Integer Ambiguity Search Technique Using Separated Gaussian Variables

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Abstract

Real-Time Kinematic GPS positioning is widely used for many applications. Resolving ambiguities is the key to precise positioning. Integer ambiguity resolution is the process of resolving the unknown cycle ambiguities of double difference carrier phase data as integers. Two important issues of resolving are efficiency and reliability. In the conventional search techniques, we generally used chi-square random variables for decision variables. Mathematically, a chi-square random variable is the sum of mutually independent, squared zero-mean unit-variance normal (Gaussian) random variables. With this base knowledge, we can separate decision variables to several normal random variables. We showed it with related equations and conceptual diagrams. With this separation, we can improve the computational efficiency of the process without losing the needed performance. If we average separated normal random variables sequentially, averaged values are also normal random variables. So we can use them as decision variables, which prevent from a sudden increase of some decision variable. With the method using averaged decision values, we can get the solution more quickly and more reliably.

To verify the performance of our proposed algorithm, we conducted simulations. We used some visual diagrams that are useful for intuitional approach. We analyzed the performance of the proposed algorithm and compared it to the conventional methods.

Key Word : RTK-GPS, ambiguity search technique, ambiguity resolution

Introduction

In RTK GPS positioning, we must solve the integer ambiguity for using precise carrier phase measurements. For this reason, we need appropriate resolution technique for integer ambiguities. Many researchers have been involved in this problem and presented a lot of possible solutions.

The key elements of performance indices for ambiguity resolution include time-to-fix, computational effectiveness, and reliability that is most critical one. In general, we need more time for more reliable solution. We can fix the solution even in one second, but we hardly can guarantee the reliability of that. On the other hand, there is no reason to idle away our time. As a expected result, we need reasonable and acceptable "trade-off" to solve this dilemma. When we apply this kind of trade-off, effectiveness of the technique becomes the key of problem. Figure 1. describes this concept. With more effective method, we can get the equally reliable solution with less time.

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Typical ambiguity search process can be divided into three parts: search space construction, sequential test, and verification processes.

The first step is the construction of initial search space. This step is critical for continued steps, especially initial phases of search process. Too many initial candidates need too much computation time. The well-known and very efficient method for this phase was presented in LAMBDA(Least-squares AMBiguity Decorrelation Adjustment) method. We used this method for first step in this paper.

Fig. 1. Time-to-fix vs. Reliability and Effectiveness

The second step is the sequential tests of candidates. In this phase, we need appropriate decision variable that efficiently represent the stochastic characteristics of candidates. With this kind of decision variable and its threshold, we check each candidate and eliminate unacceptable one. Conventionally, chi-squared random variable has been used for decision.

The third step is the verification process using the method like namely "ratio test". This test uses the ratio of minimum and second-minimum values of chi-squared variables. This radio has F-distribution. If a series of ratios exceed pre-defined threshold and satisfy the necessary condition, we can fix the solution as true.

Among these three steps, we focused on the second step. This paper proposed new decision variables and new test procedures. We used some intuitional diagrams for describing proposed concepts. To verify the performance of our proposed algorithm, we conducted simulations. We analyzed the performance of the proposed algorithm and compared it to the conventional methods.
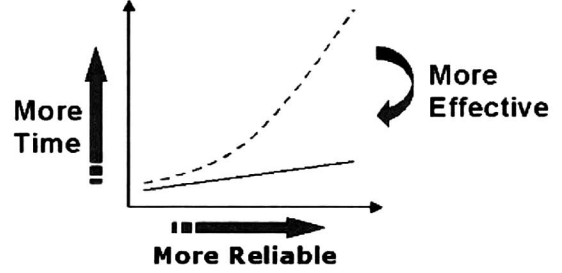
Nomenclature

$^i\triangledown^j$: difference between i-th and j-th satellites

$_r\triangle_u$: difference between reference station and user

ϕ : carrier phase measurements

b : baseline vector from reference station to user

e_u : line of sight vector from user to satellite

λ : wavelength of L1 frequency

N : integer ambiguity

ε_ϕ : measurement noise $\sim N(0, \sigma_\phi)$

σ_ϕ : standard deviation of ε_ϕ

δN : residual of integer ambiguity

x : confidence level

m_i : number of candidates of i-th Gaussian test

F : computational efforts (flops/candidate)

$\delta\hat{n}$: normalized residual of integer ambiguity

$\overline{\delta n}$: average of normalized residuals of integer ambiguity

k : number of epochs

$\{ \cdot \}_I$: independent set

$\{ \cdot \}_D$: dependent set

$\{ \cdot \}_r$: reference station

$\{ \cdot \}_u$: user

Algorithms

Conventional approach

Conventionally, we used the chi-squared random variable for decision. If we have N visible satellites, decision variable has N-4 degrees of freedom. With these variables, we check whether those are in thresholds or not. After the test, we eliminate unacceptable candidates. We repeat this process with remained candidates. next epoch. Figure 3 describes an example of chi-squared test. The solid line means ideal chi-squared distribution. The bars are histogram of decision variables of candidates. These are all about 2 degrees of freedom i.e. the case of 6 satellites.

Proposed approach

Mathematically, chi-squared random variable with N-4 degrees of freedom is equivalent to the sum of N-4 squared Gaussian random variables. Based on this idea, we can separate a conventional decision variable to several normal random variables. It can be derived by geometrical intuition or using some related equations. Because we use separated variables, tests must be done for each measurement independently. Figure 2 shows this concept. In this case, as mentioned previously, we have 6 measurements i.e. 5 double-differenced measurements, so there are two Gaussian decision variables. This figure shows ideal Gaussian distribution and histogram of new decision variables of candidates.

Circle bound of figure 2 is the threshold of chi-squared test. Square bound is the intersection of two thresholds of Gaussian tests. Each bound has the same probability. As shown by these two figures, residuals of true ambiguity can be assumed to have Gaussian distribution and also chi-squared distribution. But usually other candidates have different distribution as shown by figure 2, 3.
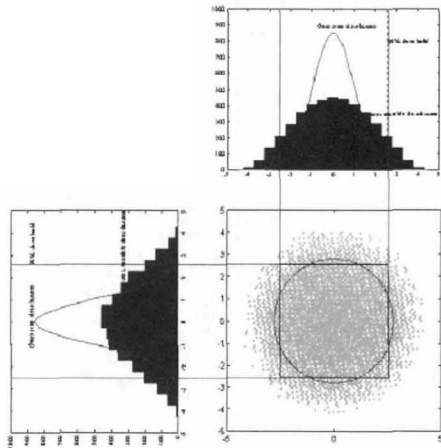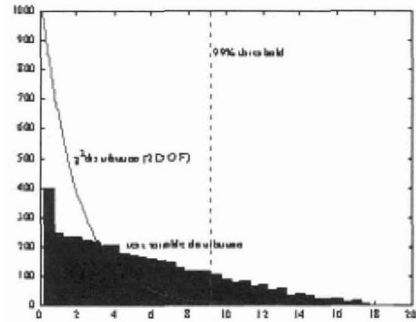

Fig. 2. Concept of Gaussian test

Fig. 3. Concept of chi-squared test

Derivation of new decision variable

We can write a double-differenced carrier phase measurement equation as follows.

$$^i\nabla^j{}_r\triangle_u\phi = \boldsymbol{b} \cdot {}^i\nabla^j\boldsymbol{e_u} + \lambda\ {}^i\nabla^j{}_r\triangle_u N + {}^i\nabla^j{}_r\triangle_u\varepsilon_\phi \tag{1}$$

Now, we accumulate equations of all measurements and rewrite them as follows.

$$\boldsymbol{z} = \boldsymbol{Hb} + \lambda\,\boldsymbol{N} + \boldsymbol{\nu} \tag{2}$$

In this equation, only three equations are independent. So, we can separate them into three independent equations and the other dependent equations as follows.

$$\begin{bmatrix} z_I \\ z_D \end{bmatrix} = \begin{bmatrix} H_I \\ H_D \end{bmatrix} b + \lambda \begin{bmatrix} N_I \\ N_D \end{bmatrix} + \nu \tag{3}$$

Using this independent set, we make initial integer candidates. With these candidates, we can compute residuals of dependent set as follows.

$$\hat{b} = H_I^{-1}(z_I - \lambda N_I) \tag{4}$$

$$\hat{N}_D = z_D - H_D \hat{b} = z_D - H_D H_I^{-1}(z_I - \lambda N_I) \tag{5}$$

$$\delta N_D = \hat{N}_D - round(\hat{N}_D) \tag{6}$$

We use these residuals as decision variables. If we assume that all noises have Gaussian distributions, we can compute variances of each residual by using following equation.

$$\sigma_{N,i}^2 = E\{\delta N_{D,i}^2\} = \left\{ \nabla e_{D,i}^T H_I^{-1} Q_I H_I^{-T} \nabla e_{D,i} - 2 \nabla e_{D,i}^T H_I^{-1} \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} + 4 \right\} \sigma_\phi^2 \Big/ \lambda^2 \tag{7}$$

Using square roots of these variances as thresholds, we check all candidates and eliminate some of them that are unacceptable. Related equation is as follows.

$$\delta N_{D,i} \leq x \sigma_{N,i} \tag{8}$$

Geometrical derivation

We can derive equation (7) by using geometrical intuition. With independent set, we can construct grids of candidates. Figure 4 shows this concept in 2D case. Each measurement that constructs grid points has its own error distribution. It can be assumed to be a Gaussian. So, each grid point makes an error ellipse as shown in figure 4. It becomes error ellipsoid in 3D case. With this error ellipse, we can compute the threshold of each dependent set.
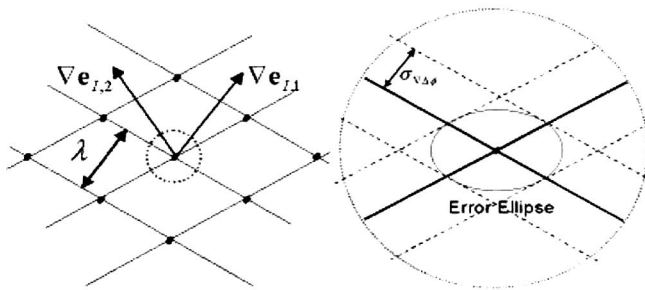

Fig. 4. Grid and its error ellipse using 2 independent measurements (2D case)
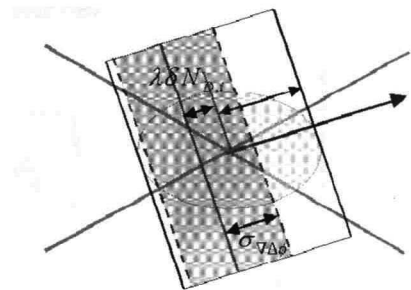

Fig. 5. 1σ bounds of grid point & measurement

Figure 5 demonstrates equation (7) conceptually. In figure 5, the width of the zone between two solid lines means one sigma of grid points with respect to the direction of a correspond measurement from dependent set. First term in equation (7) corresponds to this. This measurement from dependent set also has its own uncertainty. The width of grey zone in figure 5 indicates that and corresponds to third term in equation (7). Considering relation between these two uncertainties including their correlation that is second term of equation (7), we can finally compute the variance of decision variable.

Computational effectiveness

Separated decision variables give us the computational effectiveness. If we have m_1 candidates initially and we need F flops per candidates for computation, chi-square test simply need $m_1 F$ flops. On the other hand, approximate flops for Gaussian test can be written as following equation.

$$\sum_{k=1}^{n-4} m_k \frac{F}{n-4} = (m_1 + m_2 + \cdots + m_{n-4}) \frac{F}{n-4} \tag{9}$$

Gaussian test may obtain computational effectiveness from its measurement-by- measurement tests. Generally, this relationship can be assumed for Gaussian test.

$$m_1 \rangle m_2 \rangle \cdots \rangle m_{n-4} \tag{10}$$

From this relationship, computation efforts of Gaussian test become smaller than those of chi-squared test as shown by following equation (11)

$$m_1 F \rangle \sum_{k=1}^{n-4} m_k \frac{F}{n-4} \tag{11}$$

Vulnerability of epoch-by-epoch method

In the case of temporal exceptionally large error of some measurement, epoch-by-epoch method may encounter the situation removing "true ambiguity". Figure 6 shows the case. In this figure, we can check the time variation of decision variables for each true and wrong candidate. Figure in the circle shows the state of 10 second. At that moment, magnitude of decision variable for true candidate exceeds the confidence level while that of wrong candidate doesn't. This unwanted occurrence may cause dangerous wrong-fix. To get around this problem, we proposed "the sequential averaging of separated Gaussian variables" as follows.
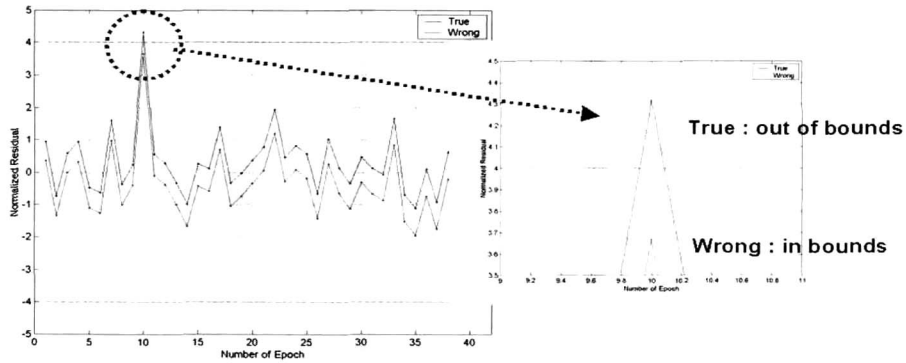

Fig. 6. Vulnerability of epoch-by-epoch method

Sequential averaging

The proposed concept is quite simple. We just average the separated Gaussian variables sequentially and get a new decision variable. Followings are related equations.

At first, we normalize decision variable with its standard deviation and get a new variable that has normal distribution.

$$\delta N_{D,i} \sim N(0, \sigma^2_{N,i}) \rightarrow \delta \hat{n}_{D,i} = \frac{\delta N_{D,i}}{\sigma_{N,i}} \sim N(0, 1^2) \tag{12}$$

With this normalized variables, we average them sequentially by using following equation.

$$\delta \overline{n}_{D,i}(1) = \delta \hat{n}_{D,i}(1) \tag{13}$$

$$\delta \overline{n}_{D,i}(k) = \frac{k-1}{k} \delta \overline{n}_{D,i}(k-1) + \frac{1}{k} \delta \hat{n}_{D,i}(k) \tag{14}$$

And we do the test & elimination again as follows.

$$\delta \overline{n}_{D,i}(k) \geq x \frac{1}{\sqrt{k}} \tag{15}$$

Using this method, true candidate remain safely and wrong candidates are removed relatively quick. In the case of wrong candidates, their biases should be accumulated and their decision variables may be increased relatively fast. With this characteristics, we can say that proposed method is more effective approach. Related results are shown in next section.

Simulation Results

For the performance analysis, we conducted simulations. We used 24-hour full measurements and tried search processes 800 times for statistical analysis. We applied short baseline assumption. In this simulation, biases caused by ionospheric delay, tropospheric delay, and orbital errors can be neglected. Using LAMBDA method, we constructed initial search space. We used four-sigma as confidence level for this step.

Computational efforts

Here come the results of analyzed computational loads for each method. Figure 6 shows the ratio results that use base values as necessary flops of Gaussian epoch-by-epoch test. This result indicates that chi-squared test needs two to four times more flops.
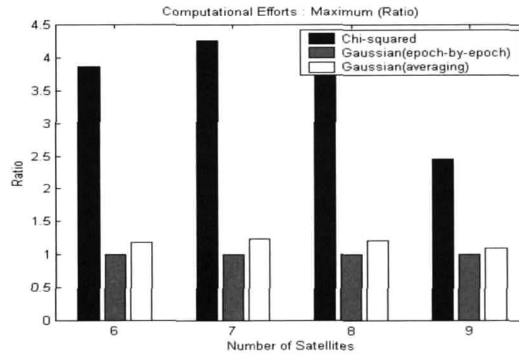

Fig. 7. Ratio of computational efforts of each method

Success rate

Figures 7 demonstrate success rate of each test. It can be used as the indicator of appropriateness. As obvious result, the larger confidence level we apply, the better success rate we can achieve. With confidence level more than five-sigma, all methods get to 100 % success rate. But we can see overall performance of averaging method is better than the others.

Time-to-fix vs. reliability

Here comes the most important result of this paper. With figures 8, we can easily indicate

the performance differences of each method. First of all, it is clear that time-to-fix is inversely proportional to the number of satellites. But important point is the increasing trend of time-to-fix with respect to confidence level that indicates reliability. Using epoch-by-epoch method, time-to-fix increase almost exponentially for getting more reliable solution. But with sequential averaging method, their trends are more like linear proportional with relatively small gradients. As mentioned earlier in this paper, effectiveness of search technique is the key of the efficient "trade-off". Proposed method can get this effectiveness as shown here. Using our proposed method, we can get equally reliable solution with less time.
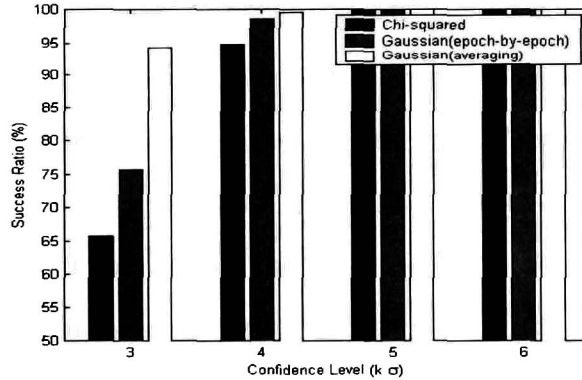

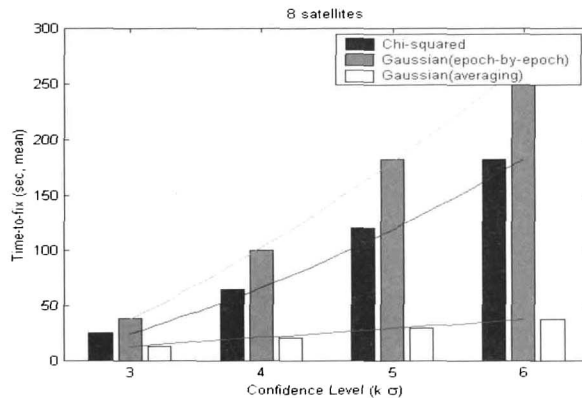**Fig. 8. Success rate (8 satellites)**

Fig. 9. Time-to-fix vs. Reliability (8 satellites)

Conclusions

We derived new decision variable by using related equations and intuitional diagrams. It is "sequentially averaged results of separated Gaussian variables". We analyzed performance of proposed approach by simulations. Proposed approach shows better performances in comparison with conventional approach. We effectively reduced computational loads to approximately 25~50% of conventional method. Especially, using sequential averaging technique, we contrived the method that can get the steadiness of desired solution and amplify the unsteadiness of the other wrong solutions. As a result, we achieved the effectiveness of sequential test process. With much less time that is about 20% of conventional technique, we obtained equally reliable solution.

Our proposed method can be the possible solution of faster and more reliable technique for

the ambiguity search. For having more confidence of this approach, experimental results will be presented instead of simulation results in the future.

Acknowledgement

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