Nonlinear Formation Guidance Law with Robust Disturbance Observer

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Abstract

Many formation guidance laws have been proposed for UAV formation flight. Since most autonomous formation flight methods require various active communication links between the vehicles to know motion information of other vehicles, damage to the receiver or the transmitter and communication delay are critical problem to achieve a given formation flight mission. Therefore, in this point of view, the method that does not need an inter-vehicle communication is preferred in the autonomous formation flight. In this paper, we first summarize the formation guidance law without an inter-vehicle communication using feedback linearization and sliding mode control proposed in previous study. We also propose the modified formation guidance law with robust disturbance observer, which can provide significantly better performance than previously mentioned guidance law in case that other vehicles maneuver with large accelerations. The robust disturbance observer can estimate uncertainties generated by acceleration of leader vehicle. By eliminating the uncertainties using the estimated uncertainties, UAVs are able to achieve the tight formation flight. The performance of the proposed approach is validated by numerical simulations.

Key Word : Formation guidance, Feedback linearization, Sliding mode control, Robust disturbance observer

Introduction

In the past several years, there has been increasing interest in autonomous formation guidance due to several advantages of formation flight. The autonomous formation flight includes two major problems: (i) how to guide each vehicle to formation positions, (ii) how to maintain tight formation. To form and keep up tight formation, it is necessary to know motion information of other vehicles. Although the method to recognize the motion information is different depending on the formation control method, communication is required between the vehicles involved in the most autonomous formation flight methods. Therefore, defect in the instruments used for formation flight and communication delay are critical problem to achieve a given formation mission. In this point of view, the formation method that does not need an inter-vehicle communication would be much preferred to the autonomous formation flight.

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In order to avoid an inter-vehicle communication for formation guidance, one possible method is to use visual sensors. Vision-based formation control has been actively studied in robotics [1]. The guidance laws for approaching and forming the formation by using only line-of-sight (LOS) angle, which can be measured by the vision sensor, is also proposed in Ref. [2]. In Ref. [3], an adaptive approach for vision-based UAV formation control assumes that LOS range can be estimated by the visual sensors. Another possible method for sensing unnecessary data communication is to use the wake produced by the leading aircraft [4].

We have proposed the autonomous formation guidance law without an inter-vehicle communication using feedback linearization and sliding mode control method in our previous research [5]. The performance of the method, however, is significantly degraded when there are large accelerations of other vehicles. In general, the accelerations of other vehicles are required to accomplish common missions and objectives of formation flight. Furthermore, large acceleration is essential to avoid various unexpected situations such as the collision between UAVs.

In this paper, we propose the modified formation guidance law with robust disturbance observer that can improve the formation performance although vehicles have large maneuver accelerations. The leader-follower structure, which is a common formation structure, is considered to derive the proposed guidance law. Note that measurements for the guidance command generation with application to the follower are only relative distance to the leader, LOS angle and their time differentiations, and own motion information to approach and form the formation flight. The modified guidance law is very useful because motion information of leader is not required for formation and the performance degradation caused by large acceleration of the leader is able to be compensated.

The performance of the proposed approach is verified by some numerical simulations. A case with small acceleration and that with large acceleration of leader are considered in numerical examples. The performance of the modified formation guidance law is also compared with the previous guidance law suggested in Ref[5].

Formation Guidance Problem

In this chapter, relative dynamic and outputs are defined to identify formation guidance problem.

Dynamic for Formation Guidance

Consider the planar motion of two vehicles on the assumption that vehicles are lag-free systems as shown in Fig. 1. The two-dimensional (2-D) point mass model is used in formation of a group of vehicles for the simplicity. In this figure, subscript F means the motion information of the follower and L denotes the information of leader's motion. ρ represents the relative distance, λ and γ , V denote the line-of-sight (LOS) angle, the flight path angle and the velocity of the vehicles, respectively. The I-frame is an inertial reference frame and all motions of vehicles are described with respect to the inertial frame.

From the formation guidance geometry, the relative distance between the vehicles and LOS angle are given by

$$\rho = \sqrt{\rho_X^2 + \rho_Y^2} \tag{1}$$

$$\lambda = \tan^{-1}(\rho_Y / \rho_X) \tag{2}$$

where $\rho_X = \rho\cos\lambda$, $\rho_Y = \rho\sin\lambda$.

Then second-order derivatives of the relative distance and the LOS angle are as follows.

$$\ddot{\rho} = \rho\dot{\lambda} - u_1 + a_{LT}\cos(\gamma_L - \lambda) - a_{LN}\sin(\gamma_L - \lambda) \tag{3}$$

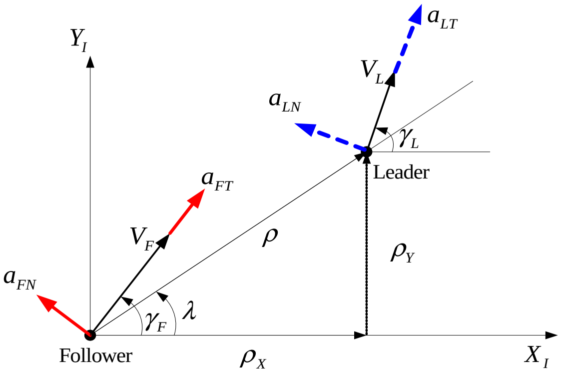

Fig. 1. Formation guidance geometry

$$\ddot{\lambda} = \left\{-2\dot{\rho}\dot{\lambda} - u_2 + a_{LT}\sin(\gamma_L - \lambda) + a_{LN}\cos(\gamma_L - \lambda)\right\}/\rho \tag{4}$$

where $\boldsymbol{u} = [u_1,\ u_2]$ are follower's guidance command and a_{LT} , a_{LN} are, respectively, the tangential and normal acceleration of the leader as shown in Fig. 1. After generation of u_1 and u_2 by using guidance law for formation, the tangential and normal acceleration of the follower can be determined from Eq. (5).

$$\begin{bmatrix} a_{FT} \\ a_{FN} \end{bmatrix} = \begin{bmatrix} \cos(\gamma_F - \lambda) & \sin(\gamma_F - \lambda) \\ -\sin(\gamma_F - \lambda) & \cos(\gamma_F - \lambda) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{5}$$

Note that γ_L can be obtained using the relative distance, relative velocity and LOS rate, which can be measured by radar or seeker, according to Eq. (6). The inter-vehicle communication is, therefore, not needed to know the flight path angle of the leader.

$$\gamma_L = \tan^{-1}\left(\frac{\rho\dot{\lambda} + V_F\sin(\gamma_F - \lambda)}{\dot{\rho} + V_F\cos(\gamma_F - \lambda)}\right) + \lambda \tag{6}$$

Output for Formation Guidance

It is important and essential to define the proper output for formation flight. Let us define the output and desired output as

$$\boldsymbol{y} = [\rho\ ,\ \theta]^T \quad , \quad \boldsymbol{y}_d = [\rho_d\ ,\ \theta_d]^T \tag{7}$$

where $\theta = \gamma_L - \lambda$, and ρ_d , θ_d denote respectively the designated relative distance and angle according to the desired formation shape. Note that the angle, θ, can be used for the output of vehicles because γ_L can be estimated from Eq. (6) and λ can be measured by seeker as mentioned in the previous section. The guidance problem for formation flight is to design the guidance law that can make the outputs track the desired output in this paper.

Formation Guidance Law

The formation guidance law using feedback linearization and sliding mode control method is introduced. The modified guidance law with robust disturbance observer is also described in this section.

Guidance Law using Feedback Linearization and Sliding Mode Control

The tracking error for the formation guidance problem is defined as

$$\boldsymbol{e} = [\,e_1\ ,\ e_2\,]^T = \boldsymbol{y} - \boldsymbol{y}_d \tag{8}$$

Clearly, the follower can fly in desired formation with the leader if the tracking error is driven to zero. To apply sliding mode control for the formation problem, it is fundamental to find the guidance law using feedback linearization. This guidance law can be easily solved by differentiating each component of the error $\boldsymbol{e}$ until at least one component of the input $\boldsymbol{u}$ explicitly appears. This treatment is adopted on the assumption that the leader′s accelerations are zero. The second-order error differential equations, on which the components of the input appear, can be obtained as

$$\begin{bmatrix} \ddot{e}_1 \\ \ddot{e}_2 \end{bmatrix} = \begin{bmatrix} \rho\dot{\lambda}^2 - \ddot{\rho}_d \\ 2\dot{\rho}\dot{\lambda}/\rho - \ddot{\theta}_d \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & 1/\rho \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{9}$$

Then, letting

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & \rho \end{bmatrix} \left(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - \begin{bmatrix} \rho\dot{\lambda}^2 - \ddot{\rho}_d \\ 2\dot{\rho}\dot{\lambda}/\rho - \ddot{\theta}_d \end{bmatrix} \right) \tag{10}$$

gives the decoupled, the closed input-output relation is as follows.

$$\begin{bmatrix} \ddot{e}_1 \\ \ddot{e}_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{11}$$

By defining the outer-control v_1 and v_2 as

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -k_1\dot{e}_1 - k_2 e_1 \\ -k_3\dot{e}_2 - k_4 e_2 \end{bmatrix} \quad , \quad k_i > 0 \quad \text{for } i = 1, \cdots, 4 \tag{12}$$

the overall guidance law consisting of Eq. (10) and Eq. (12) will asymptotically drive the tracking error to zero. The assumption that the leader′s accelerations are zero is used to derive this guidance law. Therefore, if the leader has non-zero accelerations during formation flight, the performance of the guidance is significantly degraded, so sliding mode control method is applied to compensate the performance degradation.

Under assumption the accelerations of the leader are bounded even though it is difficult to obtain, the accelerations can be set as uncertainties, $\boldsymbol{\Delta} = [\,\Delta_1\ ,\ \Delta_2\,]^T$ represented as

$$\begin{aligned} \left\| \begin{bmatrix} \Delta_1 \\ \Delta_2 \end{bmatrix} \right\| &= \left\| \begin{bmatrix} a_{LT}\cos(\gamma_L - \lambda) - a_{LN}\sin(\gamma_L - \lambda) \\ \dfrac{\dot{a}_{LN}V_L - a_{LN}a_{LT}}{V_L^2} - \dfrac{a_{LT}\sin(\gamma_L - \lambda) + a_{LN}\cos(\gamma_L - \lambda)}{\rho} \end{bmatrix} \right\| \\ &\leq \left\| \begin{bmatrix} 1.1\alpha_1 \\ 1.1\alpha_2 \end{bmatrix} \right\| \end{aligned} \tag{13}$$

where α_1 , α_2 are positive constant weightings.

From Eq. (10) and (13), we can derive the control input $\boldsymbol{u}$ as following equation by using sliding mode control method.

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & \rho \end{bmatrix} \left(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - \begin{bmatrix} \rho\dot{\lambda}^2 - \ddot{\rho}_d + 1.1\alpha_1 sgn(s_1) \\ 2\dot{\rho}\dot{\lambda}/\rho - \ddot{\theta}_d + 1.1\alpha_2 sgn(s_2) \end{bmatrix} \right) \tag{14}$$

where $[\,s_1\ ,\ s_2\,]^T$ is sliding surface defined as

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} \dot{e}_1 + k_1 e_1 + k_2 \int e_1 dt \\ \dot{e}_2 + k_3 e_2 + k_4 \int e_2 dt \end{bmatrix} \tag{15}$$

As shown in Eq. (14), the motion information of the leader is not required for the formation flight. The follower can fly in formation if sensors such as radar or seeker are equipped in the vehicle for measuring the relative distance, LOS angle and their first-order differentiation without communication between the vehicles. The stability of the guidance law using sliding mode control, however, does not guarantee in case that the uncertainties fail to satisfy the inequality constraints in Eq. (13). Although the large constant weightings guarantee the stability of the guidance, it will lead to large guidance command.

Modified Guidance Law with Robust Disturbance Observer

As described previous section, the performance of the suggested guidance law can be significantly degraded due to the leader's acceleration. This problem is critical because it is more difficult to measure or estimate the acceleration of the vehicle. The modified guidance law with robust disturbance observer is proposed to solve the problem.

Consider following first-order system expressed in Eq. (16) to understand the robust disturbance observer.

$$\dot{x} = f(x, u) + d \tag{16}$$

where x is measurable state, $f(x, u)$ is the known function of state and control, d is unknown disturbance term. If measurement information is reliable and unknown disturbance are differentiable and bounded, it is known that high order sliding mode disturbance observer can estimate disturbance term from Eq. (17) [6, 7].

$$\begin{aligned} \dot{z}_0 &= v_0 + f(x, u) \\ \dot{z}_1 &= v_1 \\ \dot{z}_2 &= v_2 \\ \dot{z}_3 &= -1.1L\, sgn(z_3 - v_2) \end{aligned} \tag{17}$$

where

$$\begin{aligned} v_0 &= -5L^{1/4}|z_0 - x|^{3/4} sgn(z_0 - x) + z_1 \\ v_1 &= -3L^{1/3}|z_1 - v_0|^{2/3} sgn(z_1 - v_0) + z_2 \\ v_2 &= -1.5L^{1/2}|z_2 - v_1|^{1/2} sgn(z_2 - v_1) + z_3 \end{aligned} \tag{18}$$

According to the theory in Ref. [6] and [7], z_1 in Eq. (17) converges to d in a finite time. Using this relation, the uncertainties, Δ can be estimated from measurements including LOS rate and relative distance. The modified guidance law with robust disturbance observer can be obtained by using the estimation of uncertainties.

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & \rho \end{bmatrix} \left(\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - \begin{bmatrix} \rho\dot{\lambda}^2 - \ddot{\rho}_d + 1.1\alpha_1 sgn(s_1) + \hat{\Delta}_1 \\ 2\dot{\rho}\dot{\lambda}/\rho - \ddot{\theta}_d + 1.1\alpha_2 sgn(s_2) + \hat{\Delta}_2 \end{bmatrix} \right) \tag{19}$$

where $[\hat{\Delta}_1 \ , \ \hat{\Delta}_2]^T$ is the estimation of Δ. Since the uncertainties are compensated by the sliding mode disturbance observer, the modified guidance method can resolve the problems of the guidance consisting of only sliding mode control.

Numerical Examples

In this section, the performance of the proposed guidance laws is validated by numerical examples about two scenarios. Initial position and flight path angle of each vehicle are same in the scenarios and represented in Table. 1. To compare the performance of two proposed guidance laws, follower 1 and 2, which are guided by the guidance law based on the only sliding mode method and the modified guidance law with additional robust disturbance observer, respectively, are implemented in the formation flight scenarios.

For the follower's formation flight, the desired output is determined as $[\,\rho_d\ ,\ \theta_d\,]^T = [\,300m\ ,\ 45^\circ\,]$ in all simulation scenarios, and the acceleration of the leader is given by

$$\text{Scenario 1 :}\ [\,a_{LT}\ ,\ a_{LN}\,]^T = [\,3\ ,\ 1\,]^T\ \ (m/s^2)$$
$$\text{Scenario 2 :}\ [\,a_{LT}\ ,\ a_{LN}\,]^T = [\,10\ ,\ 5\,]^T\ \ (m/s^2)$$

The acceleration of the followers is assumed to be bounded as

$$|[\,a_{FT}\ ,\ a_{FN}\,]^T| \leqq [\,50\ ,\ 20\,]^T\ \ (m/s^2) \tag{20}$$

In addition, a smoothing technique, which replace $sgn(s)$ by $sat(s/\epsilon)$ for $\epsilon > 0$, is adopted to alleviate the chattering due to the discontinuity of the switching function. The estimated uncertainties $[\hat{\Delta}_1\ ,\ \hat{\Delta}_2]^T$ are applied at $t = 25\,\mathrm{sec}$ to guide follower 2 since it takes some time to estimate the uncertainties.

Table 1. Initial conditions of each vehicle

	$[\,X\ ,\ Y\,]^T\,(m)$	$V\ (m/s)$	γ (deg)
Leader	$[\,0\ ,\ 0\,]$	20	30
Follwer 1 and 2	$[-1000\sqrt{2}, -1000\sqrt{2}\,]$	20	30

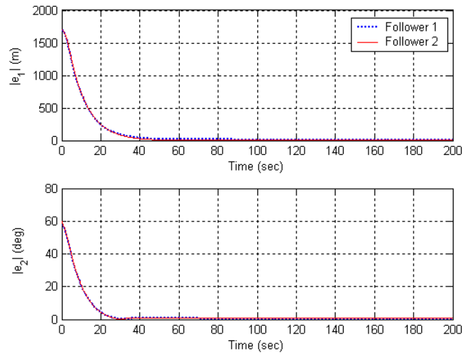

Fig. 2. Absolute output error history scenario 1

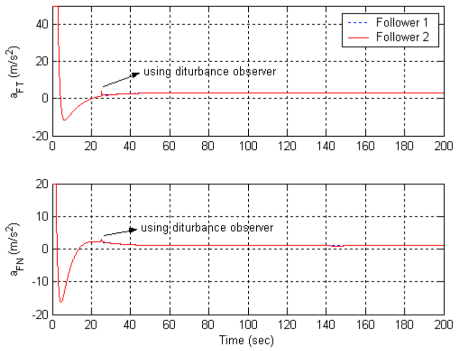

Fig. 3. Follower's acceleration command in history in scenario 1

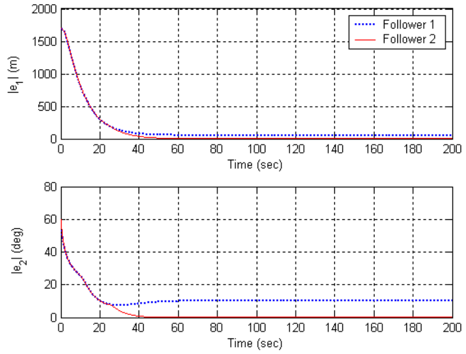

Fig. 4. Absolute output error history in scenario 2

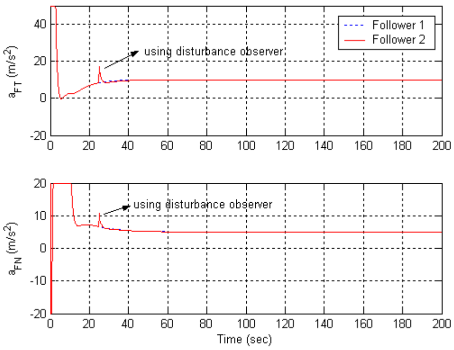

Fig. 5. Follower's acceleration command history in scenario 2

The simulation results of each scenario are represented in Fig. 2∼5. As shown in Fig. 2 and 3, if there is the small acceleration of the leader, the followers can fly in formation tightly and the performance of two guidance laws are almost similar. However, it is known that the performance of the guidance with only sliding mode control is significantly degraded due to the large uncertainties, while the tracking errors of the follower 2 using the modified guidance law go to zero as shown in Fig. 4, 5. Because the uncertainties are canceled by robust disturbance observer, we find that the performance of the modified guidance law is better than only the sliding mode method.

Conclusions

In this paper, we proposed the guidance law with robust disturbance observer for formation flight. This guidance law does not required an inter-vehicle communication, thus it can easily solve the various problems related with forma-tion flight of vehicles. By comparing with the guidance law using only sliding mode control suggested in previous study, the performance of the guidance law was evaluated through numerical simulation about two scenarios. From the simulation results, we conclude that the modified guidance law shows a satisfactory performance in spite of the vehicle maneuver with large accelerations.

Acknowledgement

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