A New Method of determining Initial Conditions for Satellite Formation Flying

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Abstract

Satellite formation flying is the placing micro-satellites with the same mission into nearby orbits to form a cluster. Clohessy-Wiltshire equations are used to describe the relative motion and control strategies between satellites within a cluster, which are known as Hill's equations. Even though Hill's equations are powerful in determining initial conditions for the satellite formation flying, they can not accurately express the relative motion under J2 perturbation. Some methods have been developed for the determination of initial conditions to avoid limits of Hill's equation. This paper gives a new method of determining initial conditions using mean elements. For this research mean elements were transformed to osculating elements using Brouwer's theory and the orbit was propagated with the consideration of J2-J8 to get a relative position. The results show that satellites within a cluster are maintained in the desired boundary for long period and the method is effective on a fuel saving for satellite formation flying.

Key words : Formation Flying, Hill's Equation, Initial Conditions, Relative Motion

Introduction

In recent years, a research on satellite formation flying has been studied by various authors. Some of the proposed missions are LISA, ST3, ORION, Auroral Lites, TechSat-21 and ION-F(http://www.aem.umn.edu/proj-prog/distributed_sc/main.html). EO-1 was launched in November 2000 and is the first satellite in NASA's NWM (New Millennium Program) Earth Observing series. It forms a cluster with Lansat-7 to get the same ground image. Formation flying system has several benefits compared to the single spacecraft system that has equivalent functions: low cost for launch and mass production, larger aperture size, greater launch flexibility, higher system reliability and easier expandibility. The concept of a satellite formation flying is frequently confused with that of a satellite constellation. As defined by the NASA, a constellation is composed of two or more spacecrafts in similar orbits with no active control to maintain a relative position. Station-keeping and orbit maintenance are performed based on geocentric states, so groups of Global Positioning System (GPS) satellites or communication satellites are considered constellations. In contrast, formation flying involves the use of an active control scheme to maintain the relative positions of the spacecraft. The

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difference lies in the active control of the relative states of the formation flying satellite(Folta and Newman, 1996).

Hill's equations are a set of linearized equations that describe the relative motion between satellites, which were used to describe a relative motion of rendezvous mechanics in the past and a satellite formation flying these days. Hill's equations are generated under the assumption that the reference orbit is circular, the Earth is spherically symmetric, and the target satellite is very close to the reference orbit. These assumptions lead that there is no external perturbing force and the nonlinear terms in the relative motion can be ignored. So Hill's equations can not accurately describe the relative motion under gravitational perturbation.

The primary perturbation is due to J2 which causes three important effects on the satellites formation flying: nodal regression, and drifts in perigee and the mean anomaly. This means that J2 perturbation causes satellites within a cluster to slowly separate and then an additional orbit control is required to maintain the formation of satellites, which reduces the life time of satellites. Therefore initial orbits of satellites within the cluster should be determined to minimize the fuel consumption for formation flight. Schaub and Alfriend(1999) developed analytical conditions for minimizing the effects by J2 perturbation using mean Hamiltonian. Vadali *et al.*(2002) calculated numerical solution by applying period-matching constraint. Schweighart and Sedwick(2002) derived the linearized equations which can capture the effect of J2 potential. Vaddi *et al.*(2003) developed initial conditions analytically by deriving a nonlinear equations including reference satellite's eccentricity. There is another method to obtain initial conditions, which is a state transition matrix that includes the gravitational perturbations and reference orbit eccentricity. For a time-explicit representation to the bounded solutions of the linearized relative motion, a state transition matrix that includes small eccentricities had been derived by Melton(2000). Gim and Alfriend(2001) derived a state transition matrix in the presence of Earth's oblateness effect.

The constraints of an in-plane and out-of-plane motion matching were applied in the analytical method developed by Schaub and Alfriend(1999). However this method cannot give good solutions for a satellite formation flying with high inclination. A numerical method using an in-plane matching constraint(Vadali *et al.*, 2002) gives better solutions than an analytical method, but it needs iterative loops for computing solutions. This paper gives a new numerical method of determining initial conditions for satellite formation flying. This new method uses the constraints of an in-plane and out-of-plane motion matching to maintain a formation under J2 perturbation. Another characteristic of this new method is that iterative loops are not necessary. TechSat-21 is tested as an example to evaluate the efficiency of the new method.

Equations for determining the initial conditions

The solutions of Hill's equations cannot describe the relative motion accurately under the perturbations such as Earth oblateness or air drag. If effects of these perturbations is not controlled, the satellite formation will break down. However, frequent orbit maneuvers will increase the fuel consumption and thus reduce the life of a satellite. So orbit design is needed to minimize fuel consumption for the formation flying. It is impossible to get a orbit which is not disturbed by all perturbations. Therefore, it is necessary in the mission design of formation flying to include at least the J2 effect, which causes the dominant perturbation on satellite orbits.

The relative orbit geometry can be described with the differences in mean orbit elements which do not show any of the short period oscillations. Some mean elements such as longitude of ascending node, argument of perigee and mean anomaly experience secular drift, short period motion and long period motion under Earth oblateness including the J2 perturbation. Other mean elements such as semi-major axis, inclination and eccentricity are possessed of only periodic motion. A secular effect of these elements due to J2, using orbit averaged elements are given as

$$\dot{\Omega} = -\frac{3nR_e^2J_2}{2a^2(1-e^2)^2}\cos(i)$$
$$\dot{\omega} = \frac{3nR_e^2J_2}{4a^2(1-e^2)^2}(4-5\sin^2(i)) \qquad (1)$$
$$\dot{M} = n + \frac{3nR_e^2J_2}{4a^2(1-e^2)^{3/2}}(3\cos^2(i)-1)\ , \quad n=\sqrt{(\mu/a^3)}$$

where Ω is the longitude of ascending node, ω is the argument of perigee, M is the mean anomaly, a is the semi-major axis, e is the eccentricity, i is the inclination, n is the mean motion, R_e is the Earth radius and μ is the gravitational constant of the Earth. Other mean elements such as semi-major axis, inclination and eccentricity are possessed of only periodic motion. On the other hand, osculating elements vary with time, which are defined at any instant in time by the corresponding position and velocity vectors. As mentioned above, mean elements are averaged over some selected time without periodic variations. Mean elements are most useful for long-range mission planning because they approximate the satellite's long-term behavior. So, it is desirable to use mean elements for determining initial conditions of satellite formation flying. Mean elements should be converted to corresponding osculating elements analytically or numerically in order to compute the inertial position and velocity. Brouwer(1959) developed the algorithm which could map between mean elements and osculating elements. The original Brouwer solution also contained singularities for orbits with small inclinations and eccentricities and at the critical inclination of 63 degree and 26 minute. Lyddane(1963) described solutions for the above problems in his paper. His method consists of formulating the perturbation theory and Hamiltonian in terms of Poincare's Elements. Felix(1981) also introduced an alternate set of variables which resulted in a solution which is more computationally efficient than the original Brouwer-Lyddane algorithm.

A rotating local-vertical-local-horizontal (LVLH) frame is used to visualize the relative motion between two satellites. The x-axis points in the radial direction, the z-axis is perpendicular to the orbital plane and points in the direction of the angular momentum vector. Finally, the y-axis points in the along-track direction. Hill's equations are given as

$$\ddot{x}-2n\dot{y}-3n^2x=a_x$$
$$\ddot{y}+2n\dot{x}=a_y \qquad (2)$$
$$\ddot{z}+n^2z=a_z$$

in the LVLH frame. The accelerations on the right hand side are non-central forces(drag, thrust, etc.). Note that the terms of the first equation are total, Coriolis and centripetal acceleration from left to right and the out-of-plane motion is decoupled from the in-plane motion. To solve Hill's equations, it is assumed that there are no perturbed forces. Periodic solutions to the above equations can be obtained by the requirement that the periods of the orbits of two satellites must be equal. Periodic solutions are very important because they can give the key in determining initial conditions for satellite formation flying. Periodic solutions of relative motion are given below(**Alfriend *et al*, 2000**).

$$x=A\sin(\Psi+\alpha) \qquad \dot{x}=An_0\cos(\Psi+\alpha)$$
$$y=2A\cos(\Psi+\alpha) \qquad \dot{y}=-2An_0\sin(\Psi+\alpha)$$
$$z=B\sin(\Psi+\beta) \qquad \dot{z}=Bn_0\cos(\Psi+\beta) \qquad (3)$$
$$\text{where,}\ A=\sqrt{x_0^2+y_0^2/4}, \qquad B=\sqrt{z_0^2+\dot{z}_0^2/n^2}$$
$$\tan\alpha=2x_0/y_0, \qquad \tan\beta=nz_0/\dot{z}_0, \quad \Psi=n_0t$$

Herein, the subscript 0 is used for the reference satellite and the subscript 1 for the target satellites.

Under the influence of the J2 perturbation, the perturbed satellite will have a different orbital period than that when unperturbed. If this discrepancy is not considered in the determination of initial conditions, the satellites in the cluster drift from the reference orbit and then fuel consumption is required to reconfigure the formation. Thus, the period of the reference orbit must be adjusted to maintain the formation. The change of period due to J2 can be found from the average J2 force(Schweighart and Sedwick, 2002). So, mean motion in the Eq. (3) should be substituted with the 'mean' mean motion considering the new period. Mean-mean motion is given by

$$\vec{\omega}\times(\vec{\omega}\times\vec{r}_{ref}) = -\frac{\mu}{r_{ref}^2}\hat{r} + \frac{1}{2\pi}\int_0^{2\pi}\vec{J_2}(\vec{r}_{ref})d\theta$$
$$where, \quad \vec{\omega} = \bar{n}\hat{z}, \quad \bar{n} = n\sqrt{1+s}, \quad s = \frac{3J_2R_e^2}{8a^2}(1+3\cos(2i)) \tag{4}$$

Constraint for matching in-plane and out-of-plane motion

TechSat-21 is a program of DoD (Department of Defense) focused on the development and on-orbit demonstration of various formation flying technologies, which will be launched in the end of 2004. It consists of three satellites which will be placed in an equilateral triangle of the circular horizontal plane orbit as Fig. 1. Each satellite will be equipped with X-band SAR(Synthetic Aperture Radar) which yields high-resolution image(about 3m) of the terrestrial surface. Thus, the relative distance between satellites is important. It is known that Techsat-21 will fly in a near circular orbit at an altitude of 550km and at a 35.4 degree of orbital inclination.

The equilateral triangle will be break down under the J2 perturbation because J2 makes the secular drifts for Ω, ω and M. Unfortunately, the rates given by Eq. (1) cannot be the same simultaneously for two satellites with different mean a, e and i(Vadali *et al*, 2000). However, the equilateral triangle can be maintained from the J2 perturbation using constraint of in-plane and out-of-plane motion matching. The equilateral triangle can not be established without out-of-plane motion which can be created using a node difference or an inclination difference between two satellites because out-of-plane motion is due solely to the fact that the orbits of two satellites are not coplanar. A maximum separation occurs at the maximum latitude in case of an inclination difference and occurs at the equator in the case of a node difference.

Under the J2 perturbation, the equilateral triangle will break down if initial values of Eq. (3) are applied. Therefore, it is necessary to impose the constraints in determining initial conditions in order to maintain the equilateral triangle. Angular rates of two satellites must be matched to enforce bounded motion in the in-plane direction and secular drifts of the longitude of ascending node must be equal to enforce bounded motion in the out-of-plane direction. For small differences in the inclinations of two satellites, the constraint of angular rate matching(ARM) is given by Eq. (5) and the constraint of secular drift matching of the longitude of ascending node (SDMAN) is given by equation (6).

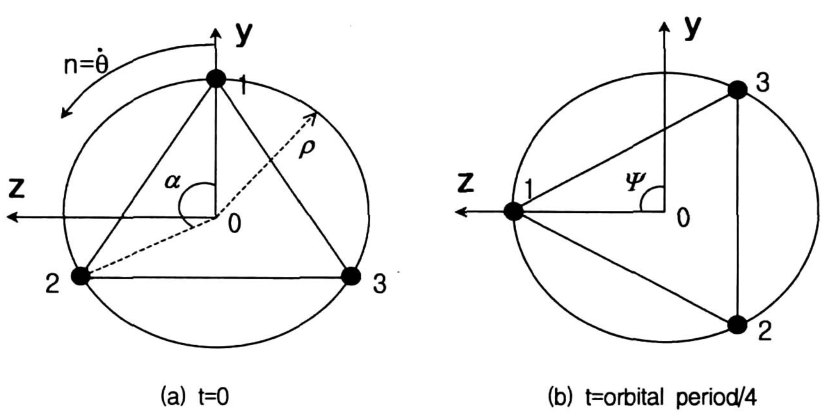


Fig. 1. Constellation of TechSat-21 in the horizontal plane (θ is sum of argument of perigee and true anomaly)

$$(\dot{\omega}_1 - \dot{\omega}_0) + (\dot{M}_1 - \dot{M}_0) + (\dot{\Omega}_1 - \dot{\Omega}_0)\cos(i_0) = \delta\dot{\omega} + \delta\dot{M} + \delta\dot{\Omega}\cos(i_0) = 0 \quad (5)$$

$$\dot{\Omega}_1 - \dot{\Omega}_0 = \delta\dot{\Omega} = 0 \quad (6)$$

In the low earth orbit, air drag is an important factor, which can disturb the formation flying. However, differential drag has a negligible effect on the relative motion compared to the perturbative effect of J2 if the two satellites have similar aerodynamic characteristics(Vadali *et al.*, 2002).

Determination of Initial Conditions

Alfriend *et al.*(2000) derived the relationship between the relative position and velocity variables and the orbital element differences. This method is called as the geometric approach which can be used to construct the state transition matrix in the gravitational perturbation. To establish the initial equilateral triangle, the following equation should be satisfied

$$\begin{aligned} & y^2 + z^2 = \rho^2 \\ & \text{where, } y = a_0(\delta\theta + \delta\Omega\cos(i)) \\ & \qquad\quad\; z = a_0(\sin(\theta_0)\delta i - \cos(\theta_0)\sin(i_0)\delta\Omega) \end{aligned} \quad (7)$$

If the initial constellation such as Figure 1(a) is set, initial conditions can be determined for δi, $\delta\Omega$, $\delta\theta$, x, y, z and the velocity of x-axis using Eqs. (3), (4) and (7). And δa and δe can be computed by the above constraints because δi is given. Ultimately relative velocities of y and z-axis can be calculated from the above parameters using geometric approach method. Note that mean-mean motion is used in the periodic solutions to determine the velocity of x-axis. In the paper of Vadali *et al.*(2002), initial conditions can be determined with the assumed mean elements of the target satellite through iterative loops which update the mean orbital elements of target satellite with conditions like as Eq. (7) and the constraint of only ARM. However, the new method suggested in this paper does not estimate the velocity of x-axis through a iterative loop because the velocity considers the effect of J2 using mean-mean motion. Therefore, initial conditions can be determined directly with ARM and SDMAN constraints. A schematic layout of determining the initial conditions and computing the inertial vector is shown in Fig. 2.

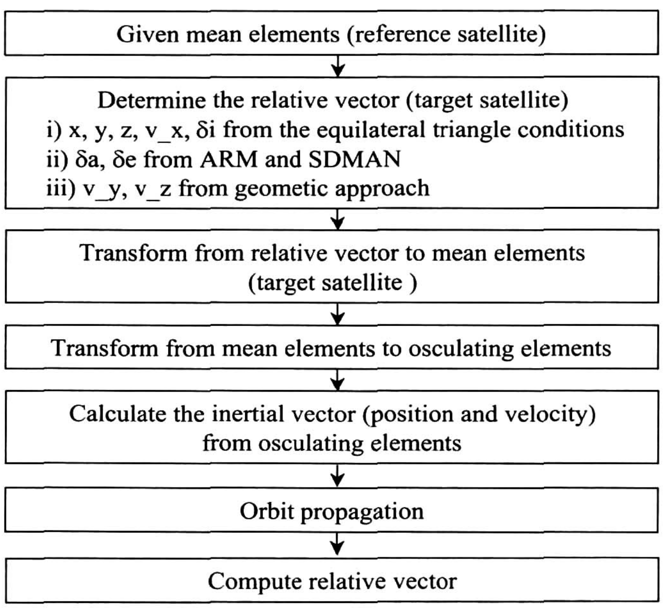


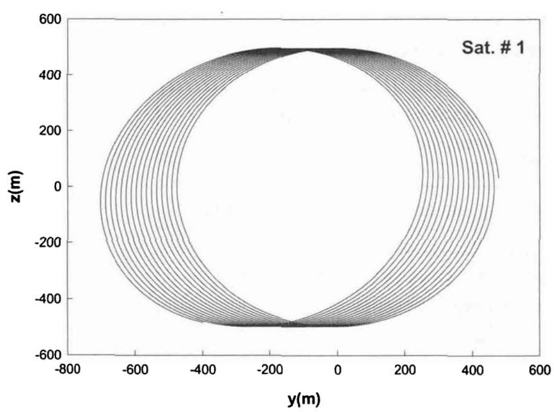
Fig. 2. Flowchart of determining the initial conditions and computing the inertial vector

Table 1. Initial conditions of TechSat-21

Relative Vector	Mean Elements	Target Sat. #1		Target Sat. #2		Target Sat. #3	
x(km)	a(km)	0.0	6928.1385	-2.16506e-1	6928.1408	2.16506e-1	6928.1408
y(km)	e	0.5000	0.00007	-0.2500	0.00005	-0.2500	-0.00013
z(km)	i(deg)	0.0	35.0041	-0.43301	34.9979	0.43301	34.9979
vx(km/s)	Ω(deg)	2.7380e-4	0.2865	-1.3690e-4	0.2847	-1.3690e-4	0.2883
vy(km/s)	ω(deg)	2.4922e-6	-0.0021	-1.2379e-6	0.0011	-1.2379e-6	0.0010
vz(km/s)	M(deg)	5.5016e-4	0.00000	-2.7508e-4	-0.0063	-2.7508e-4	0.0063

To verify the effectiveness of TechSat-21 orbit design, the resulting mean, relative initial position and velocities are given in Table 1, which are calculated with the corresponding reference orbit elements(a, e, I, Ω, ω, M : 6928.14km, 0.005, 35.0deg, 0, 0, 0). The osculating elements are calculated with considering J2-J5 effects using Brouwer's theory. As shown in the Table 1, the target satellite 2 and 3 have the same inclination. Orbits of the target satellites were propagated with the consideration of the gravitational perturbations included J2-J8 using the numerical integration method of Gauss-Jackson.

Figure 3, 4 and 5 show the relative motion trajectories with respect to the reference orbit. Two horizontal plane trajectories is shown in Fig. 3 in order to compare the results of the new method with those of Hill's equation. Obviously the new method keeps a satellite bounded in the circle for long period. However, Hill's equation can not give initial conditions which maintain an equilateral triangle. Fig. 4 shows the trajectories of three target satellites in the horizontal plane and 3-D space. Relative positions of each component are given in Fig. 5 in which each components have the same periodic variation and amplitude. This suggests that the equilateral triangle can be maintained for long period without fuel consumption for the reconfiguration of formation.



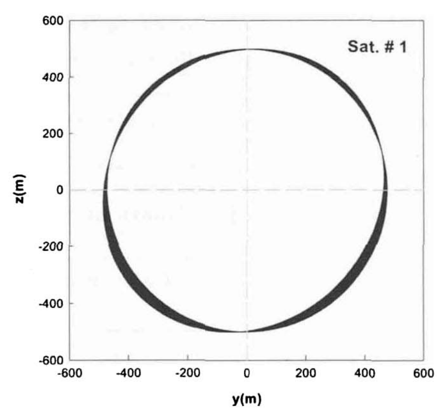


Fig. 3. Horizontal plane trajectory(for 15 periods)
(left : Hill's equation, right : the new method)

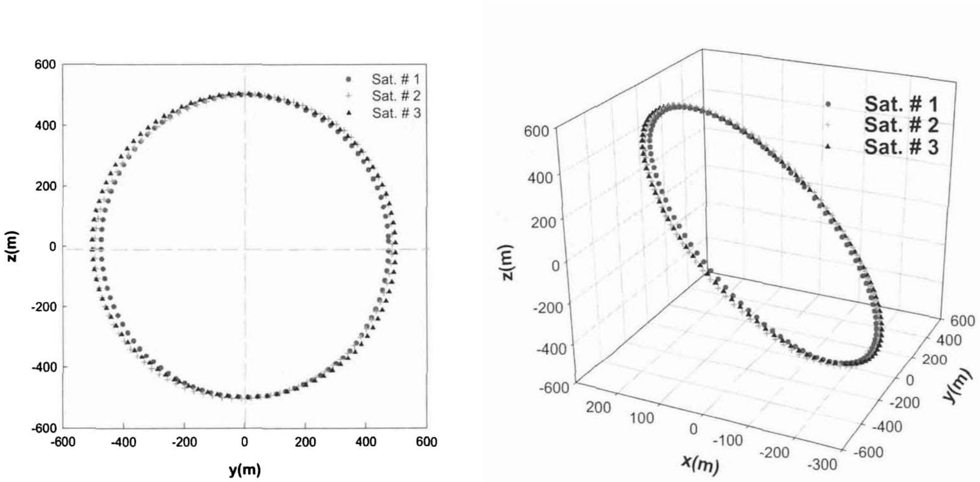


Fig. 4. Trajectory of target satellites using the new method (for one period)

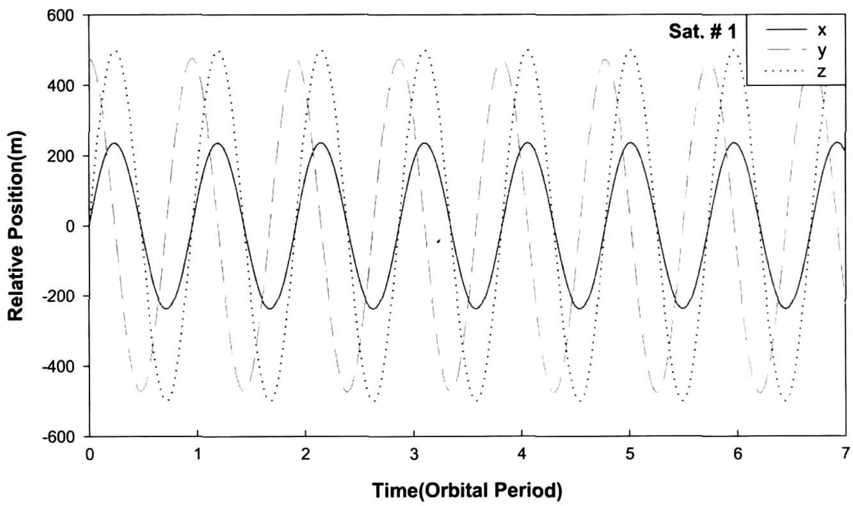


Fig. 5. Relative position using the new method

Conclusion

The technique for determining the initial conditions is utilized to develop a fuel-saving control scheme to maintain the formation. Even though Hill's equations give useful tools to determine initial conditions, they can not enforce satellites bounded in the formation. The new method for determining the initial conditions is discussed in this paper and evaluated through TechSat-21 formation flying. This method give more efficient solutions for formation flying than those of the analytic methods and contrary to the existing numerical methods, it do not need the iterative loops for computing the initial conditions but it can obtain the same results. As shown in the results, this new method can offer desired solutions for maintaining a formation during a long period without fuel consumption.

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