

# Robust Time-Optimal Control for Flexible Structure Vibration Control by Augmented Dynamics

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## Abstract

Robust optimal control problem for flexible spacecraft/structures using on-off type discrete actuators has been a subject of intensive research. Optimization by switching time parameterization can be used to minimize the maneuver time subject to equality boundary constraints. Sensitivity of the switching times with respect to modal parameters could be a critical factor degrading the performance of the time optimal solutions. A new approach for the robustness enhancement of switching times with respect to modal parameter uncertainty is introduced. A new concept so-called augmented dynamic model is added to account for the modal uncertainty. The proposed approach turns out to be similar to some previous methods, but it essentially provides some new results.

**Key Word** : Attitude control, flexible spacecraft, Lyapunov function, output feedback, three-axis attitude dynamics

## Introduction

The time optimal control for the attitude maneuver of flexible space spacecraft/structures can be achieved by control input at the maximum level[1-3]. Some input shaping methods were proposed to reduce residual vibration due to model uncertainty[4-7]. In particular, for a rest-to-rest maneuver, anti-symmetric control torque profile with symmetric switching times lead to vibration control as well as attitude changes[1]. It is well known that for a linear undamped mechanical system with  $N$  modes, there are at most  $2N-1$  switching instants for the control of every mode[1,8,9]. Various approaches have emerged to find out the switching instants(times) in such a way that the maneuver time is minimized while the induced vibration is effectively suppressed. One of the most popular methods to design the switching times is the parameter optimization technique. The switching times are parameterized as unknowns, and then they are computed by optimization algorithm with equality constraints[1,8,9]. The cost function to be optimized is the final maneuver time, while the equality constraints should be satisfied simultaneously. The equality constraints represent conditions for the final attitude by rigid motion and zero residual vibration. In some cases, other forms of cost function such as combined time-fuel optimal control have been employed.

The time optimal control solution as derived by parameter optimization is usually subject to robustness with respect to the modal characteristics of the system. Robust approaches have been investigated to overcome the issue of sensitivity of the control command with respect to the modal characteristics in some previous studies.[8-11] It turns out that adding more switching times than the exact optimal solution can lead to responses being robust about model uncertainties. The number

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of equality constraints increases due to the extra switching times. However, enhanced robustness is achievable by this approach[8-11].

In this study, we use the so-called augmented dynamics method to design robust switching profiles for time optimal control solution for flexible structures. In order to overcome the sensitivity problem, a virtual dynamic model with a modal frequency perturbed from the actual value is augmented. The virtual dynamic model does not really exist, but it is primarily used to provide robustness in modal characteristics. The augmented dynamic model also results in increased switching times and corresponding robustness. The new approach is essentially similar to the previous ones[8-10] where the sensitivity of the modal responses with respect to the natural frequencies was set to zero. In the augmented dynamic model approach, the modal parameter is selected by designer's choice. Hence, the new method leaves some room for shaping responses while still maintaining enhanced robustness by virtue of judicious selection of the modal parameters. The modal frequencies for the augmented system can be tailored to produce a desired performance. It is also verified in this study that the previous approach of employing sensitivity constraint equation could be considered as a special case of the proposed idea in this study.

## Time Optimal Control

First, a linearized second order mechanical system with a rigid-body mode and flexible vibrational modes are described as

$$M \ddot{\mathbf{q}} + K \mathbf{q} = F \mathbf{u} \quad (1)$$

where  $\mathbf{q} = [q_1, q_2, \dots, q_N]^T$  is a generalized coordinate vector,  $M, K$  are mass and stiffness matrices, and  $F, \mathbf{u}$  are input influence matrix and control input vector, respectively. For simplicity, no damping term is assumed in the governing equations of motion. The control input is a single input subject to the magnitude constraint such as  $|\mathbf{u}(t)| < N$ .

The modal coordinates(  $\eta$ ) can be used to replace the governing equation in the following form:

$$\begin{aligned} \ddot{\eta}_1 + \omega_1^2 \eta_1 &= \phi_1 \mathbf{u} \\ \ddot{\eta}_2 + \omega_2^2 \eta_2 &= \phi_2 \mathbf{u} \\ &\vdots \\ \ddot{\eta}_N + \omega_N^2 \eta_N &= \phi_N \mathbf{u} \end{aligned} \quad (2)$$

where  $\eta_1$  usually corresponds to the rigid body mode with  $\omega_1$  equal to zero. For a time-optimal attitude maneuver, in particular, rest-to-rest maneuver for a flexible model with  $N$  modes, the boundary conditions are prescribed as

$$\begin{aligned} \eta_1(0) = \dot{\eta}_1(0) = 0, \quad \eta_i(0) = \dot{\eta}_i(0) = 0 \quad \text{at } t=0 \\ \eta_1(T) = \eta_f, \quad \dot{\eta}_1(T) = 0, \quad \eta_i(T) = \dot{\eta}_i(T) = 0 \quad \text{at } t=T \end{aligned} \quad (3)$$

where  $i=1, 2, \dots, N$ .  $T$  is the maneuver time and  $\eta_f$  represents the final target displacement of the rigid body mode.

The performance index for the time-optimal control is defined as

$$J = \int_0^T dt \quad (4)$$

It is well known that for a flexible structural system with  $N$  modes of motion, the time-optimal rest-to-rest maneuver strategy should be a bang-bang input. The actuator input is saturated at

the maximum level with multiple switching times. According to the previous studies, there should be at most  $2N-1$  switching times distributed symmetrically about the half maneuver time. The control input can be represented in terms of step functions such as[8,9]

$$u(t) = \sum_{j=0}^{2N} M_j U_s(t - t_j) \quad (5)$$

where  $U_s(t)$  is a step function and  $M_j$  represents the magnitude of the control input which should be anti-symmetrically distributed in the form  $M_j = N\{1, -2, 2, -2, \dots, 2, -1\}$ .

There are switching times(  $t_1, t_2, \dots, t_{2N-1}$ ) that are symmetric about the half maneuver time  $t_h(T/2)$ .

By the control input profiles given by Eq. (6), the responses of the modal coordinates are computed as[8,9]

$$\begin{aligned} \eta_1(t) &= \frac{\phi_1}{2} \sum_{j=0}^{2N} (T - t_j)^2 M_j \\ \eta_i(t) &= -\frac{\phi_i}{\omega_i^2} \sum_{j=0}^{2N} M_j \cos \omega_i(t - t_j), \quad i = 2, \dots, N \end{aligned} \quad (6)$$

for  $t \geq T$ . The time-optimal control switching times can be formulated into a parameter optimization problem. The cost function in Eq. (4) as the total maneuver time is minimized subject to the equality constraints. The equality constraints consist of the rigid-body mode displacement and zero-residual oscillation for the flexible modes such as

$$\begin{aligned} \eta_f - \frac{\phi_1}{2} \sum_{j=0}^{2N} (T - t_j)^2 M_j &= 0 \\ -\frac{\phi_i}{\omega_i^2} \sum_{j=0}^{2N} M_j \cos \omega_i(t - t_j) &= 0, \quad i = 2, \dots, N \end{aligned} \quad (7)$$

The second equation of Eq. (7) can be rewritten, by the anti-symmetric input profile, as follows[8,9]

$$\sum_{j=0}^{2N} M_j \cos \omega_i(t_j - t_k) = 0, \quad i = 2, \dots, N \quad (8)$$

A parameter optimization algorithm can be applied to solve the optimization problem for the optimal switching times(  $t_i, i = 1, 2, \dots, 2N-1$ ). There are software tools available for the nonlinear parameter optimization problems with equality constraints. The MATLAB Optimization Toolbox could be a candidate for moderate sizes of problems.

## Robust Time-Optimal Control

It is well known that the time-optimal control solution is dependent upon the modal parameters of the system. This is quite obvious from the equality constraints; a nonlinear function of modal parameters. The switching times are sensitive to the modal parameters as it is shown in the optimization formulation. Thus some robustness measure should be taken for parameter insensitive control input design.

A promising approach introduced in the previous studies is to establish extra constraints for the purpose of robustness enhancement. For instance, the sensitivity of the modal coordinate responses to the natural frequencies is set to zero[8,9]:

$$\frac{\partial \eta_i}{\partial \omega_i} = \frac{\phi_i}{\omega_i^2} \cos \omega_i(t - t_h) \sum_{j=0}^{2N} (t_j - t_h) M_j \sin(t_j - t_k) \quad (9)$$

The above condition can be rewritten as

$$\sum_{j=0}^{2N} (t_j - t_k) M_j \sin \omega_i (t_j - t_k) = 0 \quad (10)$$

for  $i=2,3,\dots,N$ . Similarly, high order robustness constraints such as  $\partial \eta_i^m / \partial \omega_i^m = 0$  ( $m=2,3,\dots$ ) can be imposed. This approach is discussed in detail in Ref. [8].

Due to the robustness constraint, the number of constraint equations increases. Hence, the original constraint by the modal coordinate responses are combined with the robustness constraints for complete solution. In order to satisfy the extra constraints, the number of parameters, i.e, switching times should increase as well. In Refs. [8] and [9], the number of switching parameters was increased to satisfy additional constraints for the robustness constraints. If  $r$  robustness constraints are introduced, then the switching times should increase by  $2r$ . Hence, the control input in Eq. (5) with additional constraints needs to be modified as[8,9]

$$u(t) = \sum_{j=0}^{2(N+r)} M_j U_s(t - t_j) \quad (11)$$

The same parameter optimization procedure can be applied except for the increased number of switching parameters and constraint equations. It turns out that the robustness constraints and corresponding control input with increased switching times contribute to improving robustness noticeably[8]. The zero deflection derivative constraint with respect to the modal parameters is effectively satisfied to produce robustified time responses.

Another approach proposed in this study for robust control profile is the so-called augmented dynamic approach. The augmented dynamic approach is to add an auxiliary dynamic mode around a particular mode to account for model uncertainty. The extra dynamic mode is assumed to be perturbed from the nominal model. The augmented dynamic mode is not actual model, but just used to generate a robustified switching profile. For instance, consider a  $k$ th flexible mode dynamic described as

$$\ddot{\eta}_k + \omega_k^2 \eta_k = \phi_k u \quad (12)$$

Now, a augmented dynamic model for the  $k$ th flexible mode is introduced as

$$\ddot{\tilde{\eta}}_k + \tilde{\omega}_k^2 \tilde{\eta}_k = \phi_k u \quad (13)$$

where  $\tilde{\eta}_k$  represents the augmented dynamic mode and  $\tilde{\omega}_k$  is the natural frequency perturbed from the original  $\omega_k$ . Hence, the total dynamic system model including the virtual dynamic mode is written as

$$\begin{aligned} \ddot{\eta}_1 + \omega_1^2 \eta_1 &= \phi_1 u \\ &\vdots \\ \ddot{\eta}_k + \omega_k^2 \eta_k &= \phi_k u \\ \ddot{\tilde{\eta}}_k + \tilde{\omega}_k^2 \tilde{\eta}_k &= \phi_k u \\ &\vdots \\ \ddot{\eta}_{N+1} + \omega_{N+1}^2 \eta_{N+1} &= \phi_{N+1} u \end{aligned} \quad (14)$$

The number of modal coordinates increases due to the augmented mode. It is assumed that  $\tilde{\omega}_k = \omega_k \pm \delta\omega$ , where the perturbation parameter( $\delta\omega$ ) could be selected by a designer. Now, the control input profile should be modified to accommodate the augmented dynamic mode. Thus the number of switching parameters naturally increases by two for each virtually augmented mode. The

state of the virtual dynamics introduces additional constraint prescribed as

$$\tilde{\eta}_k(t) = -\frac{\phi_k}{\tilde{\omega}_k^2} \sum_{j=0}^{2(N+1)} M_j \cos \tilde{\omega}_k(t-t_h) \quad (15)$$

The augmented modal coordinate should also satisfy the terminal boundary conditions such as  $\tilde{\eta}_k(t) = 0, t \geq T$ . In other words,

$$\sum_{j=0}^{2(N+1)} M_j \cos \tilde{\omega}_k(t_j - t_h) = 0 \quad (16)$$

The new constraint equation is added to the original constraint equation (Eq. (8)) covering the augmented dynamic mode also. In order to help us to understand the physical meaning of the new constraint by the virtual dynamic mode, let us consider

$$\begin{aligned} & \sum_{j=0}^{2(N+1)} M_j \cos \tilde{\omega}_k(t_j - t_h) \\ &= \sum_{j=0}^{2(N+1)} M_j \cos(\omega_k \pm \delta\omega)(t_j - t_h) \\ &= \sum_{j=0}^{2(N+1)} M_j \cos \omega_k(t_j - t_h) \mp \delta\omega \sum_{j=0}^{2(N+1)} M_j(t_j - t_h) \sin \omega_k(t_j - t_h) \end{aligned} \quad (17)$$

From Eq. (16) and the terminal constraint on  $\eta_k$ , one can easily show that Eq. (17) can be simplified into

$$\sum_{j=1}^{2(N+1)} M_j(t_j - t_h) \sin \omega_k(t_j - t_h) = 0 \quad (18)$$

The new constraint equation is essentially equivalent to the zero derivative condition in Eq. (10). Hence, the augmented dynamic approach may be regarded as generalization for the constraint on the sensitivity given by Eq. (10). Further expansion of Eq. (18) for higher-order terms in  $\delta\omega$  yields

$$\begin{aligned} & \sum_{j=0}^{2(N+1)} M_j \cos \tilde{\omega}_k(t_j - t_h) \\ &= \sum_{j=0}^{2(N+1)} M_j \cos \omega_k(t_j - t_h) + \sum_{j=0}^{2(N+1)} M_j \sum_{m=1}^q \delta\omega^m f_m(t) \end{aligned} \quad (19)$$

where the  $m$ th term in the series expansion is equal to

$$f_m(t) = \frac{1}{m!} \sum_{j=0}^{2(N+1)} M_j \frac{d^m}{d\omega_k^m} \cos \omega_k(t_j - t_h) \quad (20)$$

with  $m = 2, \dots, q$ . Thus the optimality condition for the high order  $\delta\omega$  terms can be expressed as

$$f_m(t) = 0, \quad m = 2, 3, \dots \quad (21)$$

This equation is also identical that of Ref. [8]. Hence, we can see again that the augmented dynamic model serves the same purpose as zero sensitivity equation enhancing robustness.

To help further understanding on the virtual model and corresponding optimal solution for switching times, the constraints equations, Eqs. (8) and (16) are combined together. Consider a switching time ( $t_j$ ), which should satisfy the constraints for the original as well as augmented dynamic modes. Let us introduce the following notations for the uncertain model parameter.



for  $i=2,3,\dots,N$ . It was shown that the costate vector at the half-maneuver time is prescribed as  $\lambda(t_h)=[\lambda_1(t_h),0,\lambda_2(t_h),0,\dots,\lambda_N(t_h),0]^T$ [1].

The solution to the differential equations for the costate, therefore, becomes

$$\begin{aligned}\lambda_1(t) &= \bar{\lambda}_1(t_h) \\ \lambda_1(t) &= -(t-t_h)\lambda_1(t_h) \\ \lambda_i(t) &= \lambda_i(t_h)\cos\omega_i(t-t_h) \\ \bar{\lambda}_i(t) &= -\lambda_i(t_h)\sin\omega_i(t-t_h)\end{aligned}\quad (30)$$

for  $i=2,3,\dots,N$ . From the costate vector solution, the switching function of the system is formulated from Eq. (28) as follows

$$\xi(t) = \lambda^T B \quad (31)$$

By using the costate equation solution, the switching function becomes

$$\xi(t) = \phi_1\lambda_1(t_h)(t-t_h) + \sum_{i=2}^N \phi_i\lambda_i(t_h)\sin\omega_i(t-t_h) \quad (32)$$

For sufficient condition of optimality, the switching function should be equal to zero for  $t=t_1, t_2, \dots, t_N$ .

Now it is assumed that the virtual dynamic mode is added for the  $k$ th modal coordinate. A modal coordinate ( $\tilde{\eta}_k$ ) is added and corresponding change is accommodated in the state and costate equations. Thus, the new switching function including the  $k$ th virtual dynamic mode is given by

$$\bar{\xi}(t) = \phi_1\lambda_1(t_h)(t-t_h) + \sum_{i=2}^N \phi_i\lambda_i(t_h)\sin\omega_i(t-t_h) + \phi_k\lambda_k(t_h)\sin\tilde{\omega}_k(t-t_h) \quad (33)$$

where  $\tilde{\omega}_k = \omega_k \pm \delta\omega$  represents the perturbed natural frequency of the  $k$ th flexible mode and  $\lambda_k$  is a costate. The new switching function should also satisfy

$$\bar{\xi}(t) = 0, \quad \text{for } t = t_{k+1}, t_{k+2}, \dots, t_{2N} \quad (34)$$

together with

$$1 + \phi_1\lambda(t_h)(t-t_h) + \sum_{i=2}^{N+1} \phi_i\lambda_i(t_h)\sin\omega_i t = 0 \quad (35)$$

Eqs. (33) and (34) can be used to compute  $\lambda_1(t_k), \lambda_2(t_k), \dots, \lambda_N(t_k)$  and  $\lambda_k(t_h)$ . The sufficient condition for the optimality thus can be claimed by examining the new shaping function. The augmented dynamic method thus produces the sufficient condition of optimality. It should be noted that in the previous approaches[8], where the sufficient condition by robust solution was not dealt with. This is because extra switching times were just introduced to satisfy necessary boundary and robust constraints. In our case, the feedback control law becomes

$$u(t) = -N\text{sgn}[\bar{B}^T \bar{\xi}(t)] \quad (36)$$

where  $\bar{B}$  is modified from the original  $B$  to associate with the new mode ( $\tilde{\eta}_k$ ).

## Simulation Results

Sample simulation results are produced for a two-mass model connected by a spring as

Fig. 1.

This model has been used in Ref. [8], and the same model is adopted in this study for the purpose of benchmark test. This model represents a simplified flexible spacecraft model ; combination of center rigid body and flexible structures cantilevered to the center body. The governing equations of motion described in terms of displacement of each mass is

$$\begin{aligned} m_1 \ddot{x}_1 + k(x_1 - x_2) &= u \\ m_2 \ddot{x}_2 - k(x_1 - x_2) &= 0 \end{aligned} \quad (37)$$

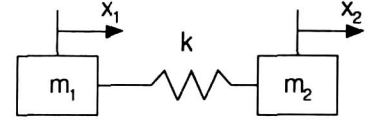


Fig. 1. A two-mass spring model

The mass and stiffness properties are given as  $m_1 = m_2 = 1kg$ , and  $k = 1N/m$ , respectively. A rest-to-rest maneuver condition is specified by the following boundary conditions

$$\begin{aligned} x_1(0) = x_2(0) = 0, \quad x_1(T) = x_2(T) = 1 \\ \dot{x}_1(0) = \dot{x}_2(0) = 0, \quad \dot{x}_1(T) = \dot{x}_2(T) = 1 \end{aligned}$$

The modal coordinate equations corresponding to the original governing equations in Eq. (35) are written as

$$\begin{aligned} \ddot{\eta}_1 + \omega_1^2 \eta_1 &= \phi_1 u \\ \ddot{\eta}_2 + \omega_2^2 \eta_2 &= \phi_2 u \end{aligned} \quad (38)$$

where it can be easily computed as  $\omega_1 = 0$ ,  $\omega_2 = \sqrt{2k/m} = \sqrt{2} rad/sec$ . There are two modes of motion including the rigid-body motion and one flexible mode. The number of switching times for the normal minimum-time rest-to-rest maneuver is therefore equal to three as illustrated in Fig. 2.

The symmetric property for the switching times can be exploited to reduce the number of switching parameters to two. The equality constraints by the boundary conditions for the modal coordinates can be shown to be [8]

$$2 + 2t_1^2 + t_2^2 - 4t_1 t_2 = 0 \quad (39)$$

$$1 - 2 \cos \omega_2(t_2 - t_1) + \cos \omega_2 t_2 = 0 \quad (40)$$

The time optimal control solution for the total maneuver time ( $2t_2$ ) can be solved in conjunction with the inequality constraints by using the MATLAB optimization Toolbox. For the given parameters, it can be shown that the solution is exactly same as Ref. [8].

Simulation results for the displacement ( $x_2$ ) of the second mass by the control input with optimized switching times are presented in Fig. 3. Different values of the material parameter ( $k$ ) are tested to examine the robustness of the system responses. As it can be shown, the system response is dependent upon  $k$  to a significant extent. In fact, Fig. 3 looks almost identical to that of Ref. [8] for benchmarking purpose.

As we can see, the response satisfies exactly the time-optimal rest-to-rest maneuver conditions with a perfect model parameter information. However, as the model parameter changes, residual oscillations are induced. With significant model error, large amount of residual error energy results. In fact, this result is identical to that of the previous study in Ref. [8].

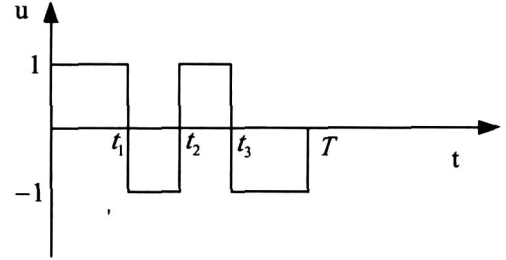


Fig. 2. Control input profile for the example model



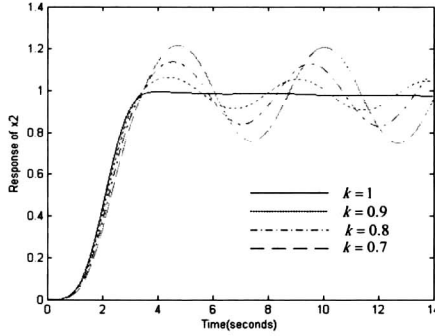


Fig. 3. Simulation results with optimized switching times

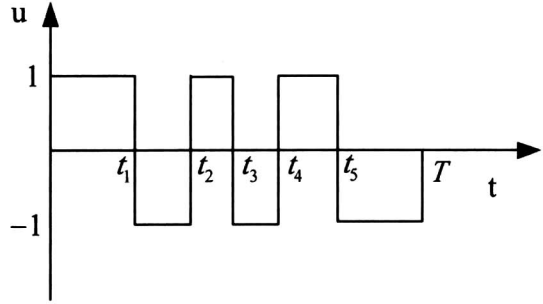


Fig. 4. Control input profile for with an augmented mode

Now we test the robust control approach by assuming uncertainty in the natural frequency  $\omega_2$ . The virtual model approach is taken again by introducing the augmented modal parameter  $\tilde{\omega}_2$ . The number of switching times increases by two, so it is equal to five as Fig. 4.

Again, symmetric property for the switching times reduces the number of switching parameters to three. Constraint equation including the virtual dynamic mode are expressed as

$$\begin{aligned} 2 + 2t_1^2 - 2t_2^2 - t_3^2 - 4t_1 t_3 + 4t_2 t_3 &= 0 \\ \cos \omega_2 t_3 - 2 \cos \omega_2 (t_3 - t_1) + 2 \cos \omega_2 (t_3 - t_2) - 1 &= 0 \\ \cos \tilde{\omega}_2 t_3 - 2 \cos \tilde{\omega}_2 (t_3 - t_1) + 2 \cos \tilde{\omega}_2 (t_3 - t_2) - 1 &= 0 \end{aligned} \quad (41)$$

The MATLAB Optimization Toolbox is used to solve the optimization problem. Several values of the model parameter ( $k$ ) are also examined for simulation. The natural frequency of the augmented dynamic is set to  $\tilde{\omega}_2 = \sqrt{1.8}$  rad/sec to check the performance of the proposed method. The switching times obtained are used to plot the responses of modal coordinates. Simulation results are presented in Fig. 5.

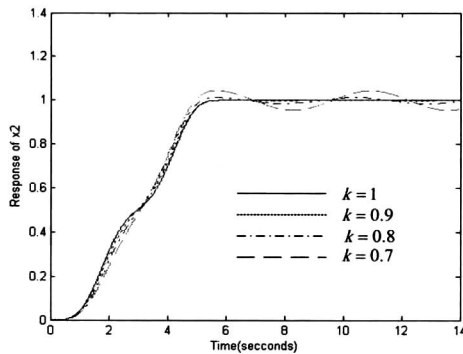


Fig. 5. Simulation results by using robust switching times ( $\tilde{\omega}_2 = \sqrt{1.8}$  rad/sec)

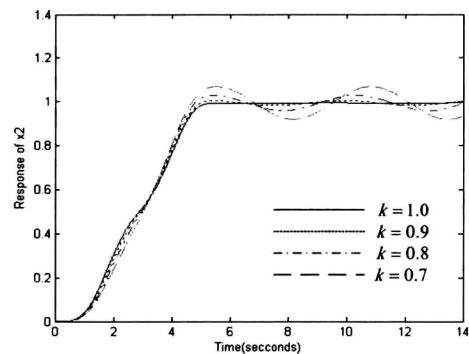


Fig. 6. Simulation results by using robust switching times ( $\tilde{\omega}_2 = \sqrt{2.2}$  rad/sec)

As it can be shown in Fig. 5, the system response looks more robust than Fig. 3 with the help of the augmented dynamic mode. The perturbation of the modal parameter for the augmented dynamic mode is made in the direction consistent with change in  $k$ . Next, we select the modal parameter

perturbation in the direction opposite to the change in  $k$ . Hence, it is selected as  $\omega_2 = \sqrt{2.2}$  rad/sec. Switching times are obtained again by optimization, and simulation results using the optimized switching times are displayed in Fig. 6.

The residual oscillations in Fig. 6 is slightly greater than those of Fig. 5. This is because the modal parameter perturbation is assumed in the wrong direction. Thus, it may be important to decide the modal parameter perturbation in a manner consistent with actual parameter change. However, still noticeable improvement in the transient response in Fig. 6 over Fig. 3 is observed. Thus modal parameter perturbation method shows inherent robustness.

Next, we assume that the modal parameter perturbation is exactly matched with the worst case uncertain model parameter ( $k = 0.7$ ). Thus, the perturbed natural frequency of the second mode is selected as  $\tilde{\omega}_2 = \sqrt{2k/m} = \sqrt{1.4}$  rad/sec. Simulation result based upon the optimization result by taking the new  $\tilde{\omega}_2$  is presented in Fig. 7

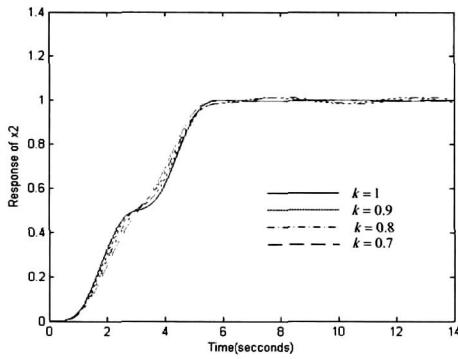


Fig. 7. Simulation results by the perturbed modal parameter exactly matching the worst case model uncertainty

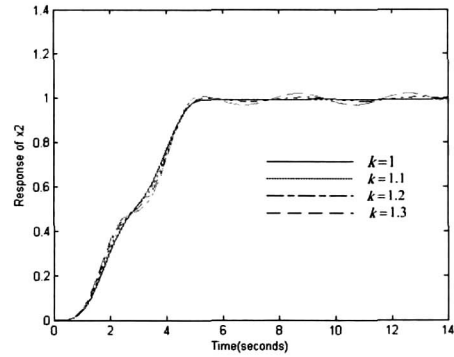


Fig. 8. Simulation results for different direction of change in  $k$  ( $\tilde{\omega}_2 = \sqrt{2.2}$  rad/sec)

The transient response in Fig. 7 is quite small, and it is because of the modal parameter perturbation ( $\tilde{\omega}_2$ ) exactly matching the worst case model parameter change.

In the simulation results, robustness of the response with respect to the model parameter change has improved significantly by augmented dynamic mode. Hence, a proper choice for the perturbed modal parameter ( $\tilde{\omega}_2$ ) can lead to satisfactory performance in the presence of significant model error. Theoretically, if one choose  $\tilde{\omega}_2$  in such a way as to exactly match the parameter  $k$ , then the best performance is assured. It should be noted only the first order condition is considered subject to a small perturbation  $\delta\omega$ . For a large value in  $\delta\omega$ , high order constraints in Eq. (23) should be imposed.

Also, perturbation in the direction of increasing  $k$  was examined. The simulation results are displayed in Fig. 8. This time, the parameter  $\tilde{\omega}_2$  is set to be equal to that of Fig. 6. Since the perturbed  $\tilde{\omega}_2$  is close to the actual variation of the stiffness ( $k$ ), the performance should be better than Fig. 6. This is illustrated well in Fig. 8.

## Concluding Remarks

Augmenting a virtual dynamic mode can increase robustness arising from uncertain model

parameter. It turns out that the new method leads to results similar to the previous approaches by constraint equations for output sensitivity with respect to model parameter. In the new method, one can choose the perturbed model parameter like a design parameter. The augmented dynamic approach results in robust output response over a moderate range of the design parameter. More design freedom may be provided, therefore, by the proposed approach for robust performance of flexible spacecraft maneuver and vibration control.

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