Near-Optimal Collision Avoidance Maneuvers for UAV

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Abstract

Collision avoidance for multiple aircraft can be stated as a problem of maintaining safe distance between aircraft in conflict. Optimal collision avoidance problem seeks to minimize the given cost function while simultaneously satisfying constraints. The cost function could be a function of time or control input. This paper addresses the trajectory time-optimization problem for collision avoidance of unmanned aerial vehicles(UAVs). The problem is difficult to handle in general due to the two-point boundary value problem subject to dynamic environments. Some simplifying algorithms are used for potential applications in on-line operation. Although under possibility of more complicated problems, a dynamic problem is transformed into a static one by prediction of the conflict time and some appropriate assumptions.

Key Word : collision avoidance, UAV, near-time optimal, performance index, on-line application

Introduction

In recent aerospace technology areas, collision avoidance for multiple aircraft have emerged as an issue of significant interest. Collision avoidance for the aircraft can be stated as a problem of maintaining a safe distance between aircraft in conflict. Optimal collision avoidance problem seeks to minimize the given cost function while simultaneously satisfying the constraints. The cost function can be a function of time and/or control input depending on the mission requirements involving specific maneuvers.

A number of studies have been conducted already on UAVs and mobile robots avoiding obstacles arriving at a destination safely[1-4] in a given time period. Various strategies associated with path planning approach were proposed. The problem formulation may differ by the type of target obstacles; moving or non-moving obstacles. There is an on-line application and optimal solution in the case of non-moving obstacles [3]. However, it becomes a quite different problem for a moving obstacle. The problem is difficult to handle and it is not easy to derive an optimal solution in on-line operation.

Piorini and Shiller investigated optimal trajectory of a mobile robot over dynamic environment [4]. They assumed that the obstacle moves straight with a constant speed. Since there is no limit on the robot speed, it is difficult to apply this method directly to the UAV. On the other hand, there is a certain lower limit in the UAV speed. Hu, *et al.* investigated collision avoidance between aircraft [5]. They did not include the aircraft dynamics and treated the problem as finding a middle waypoint.

In this paper, we consider the aircraft dynamics and construct the performance index as

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a function of time. A solution to minimize the performance index while avoiding the obstacle simultaneously is derived. Some simplifying algorithms are employed in order to apply it to a on-line operation. Although there may be more complicated problems by prediction of conflict time and some assumptions, we transform the dynamic environment problem into a static one.

The analytic solution of the time optimal trajectory for the static obstacle is derived, and expected conflict time and distance for a moving obstacle is computed also. Then the algorithm for estimation of the desired heading angle for collision avoidance maneuver and potential on-line application technique are investigated. Finally simulation study is performed to demonstrate the proposed algorithm for various situations.

Analytic Solution of Time Optimal Collision Avoidance Problem for Non-moving Obstacle

First, the optimal trajectory in the static environment is briefly discussed. A dynamic obstacle with a certain lateral acceleration level is treated as a static model by calculating the expected conflict time and distance by assumptions. This simplification is very useful in handling our problem.

Let us assume that the aircraft is a point mass in the 2-dimensional space. Then the aircraft dynamics and kinematics can be simplified as

$$\begin{aligned} \dot{x} &= V\cos\Psi \\ \dot{y} &= V\sin\Psi \\ \dot{\Psi} &= u \end{aligned} \tag{1}$$

where Ψ is the heading angle of the aircraft, V is the velocity, and x, y represent 2-dimensional position of the aircraft, respectively. A performance index is defined as the elapsed maneuver time such that

$$J = \int_{t_0}^{t_f} 1\,dt \tag{2}$$

which is subject to

$$\begin{aligned} &S(x, y) = R_p^2 - (x - O_x)^2 - (y - O_y)^2 \le 0 \\ &|u| \le C \end{aligned}$$

where R_p represents a safe distance, O_x, O_y corresponds to the position of the obstacle in the x, y axes, respectively, u is the input to the aircraft, and C is the constant limit for the control input.

This problem is an inequality constraint problem, hence the solution can be divided into two cases. One solution may lie in the feasible region and while the other is at the boundary region.

Feasible region

For the case of $S(x, y) < 0$, we define the Hamiltonian as

$$H = 1 + \lambda_x V\cos\Psi + \lambda_y V\sin\Psi + \lambda_\Psi u \tag{3}$$

According to basic optimal control theory,

$$\dot{\lambda}^T = -\frac{\partial H}{\partial x}, \quad \frac{\partial H}{\partial u} = 0$$

it follows

$$\begin{aligned} &\dot{\lambda}_x = 0, \quad \dot{\lambda}_y = 0 \\ &\dot{\lambda}_\Psi = -\lambda_x V\sin\Psi + \lambda_y V\cos\Psi \\ &\lambda_\Psi = 0 \;\Rightarrow\; \dot{\lambda}_\Psi = 0 \end{aligned} \tag{4}$$

Consequently, the optimal heading angles is the solution of

$$\tan\Psi = \frac{\lambda_y}{\lambda_x} = constant \tag{5}$$

Thus, one can easily see that

$$\Psi = constant$$

which makes sense from physical point of view.

Boundary region

In this case, the solution lies in the boundary of the constraint plane such that $S(x, y) = 0$. Fig. 1 shows the trajectory of the aircraft on the boundary of the constraint surface. Since $S(x, y)$ has no input term, one can differentiate $S(x, y)$ with respect to t until an expression explicitly dependent on input u is derived.

$$\begin{aligned} \dot{S}(x, y) &= -2(x-O_x)V\cos\Psi - 2(y-O_y)V\sin\Psi = 0 \\ \ddot{S}(x, y) &= -2V^2 + 2(x-O_x)V\sin\Psi u - 2(y-O_y)V\cos\Psi u = 0 \end{aligned} \tag{6}$$

Hence a new constraint equation in the form of $\ddot{S}(x, t)$ has been derived. The corresponding Hamiltonian with the new constraint equation can be formulated as

$$H = 1 + \lambda_x V\cos\Psi + \lambda_y V\sin\Psi + \lambda_\Psi u + \mu(-2V^2 + 2(x-O_x)V\sin\Psi u - 2(y-O_y)V\cos\Psi u) \tag{7}$$

then, the solution indeed becomes

$$\tan\Psi = -\frac{x-O_x}{y-O_y}, \qquad \mu = \frac{1}{4V^2} \tag{8}$$

Again, we differentiate the above equation with respect to time arriving at

$$\dot{\Psi} = -\frac{\dot{x}(y-O_y) - \dot{y}(x-O_x)}{(y-O_y)^2}\cos^2\Psi \tag{9}$$

with

$$\sin^2\Psi + \cos^2\Psi = 1 \tag{10}$$

Next, let us combine Eqs.(8),(10) and dynamics Eq.(1), which produces the following final results

$$\cos^2\Psi = \frac{(y-O_y)^2}{R_p^2} \tag{11}$$

$$x - O_x = \pm R_p\sin\Psi, \qquad y - O_y = \mp R_p\cos\Psi \tag{12}$$

Then Eqs.(1) and (11) are substituted into (9) that results in

$$\dot{\Psi} = \pm\frac{V}{R_p} \tag{13}$$

From the above solution, one find that the aircraft flies straight outside the safety region, and at the boundary region the aircraft flies along the boundary as illustrated in Fig. 1.

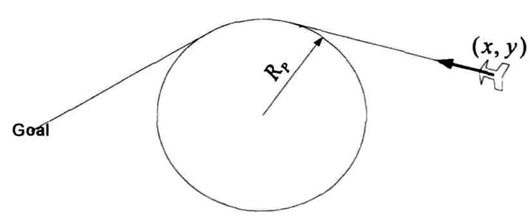

Fig. 1. Time optimal trajectory

Numerical Solution of Time Optimal Collision Avoidance Problem for Moving Obstacle

In the previous section, analytic solution of time optimal collision avoidance problem in the static environment was derived. We might use this result for solving the dynamic problem by transforming it into a static one. In this section, transformation of the dynamic problem into a static one is discussed, and a technique to solve the dynamic problem numerically is proposed.

Collision avoidance algorithm

In this part, we propose an algorithm to solve the time optimal collision avoidance problem in the dynamic environment numerically. The algorithm is summarized in Table 1.

Table 1. Summary of collision avoidance algorithm

```
Do while (Aircraft is not reach the goal)
  Calculate the expected conflict time and conflict distance.
  If (Conflict distance is greater than the safety distance)
                         u = u_goal
  else
    Calculate right and left maneuvering time
    If (Right maneuvering time is greater than left)
                         u = -min(u_max, |u_rgt|)
    else
                         u = min(u_max, |u_lft|)
    end
  end
end
```

Expected conflict time and conflict distance

The expected conflict time and conflict distance in two cases are computed herein. We can · transform the dynamic problem into static one by adopting this result. We regard the moving obstacle as non-moving obstacle, where its position is equivalent to the position at the expected conflict time.

Obstacle with straight movement at a constant speed

If an obstacle moves straight with a constant speed, the solution can be obtained easily by analytic approach. The positions of the obstacle and the aircraft are defined with respect to time as follows;

$$X_T=(x_T(t),\quad y_T(t)) \\ x_T(t)=x_{T0}+\dot{x}_T t,\quad y_T(t)=y_{T0}+\dot{y}_T t \tag{14}$$

and

$$X_I=(x_I(t),\quad y_I(t)) \\ x_I(t)=x_{I0}+\dot{x}_I t,\quad y_I(t)=y_{I0}+\dot{y}_I t \tag{15}$$

The range between the aircraft and obstacle is given by

$$R(t)=\|X_T(t)-X_I(t)\|=\sqrt{(x_I(t)-x_T(t))^2+(y_I(t)-y_T(t))^2} \tag{16}$$

The expected conflict time can be derived by minimizing the range function $R(t)$. Since $R(t)$ is a monotonic function, we define $R'(t)=R(t)^2$. The expected conflict time t_c is the time minimizing the range function $R(t)$, and it turns out to be of the form:

$$t_c=-\frac{(x_{I0}-x_{T0})(\dot{x}_I-\dot{x}_T)+(y_{I0}-y_{T0})(\dot{y}_I-\dot{y}_T)}{(\dot{x}_I-\dot{x}_T)^2+(\dot{y}_I-\dot{y}_T)^2} \tag{17}$$

Furthermore, the conflict distance is obtained by substituting Eq.(17) into Eq.(16).

Obstacle with lateral acceleration

If the obstacle moves at a certain lateral acceleration, the problem becomes quite different. First, the position of the obstacle is expressed as

$$\begin{aligned} x_T(t)&=x_{T0}+\frac{V_T^2}{a_T}\sin\Psi_{T0}-\frac{V_T^2}{a_T}\sin\Psi_T(t)\\ y_T(t)&=y_{T0}-\frac{V_T^2}{a_T}\cos\Psi_{T0}+\frac{V_T^2}{a_T}\cos\Psi_T(t)\\ \Psi_Y(t)&=\Psi_{T0}-\frac{V_T^2}{a_T}t \end{aligned} \tag{18}$$

where a_T represents the lateral acceleration of the obstacle, and V_T is the velocity of the obstacle.

The above solution possesses nonlinear terms. So when minimizes the relative distance $R(t)=\sqrt{(x_I(t)-x_T(t))^2+(y_I(t)-y_T(t))^2}$ directly to assess the expected conflict time, there may exist many local minima and it is not trivial at all to find the unique true solution in general.

Even if the lateral acceleration is constant, the result still remains the same. So we propose a new different approach to estimate the expected conflict time and distance. There are two situations between the obstacle and the aircraft trajectory. Figures 2 and 3 illustrate the two situations. One is when the trajectory of the aircraft crosses the trajectory of the obstacle, and the other is when the trajectory of the aircraft does not cross.

Case 1: The trajectory of the aircraft crosses the trajectory of the obstacle

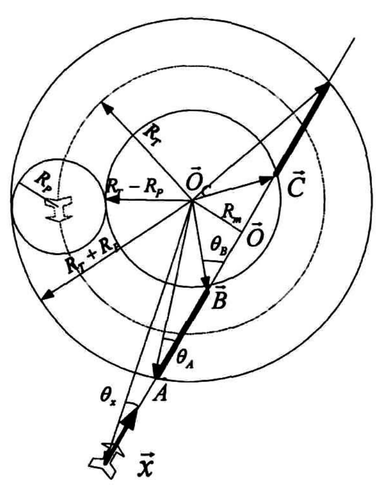

Fig. 2. Aircraft penetrates in the turn radius of obstacle

Case 2: The trajectory of the aircraft doesnt cross the trajectory of the obstacle

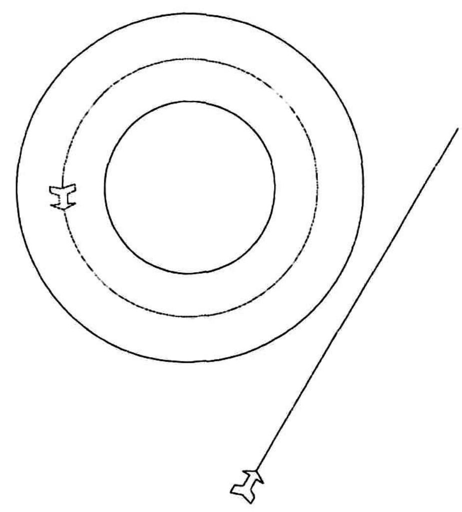

Fig. 3. Aircraft flies outside the turn radius of the obstacle

In fact, the second case could be ignored because collision situation does not occur. So only the first case is taken into account. In the first case, the time region for search is restricted as bold lines ($\vec{A}-\vec{B}$, $\vec{C}-\vec{D}$). This prevents us from calculating an incorrect solution.

Let us denote t_A, t_B, t_C, t_D as the time at $\vec{A}$, $\vec{B}$, $\vec{C}$, $\vec{D}$ in Fig.2. The center of the turn circle of the obstacle $\vec{O_c}$ is defined to be

$$\vec{O_c}=[o_{cx}, o_{cy}] \tag{19}$$

where

$$\begin{aligned} o_{cx} &= x_{T0}+R_T\sin(\Psi_{T0}) \\ o_{cy} &= y_{T0}-R_T\cos(\Psi_{T0}) \\ R_T &= \frac{V_T^2}{|a_T|} \end{aligned}$$

and R_T is the turn radius of the obstacle while Ψ_{T0} represents the initial heading angle of the obstacle. And the trajectory function of the aircraft is

$$\vec{x}(t)=[x_0+v_xt, \quad y_0+v_yt] \tag{20}$$

where v_x, v_y represent the velocity of the aircraft in the x, y axes, respectively. Moreover, from the center of the turn circle of the obstacle $\vec{O_c}$ from Eq.(19), one can obtain time t_m which minimizes the range between the aircraft and the center of the turn circle $\vec{O_c}$:

$$t_m=\frac{(o_{Cx}-x_0)v_x+(o_{Cy}-y_0)v_y}{v_x^2+v_y^2} \tag{21}$$

So the minimum range R_m between the aircraft and the center of the turn circle of the obstacle becomes

$$R_m=\sqrt{(x_0+v_xt_m-o_{Cx})^2+(y_0+v_yt_m-o_{Cy})^2} \tag{22}$$

From Fig.2, t_A represents the time to $\vec{A}$, and t_B the time to $\vec{B}$ expressed in the form

$$\begin{aligned} t_A &= \frac{R_{Ax}}{V_I}=\frac{(R_{ox}-R_{oA})}{V_I} \\ t_B &= \frac{R_{Bx}}{V_I}=\frac{(R_{ox}-R_{oB})}{V_I} \end{aligned} \tag{23}$$

where R_{oA}, R_{oB}, R_{ox} are given by

$$\begin{aligned} R_{oA} &= |\vec{O}-\vec{A}|=\sqrt{(R_T+R_p)^2-R_m^2} \\ R_{oB} &= |\vec{O}-\vec{B}|=\sqrt{(R_T-R_p)^2-R_m^2} \\ R_{ox} &= |\vec{O}-\vec{x}|=\sqrt{|\vec{O_c}-\vec{x}|^2-R_m^2} \end{aligned} \tag{24}$$

In and analogous manner, one can obtain t_C, t_D to $\vec{C}$, $\vec{D}$, respectively. However when reducing the search region, it is impossible to obtain a closed-form solution because the equations still consist of nonlinear terms.

Obstacle velocity linearization

In this section, The obstacle velocity linearization method is introduced for on-line application. It can be made possible by assuming that the turn angle of the obstacle in the search region is small.

The path function of the obstacle is already defined in Eq.(18). By rewriting this equation from $\vec{A}$ to $\vec{B}$, one arrives at

$$\begin{aligned} x_T(t) &= x_{T0} + T_T \sin \Psi_{T0} - R_T \sin[\Psi_0 - a_T/V_T t_A - a_T/V_T(t-t_A)] \\ &= x_{T0} + R_T \sin \Psi_{T0} - R_T[\sin(\Psi_T(t_A))\cos(a_T/V_T(t-t_A)) \\ &\quad - \cos(\Psi_T(t_A))\sin(a_T/V_T(t-t_A))] \\ y_T(t) &= y_{T0} + R_T \cos \Psi_{T0} + R_T[\cos(\Psi_T(t_A))\cos(a_T/V_T(t-t_A)) \\ &\quad + \sin(\Psi_T(t_A))\sin(a_T/V_T(t-t_A))] \\ \Psi_T(t_A) &= \Psi_{T0} - a_T/V_T t_A \end{aligned} \tag{25}$$

If $a_T/V_T(t-t_A)$ is very small, Eq.(25) can be rewritten in the form

$$\begin{aligned} x_T(t) &= \alpha + \beta \cdot t \\ y_T(t) &= \gamma + \delta \cdot t \end{aligned} \tag{26}$$

where each parameter is defined to be

$$\begin{aligned} \alpha &= x_{T0} + R_T \sin \Psi_{T0} - R_T \sin(\Psi_{T0} - a_T/V_T t_A) - V_T t_A \cos(\Psi_{T0} - a_T/V_T t_A) \\ \beta &= V_T \cos(\Psi_{T0} - a_T/V_T t_A) \\ \gamma &= y_{T0} + R_T \cos \Psi_{T0} + R_T \cos(\Psi_{T0} - a_T/V_T t_A) - V_T t_A \sin(\Psi_{T0} - a_T/V_T t_A) \\ \delta &= V_T \sin(\Psi_{T0} - a_T/V_T t_A) \end{aligned}$$

Substituting this equation into Eq. (17), the expected conflict time and conflict range could be calculated as follows;

$$t_{c_{AB}} = -\frac{(x_{I0} - \alpha)(\dot{x}_I - \beta) + (y_{I0} - \gamma)(\dot{y}_I - \delta)}{(\dot{x}_I - \beta)^2 + (\dot{y}_I - \delta)^2} \tag{27}$$

From $\vec{C}$ to $\vec{D}$, the expected conflict time and conflict range are computed in an analogous approach.

Maneuvering time calculation

In this section we propose a numerical algorithm to calculate the maneuvering time from present position to the goal position. The length of the trajectory to the goal should be computed in order to evaluate the maneuvering time. The desired heading angle to obtain the collision avoidance direction maneuver and input lateral acceleration are obtained by comparing left and right maneuvering times.

When there exists an obstacle such as Fig.4, first two angle variables are introduced such that

$$\begin{aligned} \theta_c &= \sin^{-1}\left(\frac{c_{cy} - y}{\sqrt{(x-o_{cx})^2 + (y-o_{cy})^2}}\right) \\ \theta &= \sin^{-1}\left(\frac{R_p}{\sqrt{(x-o_{cx})^2 + (y-o_{cy})^2}}\right) \end{aligned} \tag{28}$$

Using Eq.(28) the desired heading angle and lateral acceleration input from the present state are designed to be

$$\begin{aligned} \theta_L &= \theta_c + \theta - \Psi \\ \theta_R &= \theta_c - \theta - \Psi \\ u_{lft} &= \theta_L / \triangle t \\ u_{rgt} &= \theta_R / \triangle t \end{aligned} \tag{29}$$

where $\triangle t$ is the time interval for simulation. Furthermore, the angles related to the goal position are also obtained identically.

$$\begin{aligned} \theta_{gc} &= \sin^{-1}\left(\frac{y_f - c_{cy}}{\sqrt{(x_f-o_{cx})^2 + (y_f-o_{cy})^2}}\right) \\ \theta_{gL} = \theta_{gR} &= \sin^{-1}\left(\frac{R_p}{\sqrt{(x_f-o_{cx})^2 + (y_f-o_{cy})^2}}\right) \end{aligned} \tag{30}$$

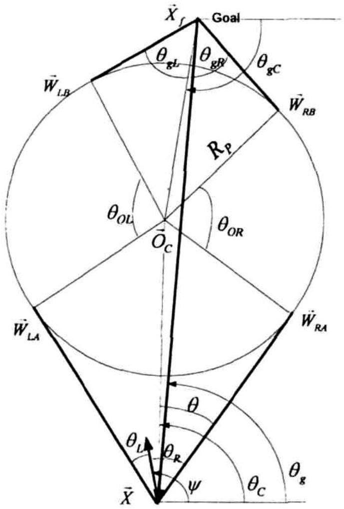

Fig. 4. Calculating desired heading angle

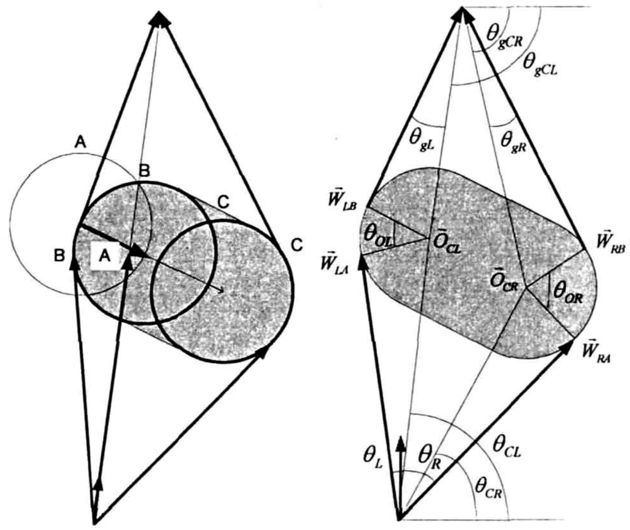

Fig. 5. Calculation of the heading angle about dynamic obstacle

From Eq.(30), the following relationships hold true

$$\begin{aligned}\theta_{OR} &= \pi - \theta_c + \theta - \theta_{gC} + \theta_{gR} \\ \theta_{OL} &= \theta_c + \theta + \theta_{gC} + \theta_{gL} - \pi\end{aligned} \tag{31}$$

Based upon the above equations, one can evaluate left and right maneuvering time from the present position to the target position as follows;

$$\begin{aligned}t_{R-man} &= (|\overrightarrow{W_{RA}} - \vec{X}| + R_p\theta_{OR} + |\vec{X_f} - \overrightarrow{W_{RB}}|)/V_I \\ t_{L-man} &= (|\overrightarrow{W_{LA}} - \vec{X}| + R_p\theta_{OL} + |\vec{X_f} - \overrightarrow{W_{LB}}|)/V_I\end{aligned} \tag{32}$$

From Eq.(32), the maneuvering direction for which the maneuver time is smaller can be decided. For a static obstacle, the direction of the obstacle avoidance maneuver and desired heading angle are determined by the above algorithm, but for a dynamic obstacle it may not be quite sufficient. Since the obstacle moves toward the aircraft performing avoidance maneuver, the obstacle region changes during the maneuver. Fig. 5 shows this situation and possible solutions to overcome such a situation.

In Fig. 5, the collision position of the obstacle is denoted as A if the aircraft does not perform avoidance maneuver, but during the left obstacle avoidance maneuver the obstacle moves to B, and during right maneuver the obstacle moves to C. So for the dynamic obstacle, the region B to C (the grey region) is regard as a type of obstacle. In the above algorithm the desired heading angle of the dynamic obstacle is expressed as

$$\begin{aligned}\theta_L &= \theta_{cL} + \theta - \Psi \\ \theta_R &= \theta_{cR} - \theta - \Psi \\ u_{lft} &= \theta_L / \triangle t \\ u_{rgt} &= \theta_R / \triangle t\end{aligned} \tag{33}$$

and the maneuvering time is

$$\begin{aligned}t_{R-man} &= (|\overrightarrow{W_{RA}} - \vec{X}| + R_p\theta_{OR} + |\vec{X_f} - \overrightarrow{W_{RB}}|)/V_I \\ t_{L-man} &= (|\overrightarrow{W_{LA}} - \vec{X}| + R_p\theta_{OL} + |\vec{X_f} - \overrightarrow{W_{LB}}|)/V_I\end{aligned} \tag{34}$$

where the angle variables satisfy

$$\theta_{OR} = \pi - \theta_{cR} + \theta - \theta_{gCR} + \theta_{gR}$$
$$\theta_{OL} = \theta_{cL} + \theta + \theta_{gCL} + \theta_{gL} - \pi \tag{35}$$

Simulation Result

It is assume that the aircraft and the obstacle are two-dimensional point mass objects. Again, the equations of motion are simply stated in the form:

$$\dot{x} = V\cos\Psi$$
$$\dot{y} = V\sin\Psi \tag{36}$$
$$\dot{\Psi} = u$$

Safety range is set to be 6,000ft and the initial position of the main aircraft is (0,0) while the final goal location is assumed as (0, 60760ft). The speed of aircraft and obstacle is selected to be 337 ft/sec.

Simulation has been performed in four cases. In the first case, the obstacle is not moving, while in the second case it moves straight at a constant forward speed. The third case is the obstacle moving with a constant lateral acceleration and constant forward speed. For the last case, the obstacle moves with changing lateral acceleration and constant forward speed.

Collision avoidance maneuver for non-moving obstacle

Two cases of simulation for a static obstacle are examined. The first case is a single obstacle and the second case is multi-obstacles case. Figures 6 and 7 shows the simulation results.

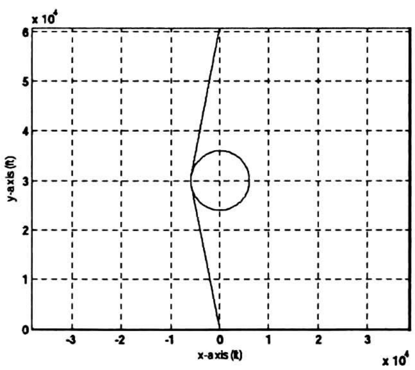

Fig. 6. One obstacle case

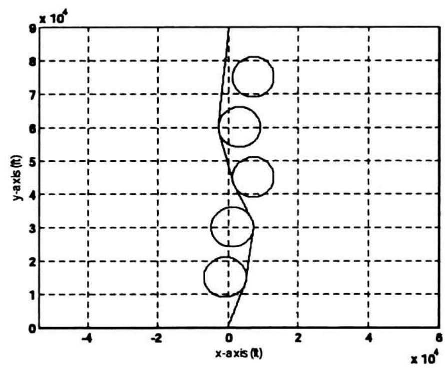

Fig. 7. Multi-obstacle case

It can be shown that the aircraft maneuvers as expected, and collision avoidance is achieved with satisfactory performance . Inequality constraints to avoid collisions are satisfied for every obstacle over the multiple obstacles.

Collision avoidance maneuver for moving obstacle

For the dynamic obstacle case, the obstacle is assumed to be a two-dimensional point mass, and equations of motion are already presented in Eq. (36).

Straight and constant speed obstacle

Simulation result against an obstacle at a constant speed is presented in Fig. 8 in the 2-dimensional plane.

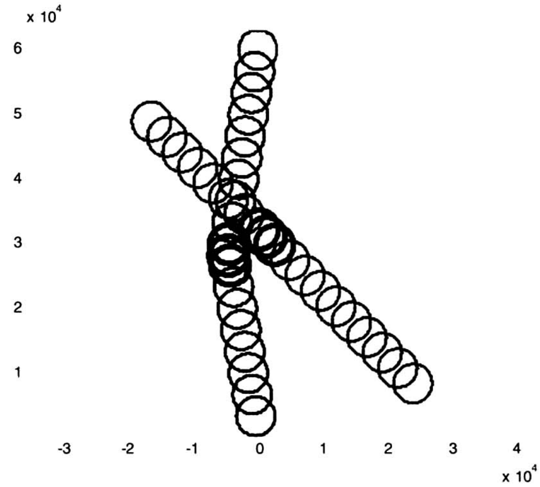

Fig. 8. Collision avoidance for straight and constant speed obstacle

The simulation results again illustrate the collision avoidance strategy works as designed. Dynamic motions of the obstacle are handled well by the designed maneuver strategy.

Moving obstacle at a constant lateral acceleration

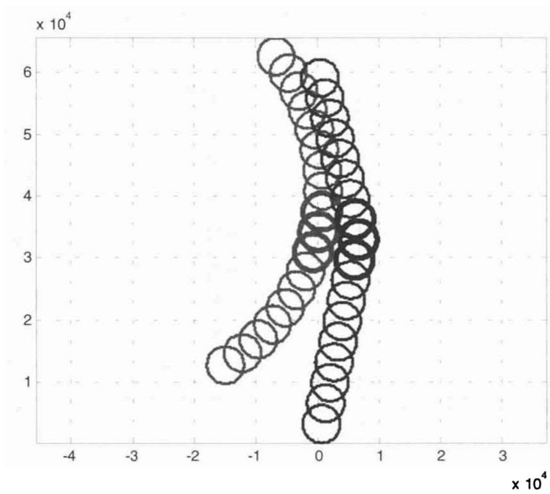

Fig. 9. Collision avoidance for obstacle with constant acceleration

Moving obstacle with a random lateral acceleration

In this simulation, the obstacle moves with changing lateral acceleration and constant speed. For the simulation, the lateral acceleration of the obstacle is settled into a cosine wave as presented in Fig. 10. The corresponding trajectory of the aircraft for avoidance is displayed in Fig. 11.

As it can be shown the collision avoidance is achieved against an obstacle with time-varying acceleration. The constraint conditions are all satisfied during the maneuver. The simulation results presented so far illustrate the practical merit of the proposed approach for collision management against various conditions of target UAVs. The formulation itself was restricted to 2-dimensional plane, but there is ample opportunity for extension into 3-dimensional space. Other performance indices will certainly provided different levels of solutions to satisfy additional mission objectives.

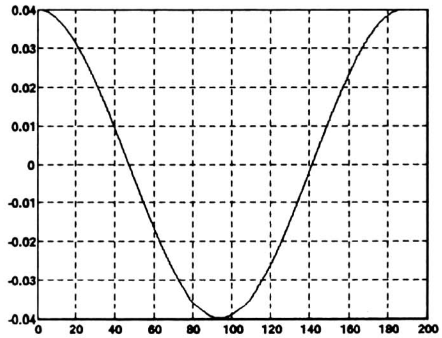

Fig. 10. Applied acceleration of obstacle

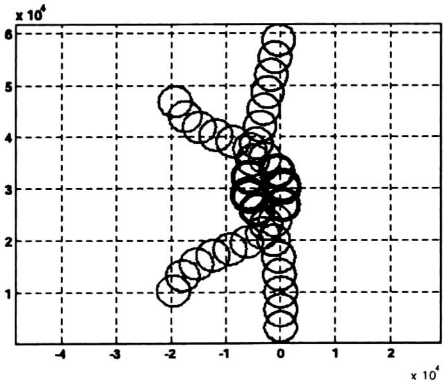

Fig. 11. Collision avoidance for obstacle with cosine wave acceleration

Conclusions

A near-minimum-time optimal collision avoidance for UAV was studied. Optimal solutions minimizing the performance index and avoiding obstacle simultaneously were derived. Analytic solution of the time optimal trajectory for a static obstacle was derived, and the expected conflict time and distance for a moving obstacle were evaluated. The moving obstacle was treated as a static object, and the numerical solution for the collision avoidance for moving obstacle was also sought. Simulations for the static and dynamic environments were conducted. Simulation results illustrate the usefulness of the proposed technique for collision avoidance of UAVs. Despite some significant results derived in this study, there remains further work to be done. First, the chattering problem in the aircraft input should be resolved. And the case of multi-moving-obstacles should be investigated for extension into three-dimensional cases.

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References

1. Kumar, B. A., and Ghose, D., 2001, "Radar-assisted collision avoidance/guidance strategy for planar flight", *IEEE Transactions on Aerospace And Electronic System,* Vol.37, No.1, pp.77-90.

2. Sundar, S. and Shiller, Z., 1997, "Optimal obstacle avoidance based on the Hamilton-Jacobi-Bellman equation", *IEEE Transaction on Robotics and Automation,* Vol.13, No.2, pp.305-310.

3. Twigg, S., Calise, A., and Johnson, E., 2003, "On-line trajectory optimization for autonomous air vehicles", *AIAA Guidance, Navigation and Control Conference,* number AIAA-2003-5522, Austin, TX.

4. Fiorini, P., and Shiller, Z., 1995, "Robot motion planning in dynamic environments", *International Symposium of Robotic Research,* Munich, Germany, pp.237-248.

5. Hu, J., Prandini, M., and Sastry, S., 2002, "Optimal coordinated maneuvers for three-dimensional aircraft conflict resolution", *Journal of Guidance, and Dynamics,* Vol.25, No.5, pp.888-900.