Modified Tomographic Estimation of the Ionosphere using Fewer Coefficients

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Abstract

Ionospheric time delay is the biggest error source for single-frequency DGPS applications, including time transfer and Wide Area Differential GPS (WADGPS). Currently, there are many attempts to develop real-time ionospheric time delay estimation techniques to reduce positioning error due to the ionospheric time delay.

Klobuchar model is now widely used for ionospheric time delay calculation for single-frequency users. It uses flat surface at night time and cosine surface at daytime[1]. However, the model was developed for worldwide ionosphere fit, it is not adequate for local area single-frequency users who want to estimate ionospheric time delay accurately[2]. Therefore, 3-D ionosphere model using tomographic estimation has been developed. 3-D tomographic inversion model shows better accuracy compared with prior algorithms[3]. But that existing 3-D model still has problem that it requires many coefficients and measurements for good accuracy. So, that algorithm has limitation with many coefficients in continuous estimation at the small region which is obliged to have fewer measurements.

In this paper, we developed an modified 3-D ionospheric time delay model using tomography, which requires only fewer coefficients. Because the combinations of our base coefficients correspond to the full coefficients of the existing model, our model has equivalent accuracy to the existing. We confirmed our algorithm by simulations. The results proved that our modified algorithm can perform continuous estimation with fewer coefficients.

Key Word : ionosphere, local area ionospheric time delay, tomography, WADGPS

Introduction

Tomography is well-known process for determining the distribution of a physical quantity from sets of path integrals through the region containing the unknown distribution[4]. It reconstructs the distribution with basis functions and their coefficients. So, we must estimate the coefficients in tomography. Fig. 1 shows the principles of tomography in medical imaging.

In ionospheric tomography, the physical quantity is the distribution of the ionosphere's electron. And path integral is TEC(Total Electron Content) measurement from satellite to receiver. Likewise, coefficients are those of horizontal function and radial functions. Fig. 2 shows ionospheric imaging by tomography.

The goal of ionospheric tomography is to find 3D function N(r). N(r) is the electron distribution function as latitude, longitude and height in ionosphere. N(r) consists of the tensor product of horizontal

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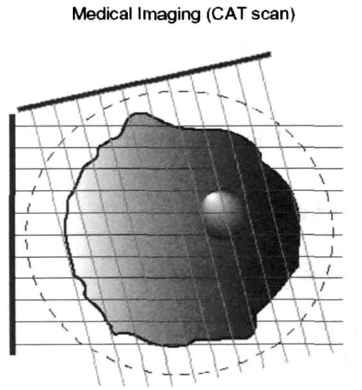


Fig. 1. Medical imaging by tomography

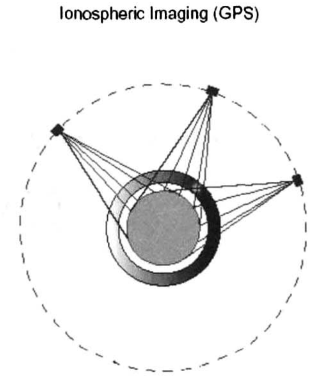


Fig. 2. Ionospheric imaging by tomography

function $Y(\theta, \phi)$ (Spherical Harmonics Function) and radial function $\Gamma(h)$ (Empirical Orthogonal Function) like equation 1.

$$N(r) = \Gamma(h) \otimes Y(\theta, \phi)$$

where,

$$\Gamma(h) = \sum a_k \Gamma_{k(h)} Y(\theta, \phi) = \sum b_l Y_l(\theta, \phi) \tag{1}$$

Consequently, we must estimate coefficients (ak, bl) of those functions to solve function N(r). k is the number of EOF and l is the number of SHF terms. For example, if we use 3 EOF and 2nd order SHF, k=3 and l=9.

Existing algorithm of ionospheric tomography has some limitations. Many coefficients must be estimated by multiple scale. For example, if k=3 and l=9, the number of coefficients are 27(3x9). But in our modified algorithm, small coefficients are required by summation scale. For example, if k=3 and l=9, the number of coefficients are 12(3+9). Besides, we can perform continuous estimation with independent terms.

Modified Tomographic Estimation

Tomographic Estimation of Ionosphere

In ionospheric tomography algorithm, we use TEC as measurement from dual frequency receiver. Equation (2) ~ (5) show that

$$TEC = \int_{R(r)}^{SV(r)} N(r)dl(r) \tag{2}$$

$$\Delta t = \frac{1}{c}\int_{R(r)}^{SV(r)} (1-n)dl(r) = \frac{1}{c}\int_{R(r)}^{SV(r)} \frac{X}{2} dl(r)$$
$$= \frac{a}{cf^2}\int_{R(r)}^{SV(r)} N(r)dl(r), \quad a = \frac{e^2}{8\pi^2 \varepsilon_0 m} \tag{3}$$

$$\delta(\Delta t) = \Delta t_{L_2} - \Delta t_{L_1} = \frac{a}{c}\int_{R(r)}^{SV(r)} Ndl(r)\left(\frac{1}{f_{L_2}^{\ 2}} - \frac{1}{f_{L_1}^{\ 2}}\right) \tag{4}$$

$$TEC = \frac{f_{L_1}^{\ 2} f_{L_2}^{\ 2}}{\left(f_{L_1}^{\ 2} - f_{L_2}^{\ 2}\right)} \times \frac{\rho_2 - \rho_1}{a} = z \tag{5}$$

Putting equation (1) to (2), equation can be calculated like equation (6).

$$\begin{aligned}\int_{R(r)}^{SV_i(r)} N(r)dl(r) &= \int_{R(r)}^{SV_i(r)} \Gamma(h) \otimes Y(\theta,\phi) dl(r) \\ &= \int_{R(r)}^{SV_i(r)} \sum a_k \Gamma_k(h) \otimes \sum b_l Y_l(\theta,\phi) dl(r) \\ &= H_{i11} a_1 b_1 + H_{i12} a_1 b_2 + \cdots + H_{inm} a_n b_m \\ &= [H_{i11}\ H_{i12} \cdots H_{i1m}\ H_{i21} \cdots\cdots H_{inm}] \underline{x} = z_i\end{aligned}$$

where,

$$\begin{aligned} H_{kl} &= \int_{R(r)}^{SV(r)} \Gamma_k(h) \cdot Y_l(\theta,\phi) dl(r) \\ \underline{x} &= [a_1 b_1\ a_1 b_2 \ldots a_1 b_m\ a_2 b_1 \ldots a_n b_m]^T \quad \underline{x} \in R^{n \times m} \end{aligned} \tag{6}$$

Accumulating k measurements, we cam make linear equation (7).

$$\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_k \end{bmatrix} = \begin{bmatrix} H_{111} & H_{112} & \cdots & H_{1nm} \\ H_{211} & H_{212} & \cdots & H_{2nm} \\ \vdots & & & \vdots \\ H_{k11} & H_{k12} & \cdots & H_{knm} \end{bmatrix} \underline{x}$$

$$\underline{Z} = H \underline{x} \tag{7}$$

And then, we can estimate x by least-square solution and reconstruct ionosphere from basis functions with coefficients x.

$$\underline{x} = (H^T H)^{-1} H^T \underline{Z} \tag{8}$$

In this algorithm, we must estimate mxn coefficients which are not independent[5].

Modified Algorithm

Equation (6) can be expressed in nonlinear form like equation (9) too.

$$\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_k \end{bmatrix} = \begin{bmatrix} H_{111} a_1 b_1 + H_{112} a_1 b_2 + \cdots + H_{1nm} a_n b_m \\ H_{211} a_1 b_1 + H_{212} a_1 b_2 + \cdots + H_{2nm} a_n b_m \\ \vdots \\ H_{k11} a_1 b_1 + H_{k12} a_1 b_2 + \cdots + H_{knm} a_n b_m \end{bmatrix}$$

$$\underline{Z} = \underline{h}'(\underline{x}')$$

where,

$$\underline{x}' = [a_1\ a_2 \ldots a_n\ b_1\ b_2 \ldots b_m]^T \tag{9}$$

In this form, coefficients are independent and their length is m+n. The coefficients can be found by nonlinear estimation. But the load of computation may be bigger than least square method.

$$\underline{M}(\underline{x}') = \underline{Z} - \underline{h}'(\underline{x}') \tag{10}$$

$$\delta\underline{x}' = \left.\frac{\partial \underline{M}}{\partial \underline{x}'}\right|^{-1}_{\underline{x}'=\underline{x}'_n} \cdot [-\underline{M}(\underline{x}')]\Big|_{\underline{x}'=\underline{x}'_n}$$

$$\underline{x}' = \underline{x}'_n + \delta\underline{x}' \tag{11}$$

We use kinematic filter for the coefficients of m+n. In filtering, we can estimate independent coefficients which have physical meaning.

$$\dot{\underline{X}} = \underline{F}\,\underline{X} + \underline{W},\ \underline{X} = \begin{bmatrix}\underline{x}' \\ \dot{\underline{x}}'\end{bmatrix} \tag{12}$$

$$\underline{Z} = \underline{h}''(\underline{X}) + \underline{V} \tag{13}$$

Advantage of Modified Algorithm

As you can see from above equations, the length of coefficients is less than existing method. So, less coefficients can have some advantages and one of those is that small data size is just required when correction data is transferred.

Simulation and Results

Simulation Scenario

In simulation, we consider IRI-95 model as true thing and estimate that[6]. We use 3 EOF and 2nd order SHF for simple example of our algorithm. So, the number of coefficients are 12(=3+9), not 27(=3x9). Continuous 3 epoch sets from 2 o'clock are used sequentially. 1 epoch set is constructed by 30 minutes. Measurement noise is included as functions of elevation angle. Reference stations are 6 in Korea. Those are Seoul, Daejun, Kwangju, Busan, Jeju island and Ulung island. So simulation was performed in Korea area. Fig. 3 shows reference stations in simulation area.

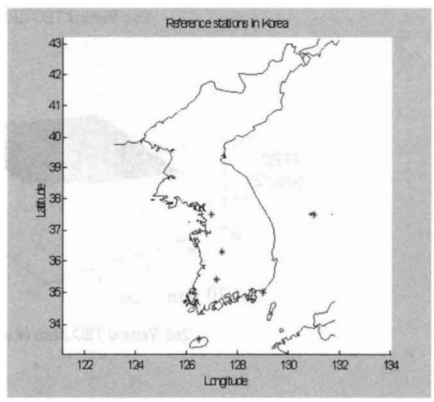


Fig. 3. Reference stations and simulation area

Simulation Result

Fig. 4 ~ Fig. 6 show simulation results. Each figure shows the result of each epoch set. Left upper graph of each figure shows vertical TEC of IRI-95 model (considered as true model) and right upper shows that of estimated model. Bottom 2 graphs show estimation errors of vertical TEC (difference between estimated minus true). Tablel 1 shows statistical result of those. The results show high accuracy with less coefficients.

Table 1. Simulation result

Estimation error (%)	mean	std	max
1st 30 min	6.30	3.51	16.5
2nd 30 min	5.21	3.04	11.9
3rd 30 min	1.61	1.04	3.77

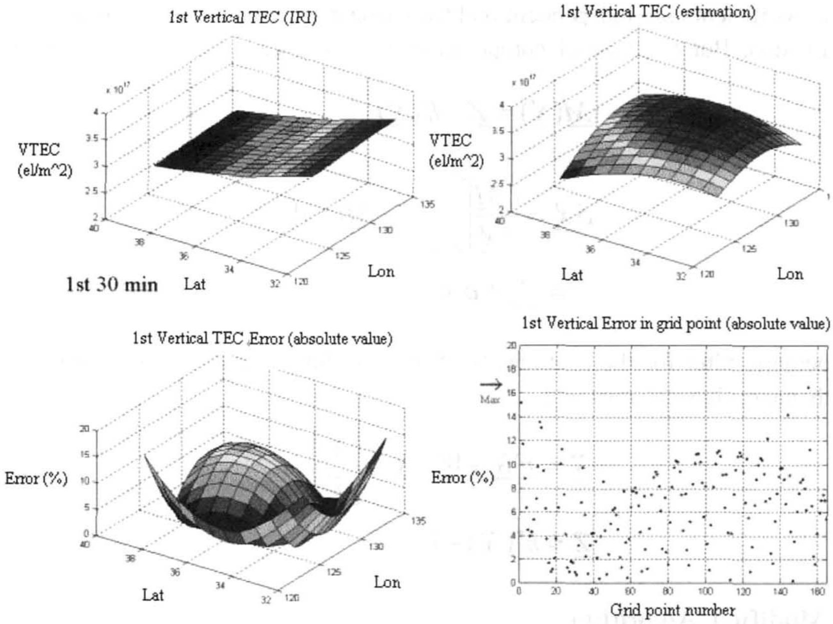


Fig. 4. The results of 1st 30 min

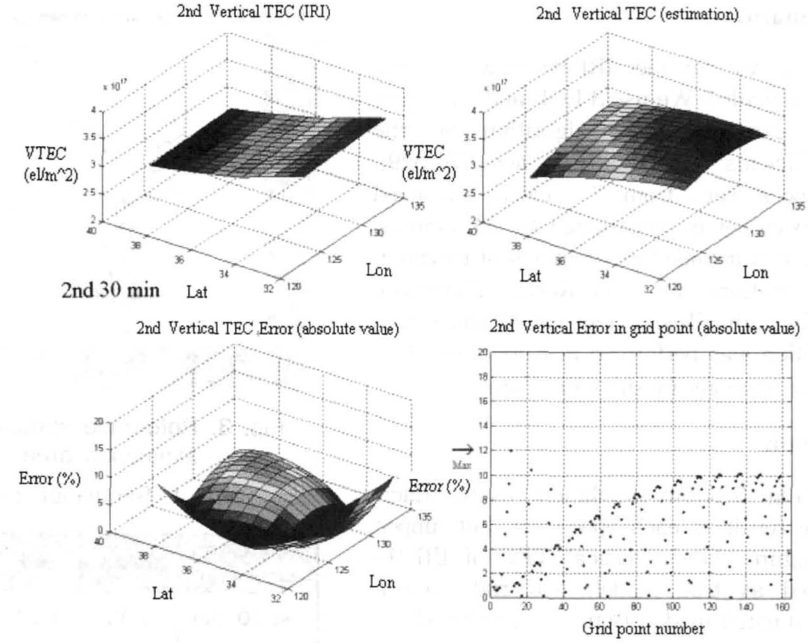


Fig. 5. The results of 2nd 30 min

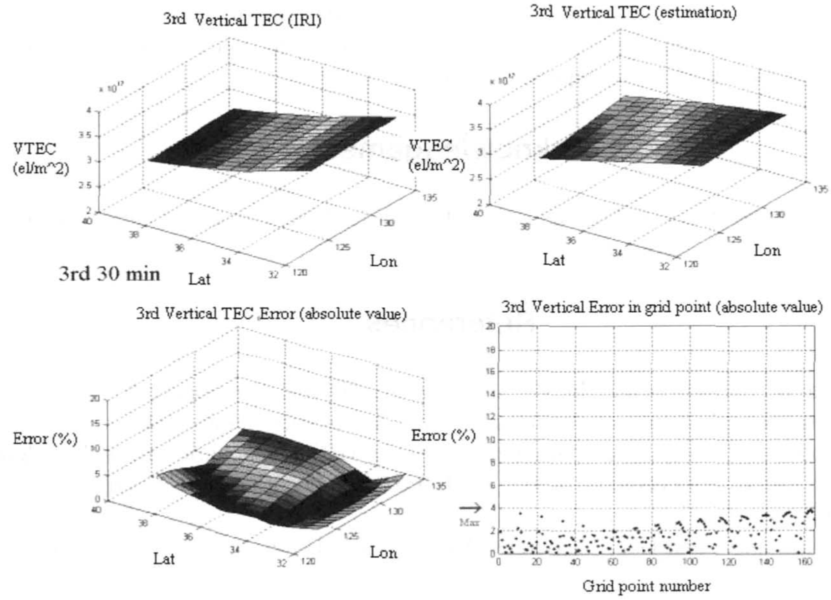
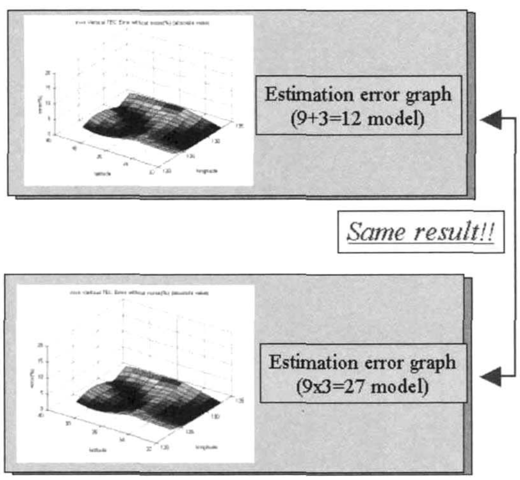


Fig. 6. The results of 3rd 30 min

We made comparison between m+n and mxn. We can see that two method show same result and accuracy. Figure 7 shows that result.



Estimation error (%) (no noise)	m+n	mxn
mean	1.01	1.01
std	0.72	0.72
max	2.93	2.93

Fig. 7. Comparison between m+n and mxn

Conclusions

In this paper, we proposed modified tomographic algorithm for ionosphere. In our algorithm, the length of coefficients is less than existing method. And we can do filtering with independent coefficients which have physical meaning. So, less coefficients can have some advantages and it is easy to describe system equation and perform continuous real time estimation.

As future work, we will consider ionospheric irregularities for better modeling of filter[7]. And we will check this algorithm with experimental data.

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