A Study on the Rotor Type Identification Method

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Abstract

The vibration characteristics of rotating machinery are directly related with its condition such as rotor types, thereby it should be acquired the information from the vibration signal so that operating conditions may be rationally decided. Accordingly, the study is to focus on developing the analysis for identifying the operational feature of rotor systems. For this purpose the complex frequency analysis for identification, which utilizes the *directional spectrum* for effective identification of rotor systems, is introduced. From this proposed method, the analysis of dynamic model of the rotors is performed including the stability behavior of the general rotor by *Floquet* theory. Through this process the excitation methodology to identify the types of rotors is investigated and the effective way to identification is also suggested.

Key Word : identification, directional spectrum, rotor types, rotor model, stability

Introduction

Rotating machinery is one of the most important and critical items of many mechanical systems such as power stations, aircraft engines, machine tools, marine propulsion, and medical equipment, etc. Predictive identification by the condition monitoring of this machinery can greatly reduces the catastrophic failure and unnecessary maintenance. Therefore, much effort has been made over past years to develop the practical diagnostic methodology and to identify the condition along with rotor types and operational feature for rotor system [1~4].

In general, a rotor-bearing system consists of rotors and stator parts. According to the non-axisymmetric properties of the rotor and stator, let the rotor types be classified as follows [1]: *isotropic (symmetric) rotor system*-both the rotor and the stator are axisymmetric; *anisotropic rotor system*-the rotor is axisymmetric but the stator is not; *asymmetric rotor system*-the stator is axisymmetric but the rotor is not; *general rotor system*-neither the rotor nor the stator is axisymmetric. Conditions for the asymmetric rotor system are as follows : the two pole generators such as a rotor with asymmetric moment of inertia, a shaft with keyway or rectangular cross section with asymmetric stiffness, a crankshaft with hydrodynamic bearings or two shafts in parallel connected by a couple of gears, and as a special case for a crack, which may develop in a open/closing or breathing status. Likewise, the anisotropic rotor system are; the typical fluid-film bearing and the magnetic or hydrostatic bearing with a comparatively heavy rotor weight, which have a anisotropic stiffness and damping properties, and the supporting structure and foundations with usually having different stiffness properties in the two orthogonal directions.

A general rotor system associated with crack and stator anisotropy is comprehensive case of above illustrated systems, which is general in practice and real in industry.Most of the rotating

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machineries have the one more combinational or even total properties of a general rotor system, which includes three types, i.e., isotropic, anisotropic, asymmetric property.

If one could detect the rotor types their conditions are easily identified, through which health monitoring, diagnostics and even detecting failure accumulations are possible. For this, the identification of such properties of a general rotor system becomes essential in gaining an adequate physical understanding of the dynamic behavior of practical rotors.

This paper proposes the efficient identification method by analyzing the *directional spectrum* characteristics by harmonic inputs for rotor systems. Also the stability of a general rotor by *Floquets* theory[5] is introduced and the methodology to identify the rotor types is to be investigated.

Complex Analysis and Equation of Motion for Rotor System

The complex modal analysis, which has been recently developed for rotor systems, utilizes the so-called *directional spectrum* analysis between complex inputs and outputs for effective identification technique[1]. The use of complex coordinates leads to reduction in the size of equations of motion by one half for axisymmetric systems and allows clear physical interpretations so that identification of modal properties of the rotor system becomes straightforward. It also defines the backward and forward modes for clear physical insight and separates them in the frequency domain, whereas they are heavily overlapped in the classical modal testing theory, so that the effective modal identification and searching more easy way to identify the properties of the rotor types are possible.

Here consider a model of simple asymmetric rotor with a rigid disk located on the shaft axis and rotating on two anisotropic stators or supports as shown in Fig. 1. Here the simple rotor means considering the translatory motion only for analytical simplicity. A rigid asymmetric disk element and the stationary coordinates of its center of mass representing translations are y, z and the rotating coordinates fixed at the shaft are ξ, η. In this model, the shaft is assumed to be comparatively light to neglect its mass. Also the stator(bearing) masses are neglected. Only the disk mass m is considered. No coupling and gyroscopic effect between y(ξ) and z(η) are assumed. The shaft stiffness and damping coefficients are used as those of lumped parameters for modeling simplicity. Note that the effect of internal damping c_ξ, c_η and shaft stiffness k_ξ, k_η include the assembled interference effects as joining or pressed on disk or sleeve to fit shaft to disk.

The equations of motion is derived from the Lagrangian equation applying the lumped parameters. For this proposed model for the simple general rotor, the equation of motion can be obtained in the following complex form as[1,4,6]

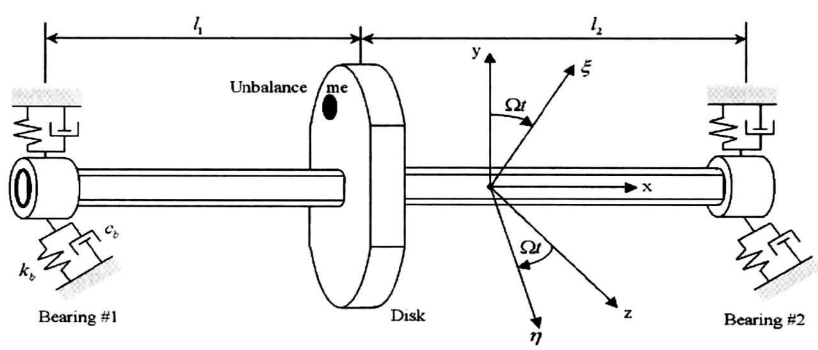

Fig. 1. Modeling a rotor system

$$\mathbf{M_f}\ddot{\mathbf{p}}+\mathbf{C_f}\dot{\mathbf{p}}+(\mathbf{C_b}+\mathbf{C_r}e^{j2\Omega t})\dot{\bar{\mathbf{p}}}+\mathbf{K_f}\mathbf{p}+(\mathbf{K_b}+\mathbf{K_r}e^{j2\Omega t})\bar{\mathbf{p}}=\mathbf{g}, \quad (1)$$

where the response and input vectors are

$$r=y+jz,\quad \mathrm{p}=[r_d,\ r_1,\ r_2]^T,\quad \mathrm{g}=[g_{y_d}+jg_{z_d},\ 0,\ 0]^T, \quad (2)$$

here overbar $\bar{\mathrm{p}}$ means complex conjugate of p, likewise, overbar notations correspond the same meaning henthforth; j means the imaginary number; the bar denotes the complex conjugate; Ω is the rotational speed; **M**, **C** and **K** are the 3x3 complex matrices representing the generalized mass, damping and stiffness, respectively; the subscripts **f**, **b**, and **r** refer to the forward or mean,anisotropic and asymmetric properties, respectively; r_d, r_1, r_2 are displacements of disk, #1,2 bearing displacements, respectively, and g_{yd}, g_{zd} are forces at disk in y-z directions, respectively; the coefficient matrices for parameters can be also written as follows

$$\mathbf{M_f}=m,\ \mathbf{C_f}=\begin{bmatrix} c_r & -\frac{l_2}{l}c_r & -\frac{l_1}{l}c_r \\ -\frac{l_2}{l}c_r & \left(\frac{1}{l}\right)^2(l_2^2c_r+l^2c_{b1}) & \left(\frac{1}{l}\right)^2 l_1l_2c_r \\ -\frac{l_1}{l}c_r & \left(\frac{1}{l}\right)^2 l_1l_2c_r & \left(\frac{1}{l}\right)^2(l_1^2c_r+l^2c_{b2}) \end{bmatrix},\ \mathbf{C_b}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta c_{b1} & 0 \\ 0 & 0 & \Delta c_{b2} \end{bmatrix},$$

$$\mathbf{C_r}=\begin{bmatrix} \Delta c_r & -\frac{l_2}{l}\Delta c_r & -\frac{l_1}{l}\Delta c_r \\ -\frac{l_2}{l}\Delta c_r & \left(\frac{1}{l}\right)^2 l_2^2\Delta c_r & \left(\frac{1}{l}\right)^2 l_1l_2\Delta c_r \\ -\frac{l_1}{l}\Delta c_r & \left(\frac{1}{l}\right)^2 l_1l_2\Delta c_r & \left(\frac{1}{l}\right)^2 l_1^2\Delta c_r \end{bmatrix},\ \mathbf{K_f}=\begin{bmatrix} k_r-j\Omega c_r & -\frac{l_2}{l}(k_r-j\Omega c_r) & -\frac{l_1}{l}(k_r-j\Omega c_r) \\ -\frac{l_2}{l}(k_r-j\Omega c_r) & \left(\frac{1}{l}\right)^2(l_2^2k_r-j\Omega l_2^2c_r+l^2k_{b1}) & \left(\frac{1}{l}\right)^2(l_1l_2k_r-j\Omega l_1l_2c_r) \\ -\frac{l_1}{l}(k_r-j\Omega c_r) & \left(\frac{1}{l}\right)^2(l_1l_2k_r-j\Omega l_1l_2c_r) & \left(\frac{1}{l}\right)^2(l_1^2k_r-j\Omega l_1^2c_r+l^2k_{b2}) \end{bmatrix},$$

$$\mathbf{K_b}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta k_{b1} & 0 \\ 0 & 0 & \Delta k_{b2} \end{bmatrix},\ \mathbf{K_r}=\begin{bmatrix} \Delta k_r+j\Omega\Delta c_r & -\frac{l_2}{l}(\Delta k_r+j\Omega\Delta c_r) & -\frac{l_1}{l}(\Delta k_r+j\Omega\Delta c_r) \\ -\frac{l_2}{l}(\Delta k_r+j\Omega\Delta c_r) & \left(\frac{1}{l}\right)^2(l_2^2\Delta k_r+j\Omega l_2^2\Delta c_r+l^2\Delta k_{b1}) & \left(\frac{1}{l}\right)^2(l_1l_2\Delta k_r+j\Omega l_1l_2\Delta c_r) \\ -\frac{l_1}{l}(\Delta k_r+j\Omega\Delta c_r) & \left(\frac{1}{l}\right)^2(l_1l_2\Delta k_r+j\Omega l_1l_2\Delta c_r) & \left(\frac{1}{l}\right)^2(l_1^2\Delta k_r+j\Omega l_1^2\Delta c_r+l^2\Delta k_{b2}) \end{bmatrix}, \quad (3)$$

where $l=l_1+l_2$, the subscripts d, 1, 2 denote disk, bearing 1 and bearing 2, respectively. and the parameters are described in the form of

$$\begin{aligned} &c_r=(c_\xi+c_\eta)/2,\ \Delta c_r=(c_\xi-c_\eta)/2,\ k_r=(k_\xi+k_\eta)/2,\ \Delta k_r=(k_\xi-k_\eta)/2, \\ &c_b=(c_y+c_z)/2,\ \Delta c_b=(c_y-c_z)/2,\ k_b=(k_y+k_z)/2,\ \Delta k_b=(k_y-k_z)/2. \end{aligned} \quad (4)$$

Note that the equation of motion (1) reduces to an isotropic rotor where an $c_b = c_r = k_b = k_r = 0$, anisotropic rotor where $c_r = k_r = 0$, asymmetric rotor where $c_b = k_b = 0$.

From Eq. (1) and its complex conjugate form, the complex equation of motion can be constructed as

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}(t)\dot{\mathbf{q}}(t) + \mathbf{K}(t)\mathbf{q}(t) = \mathbf{f}(t) \quad (5)$$

where

$$q(t) \equiv \begin{Bmatrix} p(t) \\ \bar{p}(t) \end{Bmatrix}, \quad f(t) \equiv \begin{Bmatrix} g(t) \\ \bar{g}(t) \end{Bmatrix},$$

$$M = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}, \quad C(t) = \begin{bmatrix} C_f & C_b + C_r e^{j2\Omega t} \\ \bar{C}_b + \bar{C}_r e^{-j2\Omega t} & \bar{C}_f \end{bmatrix}, \quad K(t) = \begin{bmatrix} K_f & K_b + K_r e^{j2\Omega t} \\ \bar{K}_b + \bar{K}_r e^{-j2\Omega t} & \bar{K}_f \end{bmatrix}. \quad (6)$$

Equation (1) can be rewritten in the state space form for response analysis as

$$\mathbf{A}(t)\dot{\mathbf{w}}(t) = \mathbf{B}(t)\mathbf{w}(t) + \mathbf{F}(t), \quad (7)$$

where

$$\mathbf{A}(t) = \begin{bmatrix} \mathbf{0} & \mathbf{M} \\ \mathbf{M} & \mathbf{C}(t) \end{bmatrix}, \quad \mathbf{B}(t) = \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & -\mathbf{K}(t) \end{bmatrix}, \quad \mathbf{w}(t) = \begin{Bmatrix} \dot{\mathbf{q}}(t) \\ \mathbf{q}(t) \end{Bmatrix}, \quad \mathbf{F}(t) = \begin{Bmatrix} \mathbf{0} \\ \mathbf{f}(t) \end{Bmatrix} \quad (8)$$

Now consider the responses with a numerical approach, because no closed form solution is available for a general rotor system as in Eq. (1). For this the equation form for solution of the given simple general rotor model for Runge-Kutta integration method is rearranged as

$$\dot{\mathbf{q}} = \mathbf{E}^{-1}(\mathbf{F}\mathbf{q} + \mathbf{G}) \quad (9)$$

where the inversion of $\mathbf{E}$ assumed to exist. In this case the bearing responses are condensed to eliminate the matrix $0_{2\times2}$ because of neglecting bearing masses, which causes corresponding matrix to be singular and if neglecting the damping coefficients of the shaft and bearings the corresponding $\mathbf{0}$ matrix would be condensed further, so that the response and force vectors become respectively

$$\mathbf{q} = [\mathbf{r}, \bar{\mathbf{r}}]^T, \quad \mathbf{r} = [\dot{r}_d, r_d, r_1, r_2]^T, \quad \mathbf{G} = [\mathbf{0}_{1\times2} \; \mathbf{g}_{1\times4}]^T, \quad (10)$$

and the coefficient matrices are

$$A = \begin{bmatrix} 0 & m & 0 \\ m & C_f & \\ 0 & & \end{bmatrix}, \quad B = \begin{bmatrix} m & 0 \\ 0 & -K_f \end{bmatrix}, \quad \begin{matrix} C = C_r e^{j2\Omega t} + C_b \\ D = K_r e^{j2\Omega t} + K_b \end{matrix}, \quad E = \begin{bmatrix} A & C \\ \bar{C} & \bar{A} \end{bmatrix}, \quad F = \begin{bmatrix} B & -D \\ -\bar{D} & \bar{B} \end{bmatrix}. \quad (11)$$

Stability and Directional Spectrum Analysis of Rotor System

The stability of the time-periodic system can not be determined by normal eigenvalues so that in periodic system one must resort to *Floquet* analysis[5]. In this case $\phi(t)$ is a fundamental matrix that satisfies the matrix equation with the initial condition $\phi(0) = I$. Here $\phi(T)$ is the *monodromy* matrix can be obtained after one period $T(=\pi/\Omega)$ from the initial condition $\phi(0)$ which is calculated from the numerical as Runge-Kutta method. From these premises an eigenvalue problem is derived in the form of $[\phi(T)-\lambda I]r=0$. The criterion of the stabilities is that the system is stable for the condition that all eigenvalues are $|\lambda| \leq 1$ whereas unstable if any of the eigenvalues is $|\lambda| > 1$.

The key idea for the *directional spectrum*(dS) of a general rotor system is that the harmonic components can be directly identified in the *directional spectrum* which is acquired from Fourier transform of the complex-valued signal representing responses derived in equation (5). The positive(negative) frequency components appearing in the dS physically correspond to forward(backward) whirling components. Thus from the configurations of the dS for various harmonic excitations the detection of the rotor types(isotropic, anisotropic, asymmetric and general rotor), by which the diagnosis for any defect or fault could be effectively used. The *directional* spectrum can be directly obtained from the Fourier transformation of this response[7].

Numerical Simulation Results and Discussions

For simulations, the numerical values of parameters for the proposed simple rotor model have been used as shown in Table 1 and the common parameters such as disk mass and the unbalance with 0 phase expressed by *me,* where *m* is unbalance mass and *e* is its eccentricity, are 8 kg, 10 g.cm, respectively, along with $l_1 = l_2 = 300$ *mm.*

Table 1. Values of Physical Parameters Used in Rotor Model

Part	Parameters		Isotropic Rotor	Anisotropic Rotor	Asymmetric Rotor	General Rotor
Shaft	Stiffness (N/m)	k_ξ	8x10^6	8x10^6	8x10^6	8x10^6
		k_η			6x10^6	6x10^6
Shaft/Disk	Internal Damping (Nsec/m)	c_ξ	500	500	500	500
		c_η			300	300
Bearings (#1,2)	Stiffness (N/m)	k_{by}	3x10^6	3x10^6	3x10^6	3x10^6
		k_{bz}		2x10^6		2x10^6
	Damping (Nsec/m)	c_{by}	5x10^6	5x10^6	5x10^6	5x10^6
		c_{bz}		3x10^6		3x10^6

For the stability conditions of the general rotor, Fig. 2 exhibits the unstable configuration found around 5000 rpm by *Floquet's* maximum characteristic multiplier greater than 1 so that the system shows inherently diverging, i.e., the 1st critical speed is about 5000 rpm so that the system resonance is located around that region.

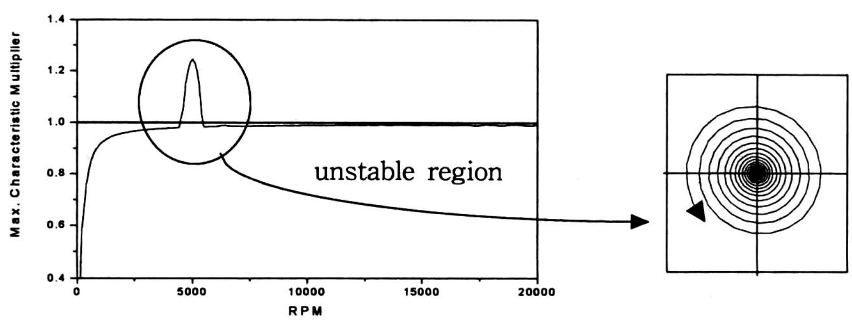

Fig. 2. Stability Check of General Rotor from Max. Characteristic Multiplier

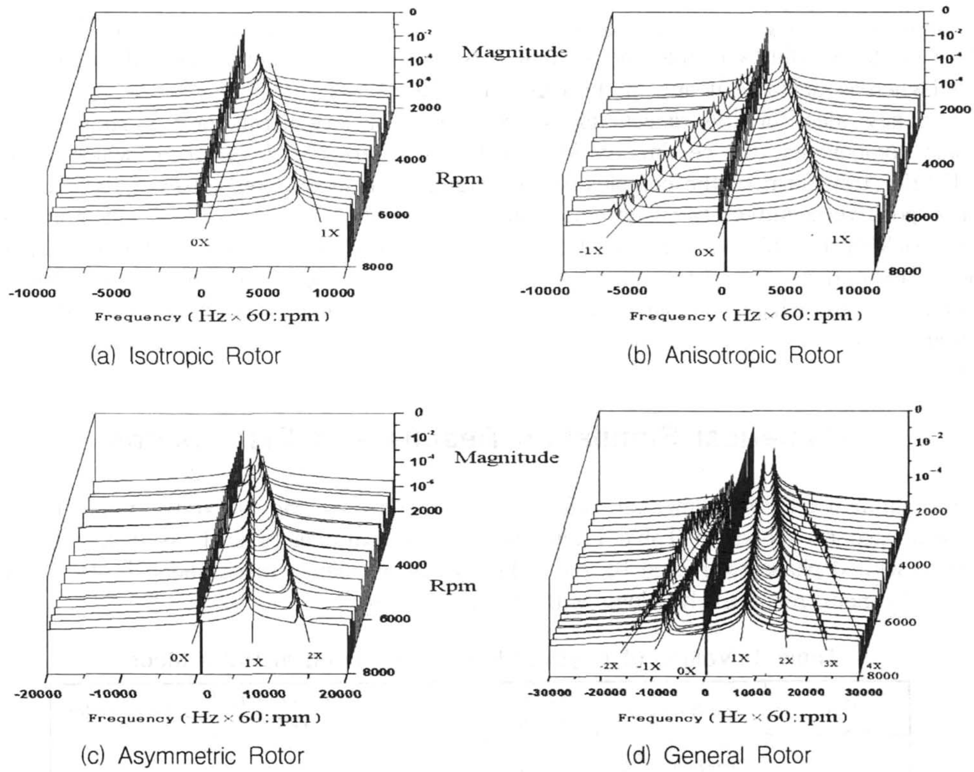

Fig. 3. ***Directional Spectra*** **of 4 Rotor Types by Unbalance/Weight Excitation**

The proposed *directional spectrum* by excitations at disk is obtained from FFT(Fast Fourier Transform) of the steady state response derived by 4th order Runge-Kutta integration method. Here the toolbox of *fft* and *shift* in *MATLAB* are used to extract FFT and shift the frequency spreads over zero position. Fig. 3 show typical cases among other ones the simulated results of 4 rotor types at disk location. They show overall *directional spectra* profiles by waterfall plots at suggested excitations.

Table 2. Simulated Results of ***Directional Spectra*** **for Identifying 4 Rotor Types by Excitations [U : Unbalance(Forward 1X), W : Weight , B : Backward 1X]**

	Isotropic Rotor	Anisotropic Rotor	Asymmetric Rotor	General Rotor
U	1X	-1X,1X	1X	-1X,1X,3X
W	0X	0X	0X,2X	-4X,-2X,0X,2X,4X
U/W/B	-1X, 0X,1X	-1X, 0X,1X	-1X, 0X,1X,2X,3X	-3X,-2X, -1X, 0X,1X,2X,3X,4X
U/B	-1X,1X	-1X,1X	-1X,1X,3X	-3X,-1X,1X,3X
U/W	0X,1X	-1X,0X,1X	0X,1X,2X	-2X,-1X,0X,1X,2X,3X
W/B	-1X,0X	-1X,0X,1X	-1X,0X,2X,3X	-3X,-2X,-1X,0X,1X,2X,3X,4X
B	-1X	-1X,1X	-1X,3X	-3X,-1X,1X,3X

▭ : Excitations Possible to Identify the 4 Rotor Types

Here in a general rotor all the modes observed in the three types of rotors are appeared along with extra modes, i.e., the 3rd and 4th order forward and backward modes. The *directional spectra* show clearly the characteristics of the 4 rotor types. From clear separations of spectra in two-side frequencies we can tell diagnostic properties in directional-wise. In a general rotor, higher harmonics above the 2nd are sporadically appeared according to rotational speed ranges. Table 2. shows the simulation results of the *directional spectra* for 4 rotor types by the various excitation techniques at disk. Here the 7 excitation types are considered to identify the 4 rotor types, through which they can be discriminated by the *directional spectra* of them. Among these the backward excitation looks theoretically best for its simplicity and observations, however in real world, every rotor system has an unbalance and weight inherently even small quantities so that pure backward excitation technique is not available. Other excitations have the similar problems. As a result the use of the unbalance and weight, which are the inherent properties in real rotor system, can be the most available excitation techniques for identifying the types of rotors in practice.

Conclusions

To investigate the feasibility of the identifying methodology by using the complex frequency analysis of a rotor system, the simple general rotor model is formulated. From that proposed model the system stability analysis is studied in advance by using *Floquets* theory. Based upon response analysis the *directional spectrum* analysis is performed, thereby the dynamic behaviors of the 4 rotor types are identified, through which the one of the most effective way to excite for identifying the rotor types is found to be the unbalance and weight, i.e. the inherent property of the practical rotor system.

Acknowledgement

This work is part of the results from the project "Small Scale Rotor for Next Generation Rotor System" supported by Korea Aerospace Research Institute, Korea.

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