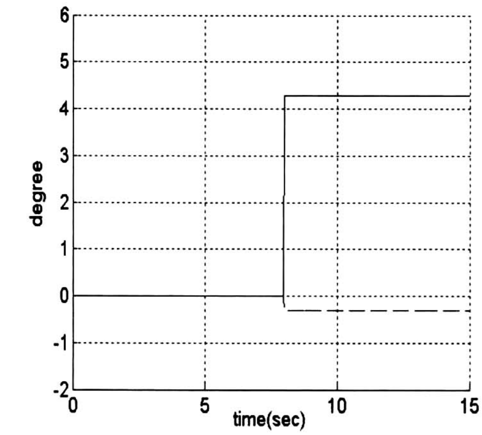


**Fig. 1. Fault scenario**

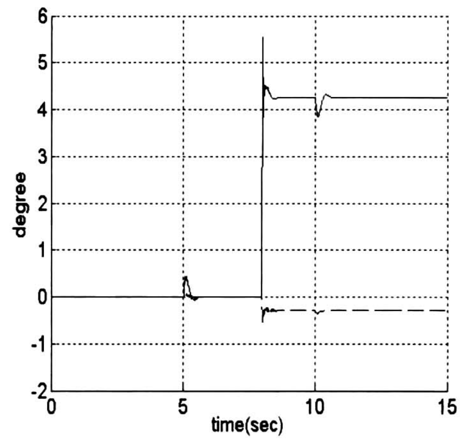


**Fig. 2. Estimated Fault**

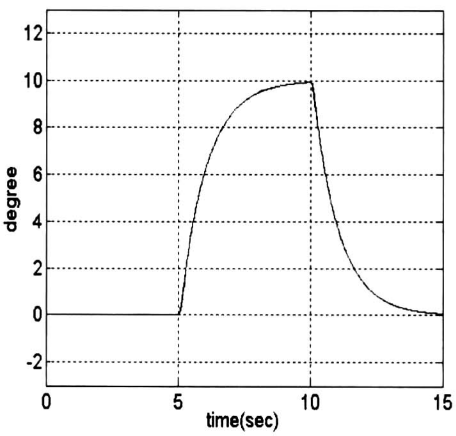


**Fig. 3. Roll Angle**

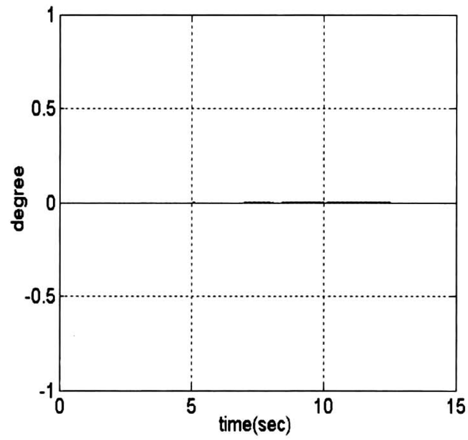


**Fig. 4. Sideslip Angle**

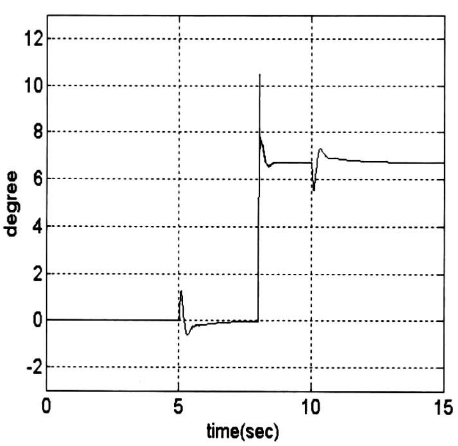


**Fig. 5. Aileron deflection**

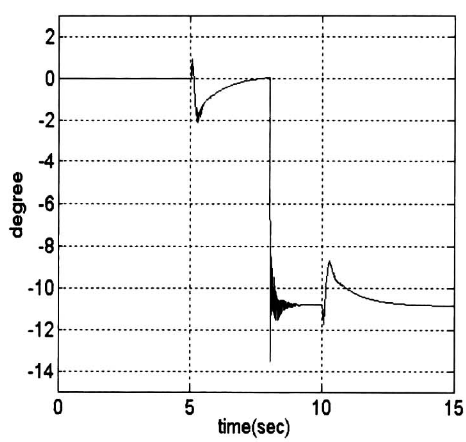


**Fig. 6. Rudder deflection**

10 seconds is caused by the abrupt roll maneuver command. Fig. 3 shows the model following performance of the proposed control method for the reference system. The response of the roll angle is represented by solid lines, and the model output is represented by circles. In spite of the elevator fault the proposed controller shows good model following performance. The model

following error convergence characteristic is achieved by the sliding mode controller and the compensation of the estimated fault. The response of the sideslip angle is represented in Fig.4. The proposed controller makes the sideslip angle remain zero during all simulation time. Figs. 5 and 6 show the aileron and rudder deflection angles. After the event of the considered elevator fault, the control input commands are rapidly converged to the new trim conditions.

## Conclusion

In this paper, a new reconfigurable flight control method based on the sliding mode control scheme is proposed. It is shown that the aircraft equations of motions can be divided into two parts by the presence of the direct input effect: the kinematic equations and the dynamic equations. The kinematic equation is used for the sliding surface design, and the model following technique is demonstrated for the sliding surface design. The dynamic equation is used for the sliding mode controller and the fault detection and isolation process. The FDI process is performed based on the sliding mode observer which can estimate the unmeasurable time derivative of the sliding variables. The proposed control method has the advantage that the fault detection and isolation process is accommodated into the sliding mode control design. Hence, the reconfiguration is simply accomplished by compensating the estimated fault vector. As a numerical example, the F-16 aircraft lateral motion is adopted. The elevator damage on the lateral aircraft motion is considered as a fault. The effectiveness of the proposed control method is discussed with numerical simulation results.

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