Identification of Anisotropic Bearing Non-linearity

Dong-Ju Han*
Research Center, Sunaerosys Co.
339-822 Songwon-ri, Nam-myeon, Yeongi-gun, Chungnam, Korea, 248-1

Abstract

Among other critical conditions in rotor systems the large non-linear vibration excited by bearing non-linearity causes the rotor failure. For reducing this catastrophic failure and predictive analysis of this phenomena the identification analysis of bearing non-linearity in an anisotropic rotor system using the higher order dFRFs are developed and are shown to be theoretically feasible as in non-rotating structures. For the identification of the anisotropic rotor with anisotropic bearing non-linearity expressed by the displacement in polynomial form, the higher order dFRFs based upon the Volterra series are investigated and depict their features by using the simple forms of the *normal* and *reverse* dFRFs. They produce additional sub-harmonic resonant peaks, which indicate the existence of higher order non-linearties, and show the energy transfer such that the modes for *normal* and *reverse* dFRFs are exchanged, which are the fundamental differences from what we can expect in linear ones.

Key Word : identification, bearing non-linearity, non-linear directional frequency response function, higher order Volterra kernel

Introduction

In general, a rotor-bearing system consists of rotor and stator parts. According to the non-axisymmetric properties of the rotor and stator, a *anisotropic rotor system* is defined as: (Lee and Joh, 1994; Lee and Joh, 1996; Lee, 1997): the rotor is axisymmetric but the stator is not. The accidental or intended presence of anisotropy in a rotor system may significantly alter its dynamic characteristics, such as the unbalance response, critical speeds and stability, from the ideal isotropic (symmetric) rotor. Thus, accurate identification of such anisotropic properties becomes essential in gaining an adequate physical understanding of the dynamic behavior of practical rotors.

The critical and local property but generally accepted feature to the defined *anisotropic rotor system* is the stator or bearing non-linearity, let alone manufacturing errors, clearances and joint surfaces. In particular the rolling element bearing, hydrodynamic bearings and squeeze film dampers are known to possess highly non-linear characteristics and their reliabilities are directly related to closer predictions and identifications of dynamic responses. Hence it is quite reasonable to investigate the effective non-linear diagnostic method as its identification for bearing non-linearity. The mathematical non-linear analysis in rotor systems whose dynamic behaviors along with the parameter may be determined, still remains some far from practical applications, however, they are useful and advantageous by explaining the physical phenomena.

As a result, the main issues of the diagnosis for non-linear properties in stator have been identifying the characteristics of its behavior from the signals in practice. In these respects, Lin

* C.T.O, Head of Research Center
E-mail : djhan@sunaerosys.com, TEL : 041-864-2177, FAX : 041-864-2035

[1993] presented a non-linear identification of complex modes with hysteretic damping model in general structure and Ozguven [1993] also introduced the similar concept for non-linear frequency response. Tiwari [1995] suggested a non-linear parameter identification of rolling element stiffness by introducing probability density function with the model of a cubic non-linearity. Liangsheng [1993] introduced the concept of the pseudo-phase diagram and spectrum from the raw signals. Also other researches have been made using non-linear time series model as NARMAX for its relatively analytical easiness though its computational efforts and contamination by noise. Most of those researches are mainly focused on the analysis of the non-linear phenomena themselves, furthermore, such a case for concentrating practical non-linear identification is mostly limited to simple general structures or simple isotropic rotor, which is in practice the same as the simple stationary structure, so that few attempts have been made in identification of the anisotropic rotor with anisotropic bearing non-linearity. Further, the advantages of the *directional spectrum* associated with *directional frequency response functions* (dFRFs) developed for linear systems as shown previously [1] have been never investigated in non-linear ones. As these reasons, in this study, in the sense that if the non-linearity can be expressed in polynomial form and the system is stable and time-invariant, the higher order frequency responses (FRFs) based upon the Volterra series [3~5,10~13] are the practically valuable tool for identifying the nature of non-linearities in wide class of structure, the higher order dFRFs for anisotropic rotor system are newly investigated and show its feasibility to further application.

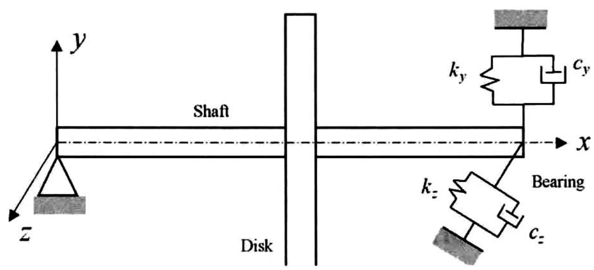

Fig. 1. Modeling a simple anisotropic rotor system

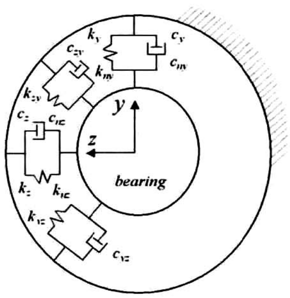

Fig. 2. Bearing Stiffness and Damping Coefficient

Representations of Bearing Non-Linearity

The feature of the bearing non-linearity is mainly caused by the dynamic motion of the fluid film between the rolling element bearing and its journal of the hydrostatic or hydrodynamic bearing. The stiffness and damping characteristic of these bearings are non-linear function of their displacements and clearances for relatively large amount of motions [2]. These two properties are coupled together by motion parameters, i.e., the displacements, clearances and rotational speeds, so that changes of the damping result in changes of the stiffness and vice-versa. For non-linear analysis of these bearing properties, however, the stiffness and damping characteristic are premised and assumed to be polynomial expressions to appropriate degrees for the closed form formulation, which is generally accepted and agreeable in past studies [3,5,7]. Hence in such a case, the trades off studies are preceded for the polynomial degree for closer access to the damping and stiffness non-linearities. The other non-linear properties of the stator (bearing) beyond these assumed polynomial model are rubbing, radial clearance, hysteric or coulomb damping, bilinear stiffness etc, in which case the polynomial assumption is heavy and rough, however, the identification of the others are naturally deduced by resulting phenomenon of the polynomial model. Based on these premises, in this study, let the identification of the polynomial non-linearity of the bearing be considered.

For the anisotropic rigid rotor, which is supposed to be simply supported at both ends and only one degree of freedom is used for the displacements in the y and z directions, with a non linearity (cubic) of dampings, c_{ny}, c_{nz} and stiffnesses, k_{ny}, k_{nz} of the bearing, (see Fig. 1, 2) which is ubiquitous in Duffing oscillator, the equation is described in the form,

$$m\ddot{y}+c_y\dot{y}+J_p\Omega\dot{z}+c_{yz}\dot{z}+c_{ny}\dot{y}^3+k_y y+k_{yz}z+k_{ny}y^3=g_y,$$
$$m\ddot{z}+c_z\dot{z}-J_p\Omega\dot{y}+c_{zy}\dot{y}+c_{nz}\dot{z}^3+k_z z+k_{zy}y+k_{nz}z^3=g_z,. \quad (1)$$

where m, c_y, c_{yz}, c_z, c_{zy}, c_{ny}, c_{nz}, k_y, k_{yz}, k_{zy}, k_{ny}, k_{nz}, J_p, Ω are rotor mass, bearing dampings and its nonlinear coefficients, bearing stiffnesses and its nonlinear coefficients, rotor polar moment of inertia normalized by squared shaft length and rotational speed, respectively.

Applying the complex notations such as $p=y+jz$, $\bar{p}=y-jz$, to these equations, the equation leads to the following complex form

$$\mathrm{M_f}\ddot{\mathrm{p}}+(\mathrm{C_f}-jJ_p\Omega)\dot{\mathrm{p}}+\mathrm{K_f}\mathrm{p}+\mathrm{C_b}\dot{\bar{\mathrm{p}}}+\mathrm{K_b}\bar{\mathrm{p}}+$$
$$\left[\mathrm{C_{nf}}(\dot{\mathrm{p}}^3+3\dot{\mathrm{p}}^2\dot{\bar{\mathrm{p}}})+\mathrm{C_{nb}}(\dot{\bar{\mathrm{p}}}^3+3\dot{\mathrm{p}}\dot{\bar{\mathrm{p}}}^2)\right]+\left[\mathrm{K_{nf}}(\mathrm{p}^3+3\mathrm{p}^2\bar{\mathrm{p}})+\mathrm{K_{nb}}(\bar{\mathrm{p}}^3+3\mathrm{p}\bar{\mathrm{p}}^2)\right]=\mathrm{g}, \quad (2)$$

where the bracket terms denote the non-linear effects and their parameters are

$$\mathrm{M_f}=m,\quad \mathrm{C_f}=(c_y+c_z)/2+j(c_{yz}-c_{zy})/2,\quad \mathrm{C_b}=(c_y-c_z)/2+j(c_{yz}+c_{zy})/2,$$
$$\mathrm{K_f}=(k_y+k_z)/2+j(k_{yz}-k_{zy})/2,\quad K_b=(k_y-k_z)/2+j(k_{yz}+k_{zy})/2,$$
$$\mathrm{C_{nf}}=(c_{ny}-c_{nz})/8,\quad \mathrm{C_{nb}}=(c_{ny}+c_{nz})/8,\quad \mathrm{K_{nf}}=(k_{ny}-k_{nz})/8,\quad \mathrm{K_{nb}}=(k_{ny}+k_{nz})/8 \quad (3)$$

In this case the damping and stiffness parameters assumed to be independent of the rotational speed for more or less higher speed range [1].

Non-Linear Response Analysis [1]

Here the output $\mathrm{P}(t)$ of a non-linear system in power of the input $\mathrm{g}(t)$ is expressed by the well-known Volterra series, which is depicted in the form of

$$\mathrm{P}(t)=\sum_{n=1}^{\infty}\int_{-\infty}^{\infty}du_1\cdots\int_{-\infty}^{\infty}du_n h_n(u_1,\cdots,u_n)\prod_{r=1}^{n}\mathrm{g}(t-u_r) \quad (4)$$

where $h_n(u_1,\ldots\ldots,u_n)$ are the nth order kernels of the system, in this case the 1st order kernel $h_n(u_1)$ is the impulse response of a linear system.

The n-fold Fourier transformation of the kernel is the nth order Volterra kernel that has analogies to the nth order transfer function, which is described as

$$\mathrm{H}_n(\omega_1,\cdots,\omega_n)=\frac{1}{n!}\int_{-\infty}^{\infty}du_1\cdots\int_{-\infty}^{\infty}du_n h_n(u_1,\cdots,u_n)\prod_{r=1}^{n}e^{-j\omega_r u_r} \quad (5)$$

where the 1st order Volterra kernel $H_n(\omega_1,\ldots\ldots,\omega_n)$ is the transfer function of a linear system.

Now to derive the general form of an expression of the Volterra series by the series of harmonic inputs such as $g(t)=\sum_{m=1}^{p}e^{j\omega_m t}$, the dummy coefficients α_s and the differential operator, which is expressed by the form as $D_\alpha^n \equiv \left.\frac{\partial^n}{\partial\alpha_1 \ldots \partial\alpha_n}\right|_{\alpha_1=\ldots=\alpha_n=0}$, is used. Applying this dummy variable and the sum of n exponential terms as the form of $A_n(\omega)=\sum_{s=1}^{n}\alpha_s e^{-j\omega u_s}$ to Eq. (5), the *nth* order Volterra kernel, or the *nth* order transfer function of the system becomes

$$H_n(\omega_1,\cdots,\omega_n)=\frac{1}{n!}\int_{-\infty}^{\infty}du_1\cdots\int_{-\infty}^{\infty}du_n h_n(u_1,\cdots,u_n)D_\alpha^n\prod_{r=1}^{n}A_n(\omega_r), \tag{6}$$

and from using the multinomial theorem [11] the terms of power by the harmonic inputs becomes

$$\begin{aligned}\prod_{r=1}^{n} g(t-u_r) &= D_\alpha^n \exp\left[\sum_{r=1}^{n}\alpha_r g(t-u_r)\right] = D_\alpha^n\frac{1}{n!}\left[\sum_{r=1}^{n}\alpha_r g(t-u_r)\right]^n \\ &= D_\alpha^n\frac{1}{n!}\left[\sum_{r=1}^{n}\sum_{m=1}\alpha_r g_m e^{j\omega_m(t-u_r)}\right]^n = D_\alpha^n\frac{1}{n!}\left[\sum_{m=1}^{p} g_m e^{j\omega_m t}A_n(\omega_m)\right]^n \\ &= D_\alpha^n\left\langle\sum_{n_1=1}^{n}\sum_{n_2=1}\cdots\sum_{n_p=1}\frac{1}{n_1!n_2!\cdots n_p!}\left\{\prod_{m=1}^{p}\left[g_m e^{j\omega_m t}A_n(\omega_m)\right]^{n_m}\right\}\right\rangle,\end{aligned} \tag{7}$$

where n_m means the mth polynomial of the nth factorial number, so that finally from Eq. (6) and (7) the general complex form of Volterra series of the output by the input of pth multipl harmonics is derived by the following equation

$$P(t)=\sum_{n=1}^{\infty}\left\langle\sum_{n_1=1}^{n}\sum_{n_2=1}\cdots\sum_{n_p=1}\frac{1}{n_1!n_2!\cdots n_p!}\left\{\prod_{m=1}^{p}\left[g_m e^{j\omega_m t}\right]^{n_m}H_{n,n_m}(\omega_m)\right\}\right\rangle \tag{8}$$

where $\sum_{i=1}^{p} n_j = n$, for all i ; $n_i \geq 0$ (i=1 ~p, i : integer) and here $H_{n,n_m}(\omega_m)$ denotes $H_n(\omega_1,\ldots,\omega_m,\ldots,\omega_p)$ with the *mth* harmonic input of the ω_i equal to $+\omega_m$ and remaining ω_{p-m} equal to $-\omega_m$, respectively.

Non-Linear *Directional* Frequency Response Analysis

Here for the case of the anisotropic rotor system by the input of sweeping single-tone excitation, $g(t)=Ge^{j\omega t}$, the solution form of the Volterra series (8) can be deduced from Eq. (1) based upon the concept of a linear solution associated with the forward (H) and backward modes ($\hat{H}$), which is depicted in the form of

$$\begin{aligned}p(t) &= P(t)+\hat{P}(t) \\ &= H_1(\omega)Ge^{j\omega t}+\hat{H}_1(\omega)\bar{G}e^{-j\omega t}+H_3(\omega,\omega,\omega)G^3e^{j3\omega t}+\hat{H}_3(\omega,\omega,\omega)\bar{G}^3e^{-j3\omega t}+ \\ &\quad H_5(\omega,\omega,\omega,\omega,\omega)G^5e^{j5\omega t}+\hat{H}_5(\omega,\omega,\omega,\omega,\omega)\bar{G}^5e^{-j5\omega t}+O(H_7,\hat{H}_7,H_9,\hat{H}_9,\cdots).\end{aligned} \tag{9}$$

where in this relation, the property of conjugate symmetry, i.e., $H_n(-\omega,\ldots,-\omega)=\widehat{H_n}(\omega,\ldots,\omega)$ is used and only the principal diagonals of the three and five dimensions are employed for simplifying the physical interpretation,, i.e., for five dimensional functions, $\omega_1 = \ldots = \omega_5 = \omega$. Substituting (9) into (2) with considering terms up to order three only and equating the coefficients of each exponential order and excitations, the successive formulations for the higher order (2nd and 3rd) transfer functions with the 1st order transfer function can be obtained. Here for notational conveniences lets simplify the terms that $H_1(\omega) = p_{f1}$, $\hat{H}_1(\omega) = p_{b1}$, $H_3(\omega, \omega, \omega) = p_{f3}$, $\hat{H}_3(\omega, \omega, \omega) = p_{b3}$, $H_5(\omega, \omega, \omega, \omega, \omega) = p_{f5}$ and $\hat{H}_5(\omega, \omega, \omega, \omega, \omega) = p_{b5}$. Then by using the harmonic probing method [10], for the terms of $Ge^{j\omega t}$, $\overline{G}e^{-j\omega t}$, $G^3e^{j3\omega t}$, $\overline{G}^3e^{-j3\omega t}$, $G^5e^{j5\omega t}$ and $\overline{G}^5e^{-j5\omega t}$, respectively, the linear equations for the 2nd and 3rd order transfer functions are successively described as

$$
\begin{aligned}
&\mathrm{D_f}(\omega)\, p_{f1} + \mathrm{D_b}(\omega)\, \overline{p}_{b1} = 1,\\
&\hat{\mathrm{D}}_\mathrm{b}(\omega)\, p_{f1} + \hat{\mathrm{D}}_\mathrm{f}(\omega)\, \overline{p}_{b1} = 0,\\
&\mathrm{D_f}(3\omega)\, p_{f3} + \mathrm{D_b}(3\omega)\, \overline{p}_{b3} = -(c_{nf} j\omega^3 + k_{nf})(p_{f1}{}^3 + 3p_{f1}{}^2 \overline{p}_{b1}) - \\
&\qquad (c_{nb} j\omega^3 + k_{nb})(\overline{p}_{b1}{}^3 + 3p_{f1}\overline{p}_{b1}{}^2),\\
&\hat{\mathrm{D}}_\mathrm{b}(3\omega)\, p_{f3} + \hat{\mathrm{D}}_\mathrm{f}(3\omega)\, \overline{p}_{b3} = -(\overline{c}_{nf} j\omega^3 + \overline{k}_{nf})(\overline{p}_{b1}{}^3 + 3p_{f1}\overline{p}_{b1}{}^2) - \\
&\qquad (\overline{c}_{nb} j\omega^3 + \overline{k}_{nb})(p_{f1}{}^3 + 3p_{f1}{}^2 \overline{p}_{b1}),\\
&\mathrm{D_f}(5\omega)\, p_{f5} + \mathrm{D_b}(5\omega)\, \overline{p}_{b5} = -3(3c_{nf} j\omega^3 + k_{nf})(p_{f1}{}^2 p_{f3} + p_{f1}{}^2 \overline{p}_{b3} + 2p_{f1}p_{f3}\overline{p}_{b1}) - \\
&\qquad 3(3c_{nb} j\omega^3 + k_{nb})(p_{f3}\overline{p}_{b1}{}^2 + \overline{p}_{b1}{}^2 \overline{p}_{b3} + 2p_{f1}\overline{p}_{b1}\overline{p}_{b3}),\\
&\hat{\mathrm{D}}_\mathrm{b}(5\omega)\, p_{f5} + \hat{\mathrm{D}}_\mathrm{f}(5\omega)\, \overline{p}_{b5} = -3(3\overline{c}_{nf} j\omega^3 + \overline{k}_{nf})(p_{f3}\overline{p}_{b1}{}^2 + \overline{p}_{b1}{}^2 \overline{p}_{b3} + 2p_{f1}\overline{p}_{b1}\overline{p}_{b3}) - \\
&\qquad 3(3\overline{c}_{nb} j\omega^3 + \overline{k}_{nb})(p_{f1}{}^2 p_{f3} + p_{f1}{}^2 \overline{p}_{b3} + 2p_{f1}p_{f3}\overline{p}_{b1}),
\end{aligned} \tag{10}
$$

where

$$
\begin{aligned}
&\mathrm{D_f}(\omega) = \mathrm{K_f} - \omega^2 \mathrm{M_f} + j\omega \mathrm{C_f} + J_p \Omega\omega, \quad \mathrm{D_b}(\omega) = \mathrm{K_b} + j\omega \mathrm{C_b}\\
&\hat{\mathrm{D}}_\mathrm{f}(\omega) = \overline{\mathrm{K}}_\mathrm{f} - \omega^2 \overline{\mathrm{M}}_\mathrm{f} + j\omega \overline{\mathrm{C}}_\mathrm{f} - J_p \Omega\omega, \quad \hat{\mathrm{D}}_\mathrm{b}(\omega) = \overline{\mathrm{K}}_\mathrm{b} + j\omega \overline{\mathrm{C}}_\mathrm{b}
\end{aligned} \tag{11}
$$

From Eq. (10), the 1st, 2nd and 3rd order transfer functions, respectively, are obtained as

$$
\begin{aligned}
&p_{f1} = D(\omega)\hat{\mathrm{D}}_\mathrm{f}(\omega),\\
&\overline{p}_{b1} = -D(\omega)\hat{\mathrm{D}}_\mathrm{b}(\omega),\\
&p_{f3} = D(3\omega)\left[f_3(p_{f1}, \overline{p}_{b1})\hat{\mathrm{D}}_\mathrm{f}(3\omega) - \overline{f}_3(p_{f1}, \overline{p}_{b1})\mathrm{D_b}(3\omega)\right],\\
&\overline{p}_{b3} = D(3\omega)\left[\overline{f}_3(p_{f1}, \overline{p}_{b1})\mathrm{D_f}(3\omega) - f_3(p_{f1}, \overline{p}_{b1})\hat{\mathrm{D}}_\mathrm{b}(3\omega)\right],\\
&p_{f5} = D(5\omega)\left[f_5(p_{f1}, \overline{p}_{b1}, p_{f3}, \overline{p}_{b3})\hat{\mathrm{D}}_\mathrm{f}(5\omega) - \overline{f}_5(p_{f1}, \overline{p}_{b1}, p_{f3}, \overline{p}_{b3})\mathrm{D_b}(5\omega)\right],\\
&\overline{p}_{b5} = D(5\omega)\left[\overline{f}_5(p_{f1}, \overline{p}_{b1}, p_{f3}, \overline{p}_{b3})\mathrm{D_f}(5\omega) - f_5(p_{f1}, \overline{p}_{b1}, p_{f3}, \overline{p}_{b3})\hat{\mathrm{D}}_\mathrm{b}(5\omega)\right],
\end{aligned} \tag{12}
$$

where

$D(\omega) = \left[\mathrm{D_f}(\omega)\hat{\mathrm{D}}_\mathrm{f}(\omega) - \mathrm{D_b}(\omega)\hat{\mathrm{D}}_\mathrm{b}(\omega)\right]^{-1}$, $f_3(p_{f1}, \overline{p}_{b1})$, $\overline{f}_3(p_{f1}, \overline{p}_{b1})$, $\overline{f}_5(p_{f1}, \overline{p}_{b1}, p_{f3}, \overline{p}_{b3})$ and $f_5(p_{f1}, \overline{p}_{b1}, p_{f3}, \overline{p}_{b3})$ are right-hand sides of the 3rd to 6th equation in Eq. (10), respectively. For the linear system, i.e., in the case of no medium bracket terms in Eq. (2), the response associated with the 1st order transfer function becomes

$$p(t) = p_{f1}\, G e^{j\omega t} + p_{b1}\, \bar{G} e^{-j\omega t}, \tag{13}$$

which becomes the same form as shown in reference [1].

From the linear complex modal analysis for rotor systems, p_{f1} denotes the linear *normal directional* FRF (dFRF) and p_{b1} denotes the *reverese* dFRF, whereas, in this non-linear analysis of the higher order frequency response function (HFRF), p_{f3} and p_{b3}(p_{f5} and p_{b5}) denote the *non-linear normal* and *reverese* dFRF of 3rd (5th) order, respectively. From these *non-linear normal* and *reverese* dFRFs, the property of the bearing non-linearity can be identified easily.

Numerical Example

In this simulation, the following numerical values have been used : m=4 kg, c_y=20 Ns/m, c_z=15 Ns/m, c_{ny} =3000 Ns/m, c_{nz}=2000 Ns/m, $c_{yz} = -c_{zy}$=20 Ns/m, $k_{yz} = k_{zy} = 0$, $k_y = 3\times10^5$ Ns/m, $k_z = 2\times10^5$ N/m, $k_{ny} = 2\times10^{10}$ N/m, $k_{nz} = 1.5\times10^{10}$ N/m, Ω=100

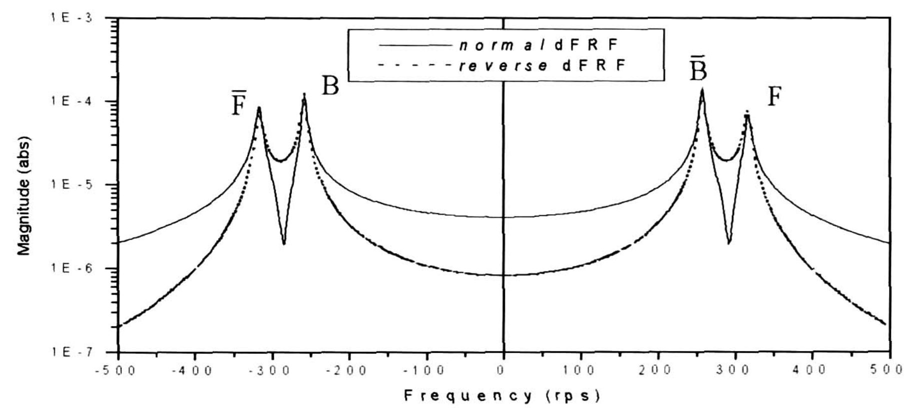

Fig. 3a. 1st Order (Linear) dFRF

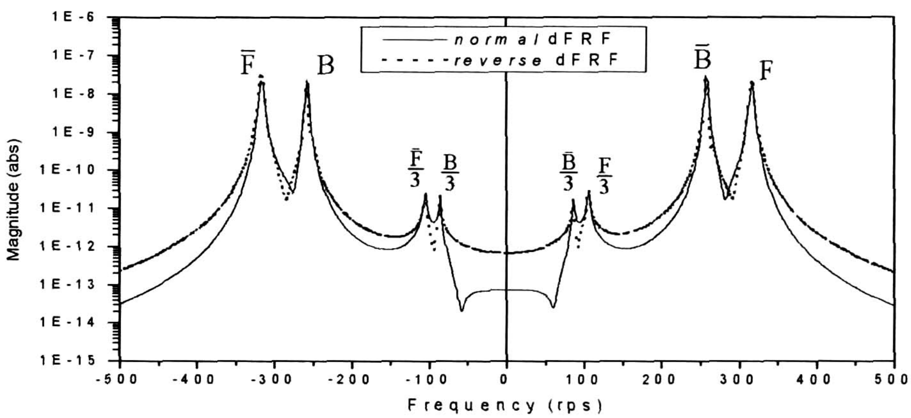

Fig. 3b. 3rd Order (Linear) dFRF

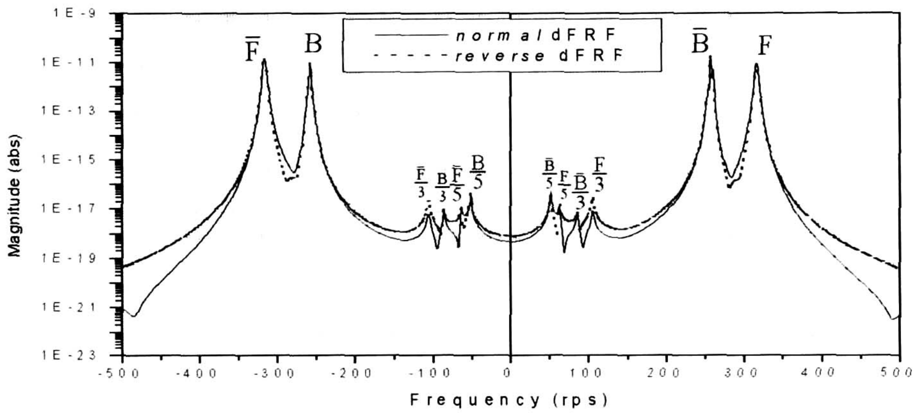

Fig. 3c. 5th Order (Linear) dFRF

Fig. 3. Nonlinear FRFs for Anisotropic Rotor with Bearing Nonlinerity

rps, $J_p = 0.0185\ kg/m^2/m^2$. Based upon these linear and non-linear parameters Fig. 3 shows the simulated frequency response functions for the 1st (linear), 3rd and 5th order, respectively. In Eq. (1), if there are no cross-coupled damping and stiffness coefficients, then the dFRFs are symmetric to negative domain in frequencies so that half of the dFRFs are sufficient, by which in this case the dFRFs shows the same profiles as the conventional FRFs [1].

From Fig. 3a, we see the typical result of linear dFRFs of the anisotropic rotor system associated with one pair of the forward (F) and backward conjugate (F) in the *normal* and *reverse* mode, whereas in Fig. 3b, for the 3rd order FRF shows two pairs of the resonant peaks, one at $\omega_n s$ and the other one at $\omega_n s/3$, which indicate the existence of a cubic non-linearty. For the 5th order FRFs, Fig. 3c shows three pairs of the resonant peaks, $\omega_n s$, $\omega_n s/3$ and $\omega_n s/5$, which indicate the existence of a linear, cubic and quintic non-linearties, respectively. The peculiar feature of the results shown in Fig. 3 is that the higher order dFRFs (3rd and 5th) do not show any distortion so that they behave like the linear FRFs. This is due to the fact that for simplifying the physical interpretation, only the principal diagonals are employed so that they represent the exact Volterra kernel transforms, which is unique and independent of the excitation level. If the extra terms of higher order transfer functions associated with additional sweeping multi-tone excitation levels are introduced, these distortional phenomena are explicitly displayed. Another important feature in higher order transfer function is that they cause energy transfer whereby an input at one frequency level influence the other frequency ones, further, the mode exchanges occur in *normal* and *reverse* ones as shown in Figs. 1 b,c. This is also clearly represented in Eq. (10) in that the *normal* dFRFs are disappeared in case of no anisotropy whereas in linear dFRFs the *reverse* ones are disappeared. From these results we see that the higher order FRFs are fundamentally different from what one can expect in linear ones and measure in practice.

Conclusions

The identification of bearing non-linearity in an anisotropic rotor system using the higher order dFRFs, which are known to represent the non-linear degree of anisotropy, is developed and shown to be theoretically feasible as in non-rotating structure or isotropic rotor. The non-linear stiffness and damping forces in rolling or journal bearings are modeled along with displacement in cubic polynomial form whose higher order transfer function can be possible from Volterra series. By using the principal diagonals of the dimensions of higher order transfer functions, the physical interpretation is simplified at one excitation level and the computational efforts are lessen so that the simple forms of the *normal* and *reverse* dFRFs can be derived. From these results, the higher order dFRFs shows additional sub-harmonic resonant peaks, which indicate the existence of higher order non-linearties. Also another feature of higher order dFRFs is that due to energy transfer the modes for the *normal* and *reverse* dFRFs are exchanged so that they suggest the fundamental differences from what one can expect in linear ones. Through this study it is suggested that the non-linear *normal* and *reverse* dFRFs display their properties effectively and show their feasibilities to further application.

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