

Efficient Time Domain Aeroelastic Analysis Using System Identification

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Abstract

The CFD coupled aeroelastic analyses have significant advantages over linear panel methods in their accuracy and usefulness for the simulation of actual aeroelastic motion after specific initial disturbance. However, in spite of their advantages, a heavy computation time is required. In this paper, a method is discussed to save a computational cost in the time domain aeroelastic analysis based on the system identification technique. The coefficients of system identification model are fit to the computed time response obtained from a previously developed aeroelastic analysis code. Because the non-dimensionalized data is only used to construct the model structure, the resulting model of the unsteady CFD solution is independent of dynamic pressure and this independency makes it possible to find the flutter dynamic pressure without the unsteady aerodynamic computation. To confirm the accuracy of the system identification methodology, the system model responses are compared with those of the CFD coupled aeroelastic analysis at the same dynamic pressure.

Key Word : Flutter, Aeroelasticity, System identification, Reduced-order modeling

Introduction

Flutter is one of the most dangerous phenomena in the flight condition because it can result in total structural failure in a matter of seconds. Hence, an accurate prediction of the aeroelastic instability is important in the design of modern aircrafts. The CFD-based aeroelastic simulations are recognized as the most accurate schemes because they have the significant advantages in their accuracy of nonlinear flow calculation especially in transonic regime. However, in spite of their accuracy, it is very time consuming work to apply this approach to an actual aircraft model. Moreover, because the aeroelastic instabilities can be predicted by simulating whether the response is decaying or diverging, the aeroelastic analyses should be performed several times until we find the neutral response. Therefore, to overcome these difficulties, noticeable researches are

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carried out for the development of reduced order models [1-5]. The reduced-order model (ROM) is common name for the efficient analysis technique to find the unsteady aerodynamic forces without the direct aerodynamic computation such as CFD.

Dowell [1] proposed an efficient unsteady aerodynamic calculation method based on the eigenmode analysis of the unsteady aerodynamics. If the eigenvalues and eigenmodes of unsteady aerodynamic flow are determined, they can be used to confirm the basic physical behavior of the flowfield. Moreover, if a small number of eigenmodes are dominant, the aeroelastic analysis can be performed with the minimum number of degree of freedom. Dowell used the Lanczos algorithm to find the eigenmodes of the Euler equation. However, even though the powerful computational resources are available, it is extremely difficult to find the eigenvalues on the 3-dimensional complex geometry. Therefore Kim et al. [2] have attempted to apply a new eigenmode approach based on the Karhunen-Loeve decomposition [3]. In this research, to avoid the direct calculation of system eigenmodes, the snapshot method that extracts the basis modes from the response of the system has been used. Tang et al. [4] introduced the proper orthogonal decomposition (POD) method to investigate the unsteady flow about an oscillating delta wing. The POD modes are determined using a time history data of the flowfield which is computed by the vortex lattice aerodynamic model. A reduced-order modeling technique based on linear and nonlinear aerodynamic impulse responses has been investigated by Silva [5]. In his research, the Volterra theory has been applied to represent the unsteady aerodynamic system. The paper also showed the results of computational efficiency test when the Navier-Stokes code had been substituted by the Volterra theory. Especially, Cowan et al. [6,7] proposed direct identification of discrete time-domain models and an Auto Regressive Moving Average (ARMA) model was identified using the 3-2-1-1 multi-step input signal and corresponding unsteady CFD response.

In the reduced-order modeling, the several approaches have been tried to substitute the unsteady aerodynamic computation and most of them showed good to save computation time. However, the more important factor is how much the scheme reduces the whole analysis time not computing time. It is also important whether the scheme is generally applicable to the actual 3-dimensional model. In this study, an efficient aeroelastic analysis based on the system modeling technique has been investigated. To perform the system modeling, we recorded input signals and corresponding output signals of the system. These inputs should be chosen to have as much as possible information of dynamic characteristics of the system. We also evaluated the accuracy of the model by comparing with the actual responses from CFD-based aeroelastic analysis at each flight conditions.

Basic Concept and Methodology

In this study, the aerodynamic system is assumed as time-invariant system. This doesn't mean the system is independent of time. Time-invariant means that the fundamental properties do not change with time. The oscillating pendulum is a good example. The full nonlinear equation of a pendulum is certainly a function of time that can exhibit nonlinear and unsteady responses. However, neither the length nor the mass at the end of the pendulum are functions of time [5]. The governing equations of aerodynamics do not have any coefficients that are an explicit function of time. As a result, the Navier-Stokes, Euler, and full potential codes are time invariant.

The aeroelastic equation of motion can be formulated by the Hamilton theorem for elastic models and can be written in matrix form as follows:

$$[M_g]\{\ddot{q}(t)\} + [C_g]\{\dot{q}(t)\} + [K_g]\{q(t)\} = \{Q(t)\} \quad (1)$$

where $\{q(t)\} = [q(t)_1, q(t)_2, \dots, q(t)_n]$ is the generalized displacement vector and $[M_g]$, $[C_g]$, and $[K_g]$ express the generalized mass, damping, and stiffness matrices, respectively. $\{Q(t)\}$ represents the generalized aerodynamic force. To solve equation (1), the state vector form is introduced for efficient numerical calculations and can be written as follows:

$$\{\dot{x}(t)\} = [A]\{x(t)\} + [B]\{u(t)\} \quad (2)$$

where

$$[A] = \begin{bmatrix} [0] & [I] \\ -[M_g]^{-1}[K_g] & -[M_g]^{-1}[C_g] \end{bmatrix}, \quad [B] = \begin{bmatrix} [0] \\ [M_g]^{-1} \end{bmatrix}$$

$$\{x(t)\} = \begin{Bmatrix} \{q(t)\} \\ \{\dot{q}(t)\} \end{Bmatrix}, \quad \{u(t)\} = \begin{Bmatrix} \{0\} \\ \{Q(t)\} \end{Bmatrix}$$

In this study, Runge-Kutta method is used to solve the equation of motion. The details of the coupled time integrated method can be found in Ref. 8.

In the CFD coupled aeroelastic system, $\{Q(t)\}$ is computed in every time step by the CFD solver and the most of execution time is exhausted to find the unsteady aerodynamic solution, $\{Q(t)\}$. Hence, how to reduce the computing time to get the aerodynamic forces is the focus of this work. To save the computing time, we substitute the unsteady aerodynamic routine by the system identification model and then, the aerodynamic forces are determined using the simple matrix algebra.

In many time-series modeling problems, linear regression approach can be used for identifying the model parameters. One of the most commonly used model structures is the ARMAX model (autoregressive moving average with exogenous input). The ARMAX models can be classified into four cases according to the system characteristics, ARMA, AR, ARX and MA. Although many different model structures can be used in system identification, the ARX is the most suitable model among them for this study, because the system inputs are prescribed and the system responses from the predetermined inputs can be recorded. It is one of a few models that can be expanded to multi-input/multi-output (MIMO) systems. The ARX model for simple linear difference equation can be written as follows:

$$y(t) = -a_1 y(t-1) - \dots - a_{na} y(t-na) + b_0 u(t) + b_1 u(t-1) + \dots + b_{nb} u(t-nb-nk+1) \quad (3)$$

In Eq. (3), the current output $y(t)$ relates to a finite number of past outputs $y(t-k)$ and inputs $u(t-k)$. The structure is entirely defined by the three integers na , nb , and nk . na is equal to the number of poles and $nb-1$ is the number of zeros, while nk is the pure time delay in the system. If there is no time delay between the input and output signals, typically nk is equal to 1.

For MIMO systems without delay, the ARX model with nu inputs and ny outputs can be written as follows:

$$\mathbf{A}(q)\mathbf{y}(t) = \mathbf{B}(q)\mathbf{u}(t) + error \quad (4)$$

Here, $\mathbf{A}(q)$ and $\mathbf{B}(q)$ are system parameter matrices and can be represented as follows:

$$\mathbf{A}(q) = \mathbf{I} + \mathbf{A}_1 q^{-1} + \dots + \mathbf{A}_{na} q^{-na}, \quad \mathbf{B}(q) = \mathbf{B}_0 + \mathbf{B}_1 q^{-1} + \dots + \mathbf{B}_{nb} q^{-nb} \quad (5)$$

or

$$\mathbf{A}(q) = \begin{bmatrix} a_{11}(q) & a_{12}(q) & \dots & a_{1ny}(q) \\ a_{21}(q) & a_{22}(q) & \dots & a_{2ny}(q) \\ \vdots & \vdots & \ddots & \vdots \\ a_{ny1}(q) & a_{ny2}(q) & \dots & a_{nyny}(q) \end{bmatrix}, \quad \mathbf{B}(q) = \begin{bmatrix} b_{11}(q) & b_{12}(q) & \dots & b_{1nu}(q) \\ b_{21}(q) & b_{22}(q) & \dots & b_{2nu}(q) \\ \vdots & \vdots & \ddots & \vdots \\ b_{ny1}(q) & b_{ny2}(q) & \dots & b_{nynu}(q) \end{bmatrix}$$

where, q^{-1} means the delay operator and all the element of $\mathbf{A}(q)$ and $\mathbf{B}(q)$ is polynomials of q^{-1} . The system parameter matrix \mathbf{A} and \mathbf{B} are 3-D arrays of $ny \times ny \times (na_{\max} + 1)$ and $ny \times nu \times (nb_{\max} + 1)$ respectively. The model orders, na and nb are given by the matrix form of dimension $ny \times ny$ and $ny \times nu$, respectively. The least-square method is applied to determine the system parameter matrices, $\mathbf{A}(q)$ and $\mathbf{B}(q)$.

$$\min_{A, B} \sum |y - y_m|^2 \quad (6)$$

where y_m is simulated output and y is measured output. The least-square estimates can be obtained by straightforward matrix algebra and their solutions to the linear regression problems have excellent properties in cases where the disturbances at different times are uncorrelated. However, when noise with nonzero mean or correlated disturbances exist, it makes the system errors increase in the parameter estimation[9].

Figure 1 shows the aeroelastic analysis procedure based on the system identification methodology. This procedure can be divided into three parts. First of all, various different input signals should be tested in order to find the best inputs for a given problem. Then, The unsteady aerodynamic responses are obtained using these inputs. To get better inputs to excite the system, the selected inputs must have the bandwidth wide enough to cover the sufficient modal frequencies and the appropriate relative amplitude of each signal contents. However, it is an unessential effort to use the trial and error method to find the suitable training data. An aeroelastic modal response can be used as a way to avoid this anxiety. Because the aeroelastic responses are products of the system dynamics, it is not necessary to worry about whether or not the training data contain the system characteristics sufficiently.

Secondly, model training is performed using the input and output data sets which are established in the previous procedure. In the parametric methods, such as ARMAX, a specific model structure must be assumed before model training. It means that the model order na and nb in Eq. (5) must be determined previously. However, there is no clear guideline to find the optimum model order because it strongly depends on which training data has been used. We only

have several recommendations; na should be less than nb . If na is larger than nb , the model training can fail or system can be unstable. To determine the appropriate model order, we start from the low order (usually 2nd or 3rd order) and increase the order gradually, because the accuracy can be aggravated when the model order is not only too low, but also too high. Once the optimum model has been completed, it is ready to be implemented in an unsteady solution.

Finally, the system model, such as ARX, substitutes the unsteady aerodynamic routine of the aeroelastic analysis program. This resulting model is independent of dynamic pressure because the model training only uses the non-dimensionalized data. Once the system model has been implemented in the aeroelastic analysis program, we can find the flutter boundary very quickly. We can also verify the flutter boundary by recalculation using the existing time marching code and check the accuracy. Usually, to find the flutter boundary using the coupled time integration method, unsteady aerodynamics should be calculated several times for different dynamic pressure at each Mach number. However, if we use the present system model, the flutter boundary can be predicted using just two step computation of unsteady aerodynamics; One is to get the training data, the other is to verify the estimated flutter boundary.

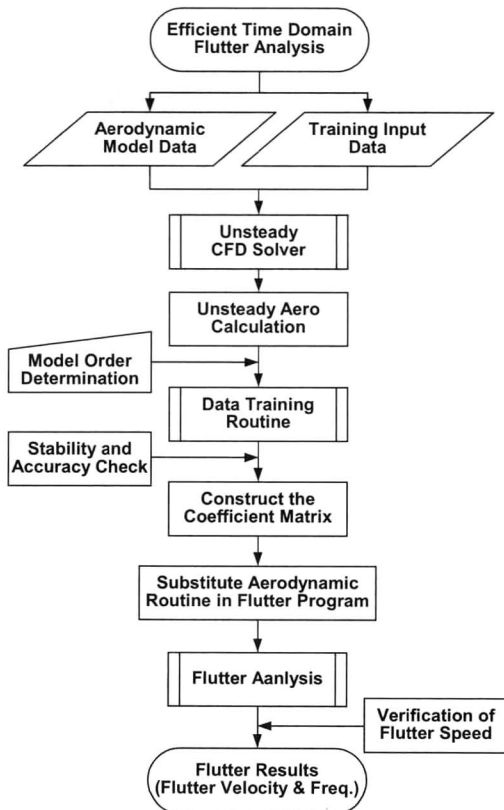


Fig. 1. Flow chart of aeroelastic analysis based on the system modeling

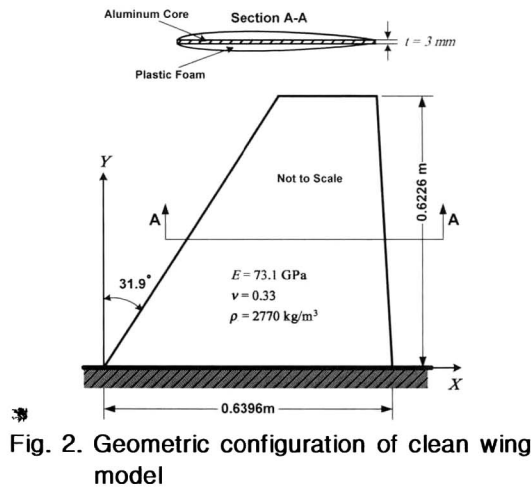


Fig. 2. Geometric configuration of clean wing model

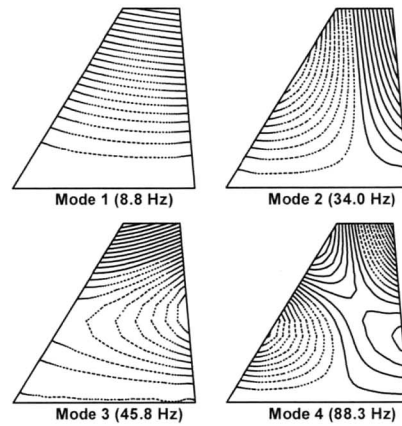
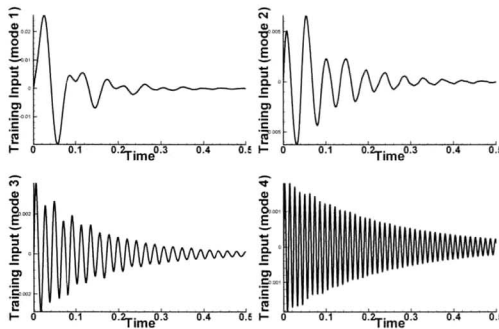
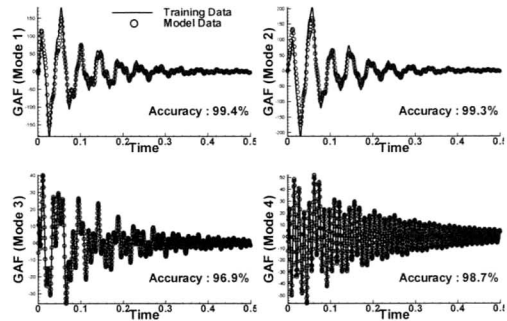


Fig. 3. Natural mode shapes and frequencies of clean wing model



(a) Aeroelastic modal displacements



(b) Generalized aerodynamic responses

Fig. 4. Modal based aeroelastic responses for model training at Mach = 0.7

Results and Discussion

Application to Clean Wing Configuration

In this study, a clean wing configuration is considered to perform the aeroelastic analysis and the unsteady aerodynamic solver is substituted by the ARX model. Figure 2 shows the geometric configuration of the wing; the sweep back angle is 31.9° at the leading edge, the taper ratio is 0.31, and the aspect ratio is 2.98. The wing section has the shape of the 64A004.8 airfoil. The free vibration analyses are accomplished by MSC/NASTRAN (Ver.70.5) and the first four modes are used in the aeroelastic analyses. The natural mode shapes and frequencies are presented in Fig. 3.

To estimate the parameters of multivariable ARX model, suitable training data will be necessary. In this study, aeroelastic responses after initial disturbance are used for training data set. Figure 4 (a) shows the aeroelastic modal responses after initial velocity disturbances and Fig. 4 (b) shows the generalized aerodynamic responses as measured output. These aerodynamic outputs are also compared with modeled outputs to confirm the model accuracy. In Fig. 4 (b), solid lines are generalized aerodynamic responses corresponding to the aeroelastic modal displacements, and the symbols represent the simulated results of unsteady aerodynamics using the ARX model. We also tested the accuracy for various model order (na and nb) and when $na=5$ and $nb=7$, the

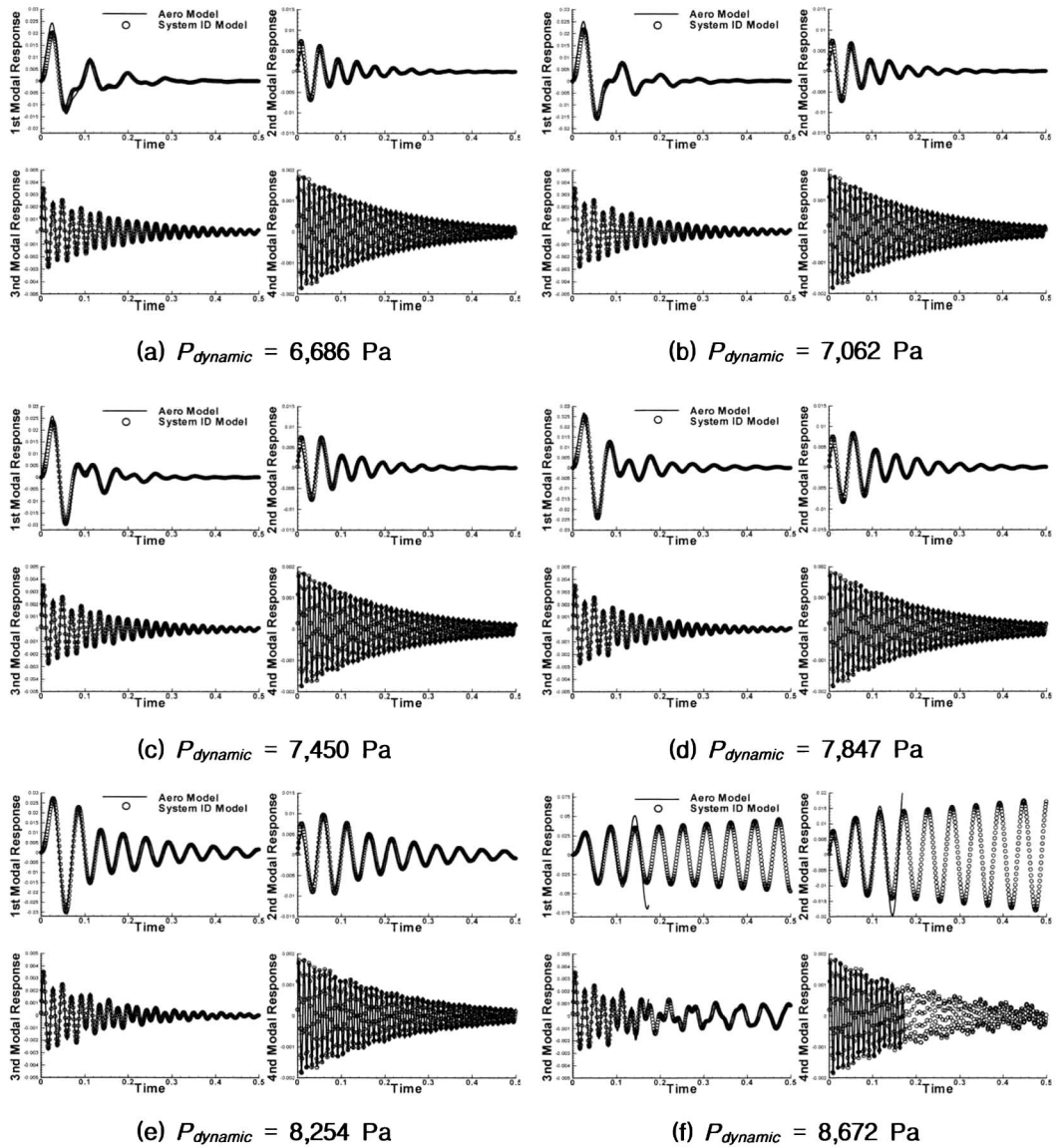


Fig. 5. Aeroelastic simulation results of clean wing model using multivariable ARX and comparisons with the results using unsteady aerodynamic solver at Mach 0.7

model is the most accurate. The percentages of accuracy are calculated using Eq. (7).

$$100 \times \frac{(1 - \text{norm}(y_m - y))}{\text{norm}(y - E(y))} \quad (7)$$

where, y_m is the simulated output; y is the measured output; and $E(y)$ is the mean value of y .

Figure 5 shows the aeroelastic responses at various dynamic pressures based on the ARX model. These responses are also compared with the results of CFD coupled aeroelastic analysis directly. In Fig. 5, although a little discrepancy appears at Fig. 5 (f), most responses of the present ARX model show good agreement with CFD coupled responses at the same dynamic pressure. This discrepancy is due to the numerical overflow error, which commonly occurs before

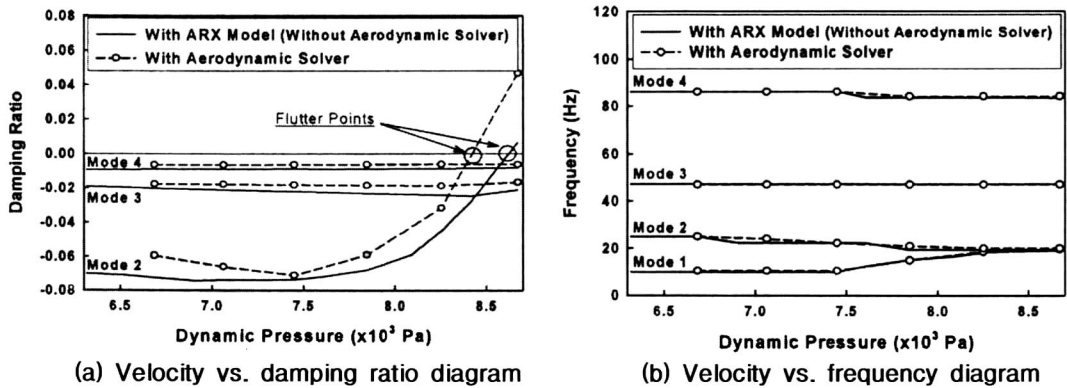


Fig. 6. Damping ratio and Frequency change of clean wing model at various dynamic pressures

CFD solver is failed. Figure 6 shows the variation of frequency and damping ratio at Mach 0.7. In this figure, the flutter occurs in the second mode which is first torsion mode. These figures also show that the ARX model can chase the variation of the damping ratio and frequency with the increase of dynamic pressure successfully. Of course, the flutter point which is obtained based on the ARX model is slightly different from the results using unsteady aerodynamic solver. However, if we consider the computing time is reduced to one-fifth when the aeroelastic analysis is performed without unsteady aerodynamic calculation, this methodology is very useful to save the computational cost of time domain aeroelastic analysis.

Application to Wing with Control Surface Configuration

This technology is also applied to the control surface model. Figure 7 shows geometric configuration of this model; the swept back angle is 44° at the leading edge, the taper ratio is 0.305, and the aspect ratio is 2.63. The wing section has the shape of biconvex airfoil with 6% thickness at the root and 4% thickness at the tip. The chord of control surface is 30% of the wing chord length. In this figure, CL means the chord length of the wing model, hence 0.855 CL means 0.855 times chord length. In this study, 14 natural vibration modes are applied for aeroelastic analysis. First 4 modes of them are described in Fig. 8. As shown in this figure, the first mode is a wing bending mode and higher modes contain large control surface motion.

The unsteady aerodynamic solution during the aeroelastic analysis has been substituted by the multivariable ARX model which has the model order of $na=3$ and $nb=4$. The higher model order has been also applied to get the more accurate results but the improvement of the accuracy is negligible. Figure 9 shows the aeroelastic analysis results using ARX model instead of the unsteady aerodynamic solver. Even though these figures just show the responses at Mach 0.9, the aeroelastic analysis has been performed at Mach 0.7, 0.9, 0.95, 1.05 and 1.3. To test the accuracy, the responses are compared with the results which are obtained using the unsteady aerodynamic computation at each time step. The solid lines are responses using the aerodynamic solver and the symbols describe the responses using the ARX model. The results of comparison show good agreement between them.

Figure 10 shows flutter boundaries of the wing with control surface model. The dotted line is the flutter boundaries which are calculated based on the ARX model instead of unsteady aerodynamic solver and the solid line is the results using the unsteady aerodynamic solver. In this figure, the two results show very good correlation in the subsonic flow region, but there is small discrepancy in the supersonic flow region. In spite of the discrepancy, the trend of the flutter boundary in the supersonic regime can be well predicted using the ARX model.

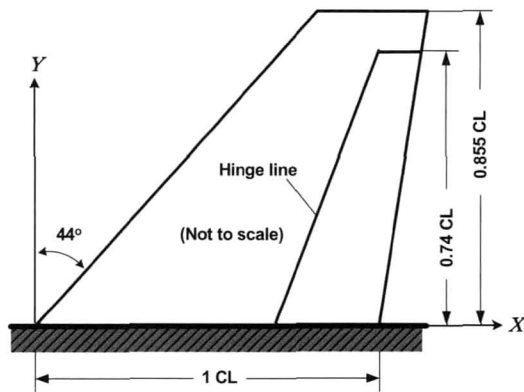


Fig. 7. Geometric configuration of wing with control surface model

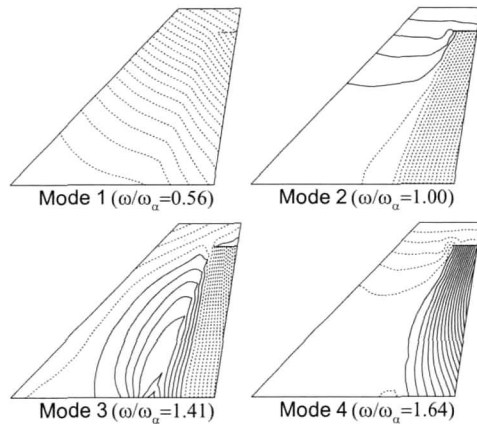


Fig. 8. Natural mode shapes and frequencies of wing with control surface model

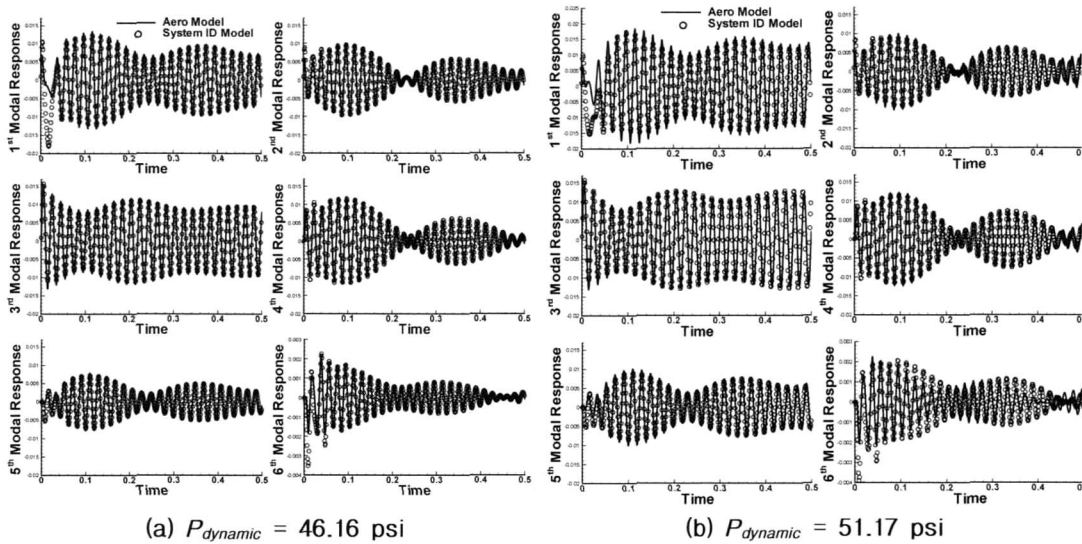


Fig. 9. Aeroelastic simulation results of wing with control surface model using multivariable ARX and comparisons with the results using unsteady aerodynamic solver at Mach 0.9

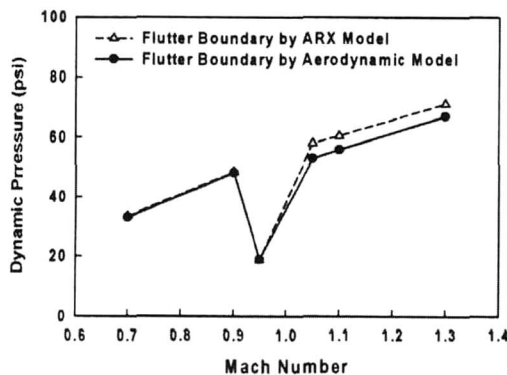


Fig. 10. Flutter boundaries of wing with control surface model

Conclusions

In this study, the system modeling technique was proposed to increase the efficiency of conventional time domain aeroelastic analysis. A multivariable ARX model substituted the unsteady aerodynamic solver in the CFD-coupled aeroelastic analysis program. This parametric model is applied to the simulation of the aeroelastic responses and the estimation of the flutter boundaries. The system model was validated by reproduction of CFD coupled responses at the same dynamic pressure. The comparison results between the responses of

ARX model and CFD-coupled program show good correlations. The aeroelastic analysis using the parametric model has an obvious limitation. That is this technique cannot be used solely without the aid of unsteady aerodynamic solver because unsteady aerodynamic responses are required during the model training process. However, the number of repeats of aeroelastic analysis until getting the neutral response, which means flutter point, can be largely reduced by estimating the flutter point based on this technique.

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