Development of Attitude Constraints for Real-time Attitude Determination System using GPS carrier phase

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Abstract

As one of validation tool for attitude determination system, we have used various constraints using priori information which is known through base vector set up. However these conventional constraints cannot guarantee validity in terms of final solutions such as Euler angle. So we suggest attitude boundary concept to verify the final attitude solution on the flying airplane, it is based on the combination of velocity based attitude estimation technique and ambiguity resolution. we can say it can check invalid solution effectively at just one epoch without repeatability test of resolved cycle ambiguity. In this paper we show that the suggested constraint can effectively reject incorrectly resolved cycle ambiguity the conventional constraints have missed.

Key Word : Constraint, Validation, Attitude Boundary

Introduction

General way to determine attitude of vehicle using GPS is to utilize ambiguity resolution method by carrier phase measurements[2,4,5]. Of course some researchers use dynamic method especially without the cycle ambiguity resolution as initialization of cycle ambiguity through taxing on the runway or as rotational motion of satellite[1]. There are researchers who use Kalman filter and they usually compose the filter to integrate GPS sensor with other sensors. However we can say that the ambiguity resolution is inevitable for reacquisition of ambiguity on the fly, because the rotational method need restricted motions and enough epochs. And to reduce cost of navigation sensor including attitude and positioning, it prefer to use the ambiguity resolution algorithm with GPS sensor only.

Though many researchers developed various algorithms to resolve ambiguity, we still experience difficulties caused by confidence of resolved solution[2,4]. In other words though the ambiguity is resolved using stochastic verification with high confidence level, there is no special way to decide the resolved ambiguity is really true especially in the case of low cost GPS L1 receiver. Moreover there is no way to know attitude before ambiguity fixing if instance ambiguity resolution is difficult by low visibility, severe multi-path and so on.

Attitude of aircraft on the fly can be estimated without ambiguity resolution by using GPS sensor only. It is a single GPS antenna method based on GPS velocity estimation[6,7]. But it shows overshoot and time delay by filtering and includes angle of attack and sideslip angles in attitude. So it makes limitation to get precise attitude. However it can be used to give geometrical or numerical boundaries to the verification procedure of the ambiguity resolution. Based on this motivation we have developed integration scheme to improve performance of attitude determination system.

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We have derived pseudo-attitude error in one order Taylor series approximation and have defined attitude boundary constraint in which the difference between traditional attitude of multiple antennas and pseudo-attitude of single antenna has to be stay for a true cycle ambiguity. Though a stochastic verification failed to catch resolving wrong solution, the boundary constraint will be able to catch the failure successfully. To use the boundary in the condition of uncoordinated flight, the pseudo-attitude should be as accurate as possible. So we have generated angular rate using Doppler measurement of antennas and pseudo-attitude estimate and then have composed Kalman filter to update the pseudo-attitude.

Conventional Constraints

Conventionally for ambiguity validation in attitude determination, baseline length constraint and fixed angular constraint are used with stochastic verifications such as chi-square test and F-test [2]. These constraints have a effective roll when there are cycle slips or low visibility conditions and have the forms as follows,

$$\mathrm{var}(\|\hat{X}\|_{norm}) \le \sigma^2 \max[eig\{(\mathbf{H}^T\mathbf{Q}^{-1}\mathbf{H})^{-1}\}] \tag{1}$$

$$l_a l_b \cos\theta - \kappa\sigma_{X_a X_b} \le X_a{}^T X_b \le l_a l_b \cos\theta + \kappa\sigma_{X_a X_b} \tag{2}$$

with $\sigma^2_{X_a X_b} \equiv \mathrm{var}(\hat{X}_a{}^T \hat{X}_b) = \sigma^2 X_a^T (\mathbf{H}^\mathbf{T}\mathbf{Q}^\mathbf{1}\mathbf{H})^{-1} X_a + \sigma^2 X_b^T (\mathbf{H}^\mathbf{T}\mathbf{Q}^\mathbf{1}\mathbf{H})^{-1} X_b$

where **H, Q** denotes line of sight matrix and covariance matrix of measurements each. l_a, l_b means length of base vector X_a, X_b and θ is the angle between the vectors. Though these constraints can check consistency of geometrical relationships, it can not say final solution is in the reasonable area. So we introduce the attitude boundary concept from next chapter as new constraint for more robust validity check.

Attitude Constraints (Attitude Boundaries)

Euler angles by single GPS antenna attitude determination system is defined in the wind axes instead of the body axes. And the relations between Euler angles of the body frames and the wind frames are expressed by:

$$R(\psi,\theta,\phi) = R(-\beta,\alpha,0)\cdot R(\psi_W,\theta_W,\phi_W) \tag{3}$$

Where α is the angle of attack, β is the sideslip angle and $(\ )_w$ is the subscript which means wind frame. If we consider wind frame only, the differences are not error in itself. But we use the Euler angle at body axes to check validity of traditional attitude by multiple antennas, so the differences should be understood as attitude error in terms of the body frame.

From equation 3, we can define the maximum attitude errors based on single GPS antenna attitude result as equation 4 ~ 6.

$$|\Delta\phi| \le \tan\tilde{\theta}_W\,(\alpha_{\max}\cdot|\sin\tilde{\phi}_W| + \beta_{\max}\cdot\cos\tilde{\phi}_W) + |\phi_W - \tilde{\phi}_W|_{\max} + \sigma_\phi \tag{4}$$

$$|\Delta\theta| \le \alpha_{\max}\cdot\cos\tilde{\phi}_W + \beta_{\max}\cdot|\sin\tilde{\phi}_W| + \sigma_\theta \tag{5}$$

$$|\Delta\psi| \le \frac{\alpha_{\max}|\sin\tilde{\phi}_W| + \beta_{\max}\cos\tilde{\phi}_W}{\cos\tilde{\theta}_W} + \sigma_\psi \tag{6}$$

with $\tilde{\phi}_W, \tilde{\theta}_W, \tilde{\psi}_W$: the pseudo-roll, pseudo-pitch and pseudo-yaw.

Note that in the equation 6 pseudo-pitch angle in the wind frame is used instead of the traditional pitch angle. The reason is we do not know traditional pitch angle. The boundaries can be used as geometrical constraints for traditional Euler angles by multiple GPS antennas for the given resolved cycle ambiguities.

Uncoordinated Flight Condition Problems

Though coefficients are set up well for the filter, the attitude estimate experiences overshoot inevitably, especially in the condition of relatively rapid attitude changes and gust inputs. Unfortunately it is a not included in the attitude boundaries analytically which is defined in the previous section.

More significant problem is occurred by uncoordinated flight conditions such as motions with accelerations in the degree of near the gravity force. In those conditions roll angle information can be degenerated by single GPS attitude algorithm itself. As a result true cycle ambiguity can be treated as unacceptable candidate due to the abnormally estimated pseudo-attitude. Therefore we have composed Kalman filter which use angular rate estimate to calibrate them in this paper. We use Doppler measurements and pseudo-attitude of current epoch to generate angular rates.

System Flow

The boundaries and the cycle ambiguity validation checking can be processed in the following method. We can estimate the ambiguity using ambiguity resolution engine and then use boundaries to check the decision of stochastic verification model as a flag. If the ambiguity with the minimum objective function which satisfy conventional constraints is within threshold of chi square test and satisfy ratio test using Fisher's distribution, the Euler angles within the boundaries mean true cycle ambiguity is resolved by ambiguity resolution procedure. Though stochastic verification result indicates estimated ambiguity as true, we do not use it as the solution. Instead we use pseudo-attitude until boundary check flag say 'yes'. See Fig. 1 which shows overall flow of the algorithm.

As ambiguity resolution algorithm for multiple antennas, we have used SNUGLAD (Seoul National University GPS Lab. Attitude Determination) [2] which is designed to reduce

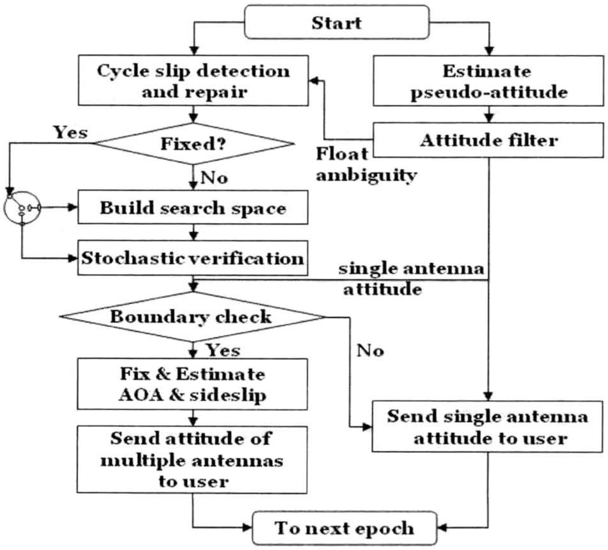

Fig. 1. the flow chart of new algorithm

computational burden as low as possible. In this paper algorithm for ambiguity resolution (AR) is not main subject, so we do not include the details for the AR.

Test Results

To show roll information degeneration effects caused by uncoordinated flight, we plotted flight experiment result in Fig. 2. we can see pseudo-roll estimates show abnormal values when there are high accelerations in direction of down. In that case, Kinematic Kalman filter using pseudo-angle and angular rates can reduce degeneration effects effectively as shown in dashed line. In the figure the traditional roll means reference roll angle.

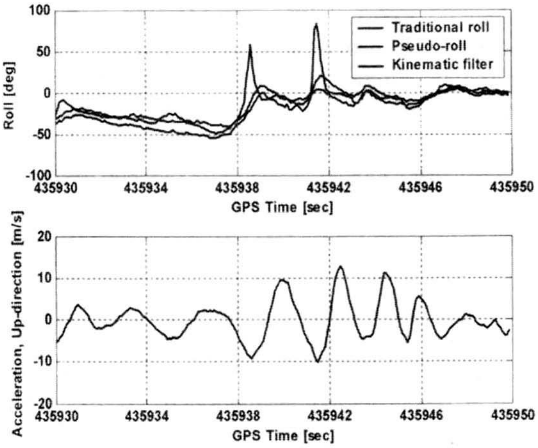

Fig. 2. the uncoordinated flight effects

As constraint we have defined the attitude boundaries using filtered pseudo Euler angles in equation 4 to 6. To see whether a new pseudo-attitude by filter stays within the boundary stably, we plotted attitude error in Fig. 3. the error means difference between traditional angle based on resolved cycle ambiguity and pseudo-angle. If resolved cycle ambiguity is correct one then the difference between the attitudes should be within the boundaries such as the figure.

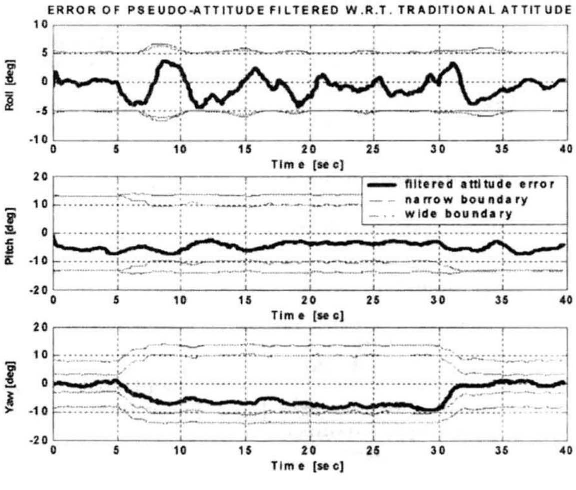

Fig. 3. the pseudo-attitude errors which have been passed Kalman filter using angular rate. narrow mean zero sideslip angle assumption and wide does non-zero

To keep roll error of single antenna solution within 2 deg in the moderate coordinated flight condition, we have set the maximum values of parameter α, θ_w, ϕ_w as 10, 15, 45 degrees [6]. For the wide boundary with non-zero side slip angle, we have assumed side gust velocity of 0.7 m/s (the side slip with 5 deg in 3 sigma) [3]. However if the aircraft is supposed to experience uncoordinated flight condition, we should redefine the above maximum parameters.

Finally we have performed simulation based on flow chart of Fig. 1 as epoch by epoch and we have defined the procedure for the test analysis. The Fig. 4 shows the procedure. The goal of this paper is to reject incorrect fix of cycle ambiguity effectively using both the stochastic verification and boundary test. So if ambiguity resolution determines wrong solution as true, then the reject counter should be increased. And the valid counter should count only true estimates. In example for the resolution trials of one hundred, if 99 epochs are resolved as true ambiguity, then one epoch should be counted by the reject counter. If boundary failure counter increase it means the failure of the boundary check definition.

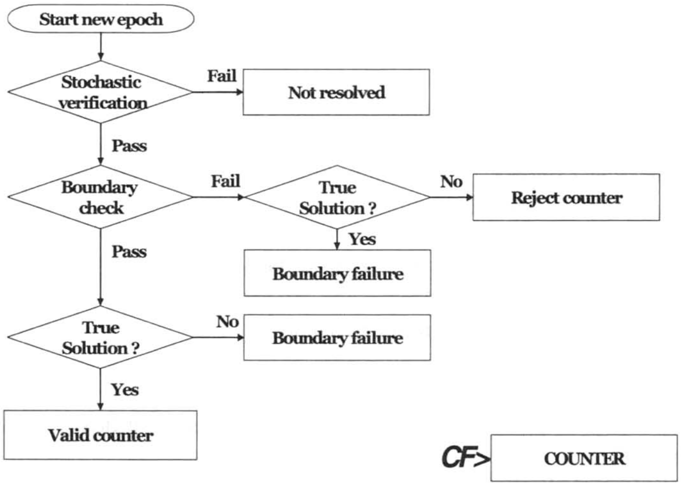

Fig. 4. the Procedure for the performance test
Yellow box means counter of epochs for the test

For 40 sec, full search of 399 epochs have been performed and 380 epochs have been passed stochastic test which include the conventional constraints. Ambiguity resolution algorithm has determined wrong solution as true one at one epoch out of them as shown in Table 1. However we can see that attitude constraint/boundaries using single antenna solution has rejected the false fix effectively. As a result we could fix the ambiguity to get precise attitude using 379 epochs, which were all true.

Table 1. the cycle ambiguity resolution result by new algorithm

Stochastic verification passed				Stochastic verification failed
380 epochs				19 epochs
Attitude boundary passed		Attitude boundary failed		-
379 epochs		1 epochs		
True solution	False solution	True solution	False solution	-
379	0	0	1	

Conclusions

In this paper we have developed the constraint in domain of Euler angle using the advantage of multiple antenna algorithm and single antenna algorithm together. Basic idea is to define attitude boundary in which multiple antenna attitude have to stay for a correct cycle ambiguity.

To do so we have analyzed simplified single GPS antenna attitude error w.r.t. traditional attitude at first. And to resolve degeneration problem by uncoordinated flight condition, we have composed the filter which has used angular rates generated by Doppler measurements of multiple antennas and pseudo-attitude. We could have resolved correct cycle ambiguities perfectly using the boundary because the boundary has rejected false fix which have been determined as correct one in the stochastic verification process. So we can say that the suggested constraint by velocity based attitude estimation with single antenna can increase the confidence of resolved solution effectively.

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