Sliding Mode Control for the Configuration of Satellite Formation Flying using Potential Functions

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Abstract

Some methods have been presented to avoid collisions among satellites for satellite formation flying mission. The potential function method based on Lyapunov's theory is known as a powerful tool for collision avoidance in the robotic system because of its robustness and flexibility. During the last decade, a potential function has also been applied to UAV's and spacecraft operations, which consists of repulsive and attractive potential. In this study, the controller is designed using a potential function via sliding mode technique for the configuration of satellite formation flying. The strategy is based on enforcing the satellite to move along the gradient of a given potential function. The new scalar velocity function is introduced such that all satellites reach the goal points simultaneously. Simulation results show that the controller drives the satellite toward the desired point along the gradient of the potential function and is robust against external disturbances.

Key Word : satellite formation flying, collision avoidance, potential function, sliding mode control

Introduction

Recently, satellite formation flying (SFF) has been a topic of significant research interest in aerospace society because it provides potential benefits compared to a single large spacecraft. The spatial separation between spacecrafts can range from a few meters to several kilometers for some SFF missions. Thus collision avoidance is a critical requirement for the configuration or reconfiguration maneuver of SFF which involves multiple satellites. Many techniques have been developed to solve the problem of path planning and collision avoidance for SFF missions. Especially the potential function technique is regarded as a very powerful tool for collision avoidance in the robotic system, which is based on the Lyapunov's stability theory. This technique has been generalized to spacecraft applications[1-3] because of simplicity of handling

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collision avoidance constraints. Other techniques have been developed for path-planning with collision avoidance constraints such as randomized algorithms[4], splines[5] and a mixed-integer linear programming[6]. Richards *et al.*[6] introduced a method of finding fuel-optimal trajectories considering collision avoidances and plume impingements based on a mixed-integer linear programming for the satellite formation reconfiguration. In recent years, the parameter optimization technique was presented for optimal formation trajectory planning[7]. Lim *et al.*[7] developed the new constraints of nonlinear equality for final configuration and nonlinear inequality for collision avoidance.

Most SFF control laws have been designed on the base of the simplified relative dynamic equations such as Hill's equations. These equations cannot capture the J2 perturbation effect because they are derived under the assumption that the reference orbit is circular, the Earth is spherically symmetric, and the target satellite is very close to the reference orbit. Control designs based on Hill's equations require high fuel consumption and can imperil the formation flying mission with long duration and large separation between satellites since Hill's equations disregard the perturbation and nonlinear terms on the relative motion dynamics. So many nonlinear control theories have been researched in SFF. Queiroz *et al.*[8] developed a nonlinear adaptive control law for the relative position tracking of multiple satellites. Gurfil *et al.*[9] proposed a nonlinear adaptive neural control methodology for deep-space SFF. Sliding mode controller was developed to track the desired trajectories with the extended Kalman filter for estimating the state vector based on measurements of relative distance between two satellites[10]. Pongvthithum *et al.*[11] developed the tracking control law using the universal adaptive control scheme. Currently, most nonlinear control schemes for SFF use full state feedback controllers, which require both position and velocity sensors. Wong *et al.*[12] designed a adaptive output feedback tracking control in the absence of velocity measurements.

In this paper, the sliding mode controller is developed for a problem of SFF configuration using a potential function which is used for collision avoidance and gives the global minimum at the desired point. It is not sure that all satellites arrive at their desired positions simultaneously. Thus, The scalar velocity function is introduced to the sliding manifold to overcome this problem.

System Dynamics and Potential Function

A rotating local-vertical-local-horizontal (LVLH) frame is used to visualize the relative motion with respect to the reference satellite. The x-axis points in the radial direction, the z-axis is perpendicular to the orbital plane and points in the direction of the angular momentum vector. Finally, the y-axis points in the along-track direction. The relative motion dynamics can be derived using the Lagrangian mechanics based on the LVLH frame[12]. This derivation utilizes the fact that the total energy(sum of potential and kinetic energy) of the satellite is conserved under the gravitational field. The relative dynamics for an eccentric reference orbit is given by

$$\ddot{x} - 2\dot{\theta}\dot{y} - \ddot{\theta}y - \dot{\theta}^2 x = -\frac{\mu(R+x)}{[(R+x)^2 + y^2 + z^2]^{3/2}} + \frac{\mu}{R^2} + D_x(t) + u_x$$

$$\ddot{y} + 2\dot{\theta}\dot{x} + \ddot{\theta}x - \dot{\theta}^2 y = -\frac{\mu y}{[(R+x)^2 + y^2 + z^2]^{3/2}} + D_y(t) + u_y \qquad (1)$$

$$\ddot{z} = -\frac{\mu z}{[(R+x)^2 + y^2 + z^2]^{3/2}} + D_z(t) + u_z$$

where $\mathbf{x} = [x, y, z] \in \mathbf{R}^3$ is a vector denoting the position of the follower satellite, $\mathbf{D} = [D_x, D_y, D_z] \in \mathbf{R}^3$ denotes external disturbances, $\mathbf{u} = [u_x, u_y, u_z] \in \mathbf{R}^3$ describes the control inputs, R presents the radius of the reference satellite, and μ is the gravitational constant. θ refers to the latitude angle of the reference satellite in Eq. (1) which describes the circular orbit

for the case of $\dot{\theta} = \text{constant}$, i.e. $\ddot{\theta} = 0$.

The potential function method is based on Lyapunov's theory which determines the stability properties of a nonlinear system by constructing a scalar energy-like function. The potential function method has been extensively investigated in the field of robot motion planning because it is a powerful tool for avoiding collision between collaborative systems. Some approaches based on potential functions were developed for spacecraft applications to handle collision avoidance strategies. The basic idea of the potential function method is to establish a potential field with a global minimum at the desired state and local maxima in the obstacles. So, the potential function is composed of attractive and repulsive terms. According to the Lyapunov's stability theory, if the gradient of the potential function of the system is always negative, the system can be driven to the desired state without collision of obstacles. Thus, controller is designed such that the velocity of the system is pointed along the negative gradient of the potential function.

Numerous potential functions have been proposed in the past decade. However, the problem lies in that the formulations of potential function suffer from local minima which can lead to the robot to stay at the undesired locations. In many papers, a harmonic function is proposed to avoid this local minima problem as a potential function. A harmonic function gives a unique global minimum because it satisfies the Laplace equation:

$$\nabla^2 \Phi = \sum_{i=1}^{n} \frac{\partial^2 \Phi}{\partial x_i^2} = 0 \tag{2}$$

where Φ represents potential function. In this study, the potential function of the ith satellite[1] is defined as

$$\Phi_i = \frac{1}{2}(\mathbf{x}_i - \mathbf{x}_i^{\mathrm{d}})^{\mathrm{T}}\mathrm{M}(\mathbf{x}_i - \mathbf{x}_i^{\mathrm{d}}) + \Psi_i(\mathbf{x}_i ; \lambda) \tag{3}$$

where,

$$\Psi_i(\mathbf{x}_i ; \lambda) = \lambda_1 \sum_{j=1, j \neq i}^{N} \left\{ \exp\left[-\lambda_2(\boldsymbol{x}_i - \mathbf{x}_j^{\mathrm{T}})\mathrm{Q}(\mathbf{x}_i - \mathbf{x}_j)\right]\right\}$$

The subscript i denotes the ith satellite, and the superscript d represents the desired state which the satellite can reach in the finite time, and N is the total number of satellites in SFF missions. The matrix M and Q is taken to be positive definite, and the constant λ is also positive definite. The first term of Eq. (3) describes the attractive potential with a global minimum at the desired state, whereas the second term represents the repulsive potential for collision avoidance against other satellites.

Design of Sliding Mode Controller

In this section, a sliding mode controller is designed for tracking the gradient of the potential function defined in Eq. (3). It is known that a sliding mode control generally yields exact tracking performance of the gradient lines and is robust with respect to parametric uncertainty and disturbances in system dynamics. The strategy of designing the controller is based on enforcing the satellite to move along the gradient of a given potential function. Let us assume that the reference satellite moves in the circular orbit and the external disturbances are unknown and bounded, i.e. $|\mathbf{D}(\mathrm{t})| \leq \overline{\mathbf{D}}$. Then, the nonlinear system dynamics can be rewritten by

$$\ddot{\mathbf{x}} = \mathbf{F}(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{D}(t) + \mathbf{u} \tag{4}$$

where

$$\mathbf{F}(\mathbf{x}, \dot{\mathbf{x}}) = \begin{bmatrix} 2\dot{\theta}\dot{y} + \dot{\theta}^2 x - \dfrac{\mu(R+x)}{[(R+x)^2 + y^2 + z^2]^{3/2}} + \dfrac{\mu}{R^2} \\[2ex] -2\dot{\theta}\dot{x} + \dot{\theta}^2 y - \dfrac{\mu y}{[(R+x)^2 + y^2 + z^2]^{3/2}} \\[2ex] -\dfrac{\mu z}{[(R+x)^2 + y^2 + z^2]^{3/2}} \end{bmatrix}$$

To enforce the velocity of the system along the negative gradient of the potential function, the sliding manifold of the ith spacecraft is defined as

$$\mathbf{s}_i = \dot{\mathbf{x}}_i - \kappa_i(\mathbf{x}) \frac{\nabla \Phi_i}{|\nabla \Phi_i|} \tag{5}$$

where

$$\kappa_i(\mathbf{x}) = \alpha \left\{ \tanh\left(\frac{N|\mathbf{x}_i - \mathbf{x}_i^d|}{\displaystyle\sum_{j=1}^{N} |\mathbf{x}_j - \mathbf{x}_j^d|} - 1 \right) + 1 \right\}$$

The scalar velocity function, $\kappa(\mathbf{x})$ is introduced such that all satellites reach the desired point simultaneously, which determines the magnitude of the velocity. The gradient of the potential function gives the direction of motion, whereas the velocity of a satellite can be controlled through the function $\kappa(\mathbf{x})$. α is a scalar constant velocity, and the function $\kappa(\mathbf{x})$ is bounded because of the properties of a hyperbolic tangent, i.e. $0 < \kappa(\mathbf{x}) < 2\alpha$. Furthermore, let us define the Lyapunov function

$$V_i = \frac{1}{2} s_i^{\ T} s_i \tag{6}$$

Using Eq. (4) and (5), the derivative of V_i can be derived as

$$\begin{aligned} \dot{V}_i &= \mathbf{s}_i^{\mathrm{T}} \dot{\mathbf{s}}_i = \mathbf{s}_i^{\mathrm{T}} \left(\ddot{\mathbf{x}}_i - \frac{d}{dt}\left[\kappa_i(\mathbf{x}) \frac{\nabla \Phi_i}{|\nabla \Phi_i|} \right] \right) \\ &= \mathbf{s}_i^{\mathrm{T}} \left(\mathbf{F}_i(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{D}_i(t) + \mathbf{u}_i - \frac{d}{dt}\left[\kappa_i(\mathbf{x}) \frac{\nabla \Phi_i}{|\nabla \Phi_i|} \right] \right) \end{aligned} \tag{7}$$

If the last term of Eq. (7) is assumed to be bounded for all $\mathbf{x} \in \mathrm{R}^3$ (refer to [13] for the boundness), then we can derive the following result:

$$\left| \mathbf{D}_i(t) - \frac{d}{dt}\left[\kappa_i(\mathbf{x}) \frac{\Phi_i}{|\nabla \Phi_i|} \right] \right| \leq \overline{\Omega}_i \tag{8}$$

Thus we can choose the control law such that the derivative of the Lyapunov function is negative. In particular, by choosing

$$\mathbf{u}_i = -\mathbf{u}_{i0}\, \mathrm{sgn}(\mathbf{s}_i) - \mathbf{F}_i(\mathbf{x}, \dot{\mathbf{x}}) \tag{9}$$

where $\mathrm{sgn}(\,\cdot\,)$ means the signum function. Thus, the derivative of Lyapunov function can be obtained by

$$\begin{aligned} \dot{V}_i &= \mathbf{s}_i^{\mathrm{T}} \left(-\mathbf{u}_{i0}\, \mathrm{sgn}(\mathbf{s}_i) + \mathbf{D}_i(t) - \frac{d}{dt}\left[\kappa_i(\mathbf{x}) \frac{\nabla \Phi_i}{|\nabla \Phi_i|} \right] \right) \\ &\leq -\left| \mathbf{s}_i^{\mathrm{T}} \right| (\mathbf{u}_{i0} - \overline{\Omega}) \\ &\leq 0 \end{aligned} \tag{10}$$

where the gain $\mathbf{u}_{i0}$ of the control inputs is chosen to be $\mathbf{u}_{i0} > \overline{\Omega_i}$. Eq. (10) ensures that all trajectories starting off the sliding manifold reach it in finite time and those on the sliding manifold cannot leave it. The sliding mode control law causes chattering phenomenon since it is discontinuous across the sliding manifold. Chattering describes rapid control signal switching between positive negative values because the control input enforces the system to reach to the sliding manifold. To eliminate the unwanted chattering, the signum function in the control law is substituted with a hyperbolic tangent:

$$\mathbf{u}_i = -\mathbf{u}_{i0} \tanh(\beta \mathbf{s}_i) - \mathbf{F}_i(\mathbf{x}, \dot{\mathbf{x}}) \tag{11}$$

Simulation and Results

The configuration problem of SFF consisting of three satellites is simulated in the only x–y plane of the LVLH frame as an example and the results are presented. The final configuration is a equilateral triangle of the projected circular orbit with 300 m radius. The relative dynamics is assumed to be a circular orbit but external disturbances are considered. The sliding mode controller works to minimize the difference between the velocity and the normalized gradient of the potential function multiplied by the scalar function. Thus, all satellites arrive at the desired points in finite time because the global minimums of the potential function are the desired points. If all satellites are in the circle with 0.1 m radius from the desired points, simulation will be finished. The sliding manifold and control inputs are computed with the interval of 0.5 second to track the gradient of the potential function. All numerical data is given in Table 1 for the simulation of the sliding mode control law.

Table 1. Numerical data for the simulation

Satellites	Radius of reference satellite [km]	R=7178.137
	Initial Position [m]	$\mathbf{x}_1 = [800, 1200]^T$ $\mathbf{x}_2 = [1000, 1000]^T$ $\mathbf{x}_3 = [1200, 800]^T$
	Initial velocity [m/s]	$\mathbf{v}_1 = \mathbf{v}_2 = \mathbf{v}_3 = [0, 0]^T$
	Desired Position [m]	$\mathbf{x}_1^d = 300 \times [\cos(0), \sin(0)]^T$ $\mathbf{x}_2^d = 300 \times [\cos(2\pi/3), \sin(2\pi/3)]^T$ $\mathbf{x}_3^d = 300 \times [\cos(4\pi/3), \sin(4\pi/3)]^T$
	External disturbances [m/s²]	$\sin(2\pi \times \theta \times t) \times \begin{matrix} 1.9106E-5 \\ -1.1517E-5 \end{matrix}$
	$\mathbf{u}_{i0}$ [m/s²]	$[0.002, 0.002]^T$
Parameters	M, Q	$M = 10 \times I_{2 \times 2}$, $Q = I_{2 \times 2}$
	λ_1, λ_2	$\lambda_1 = 1.0E6$, $\lambda_2 = 1.0E-4$
	α, β, u_{ij}	$\alpha = 150 \times \dot{\theta}$, $\beta = 20$

Fig. 1 shows the initial potential field of the 1st satellite, in which the star mark denotes the desired position. The potential function has the high values near the position of other satellites due to the repulsive term and the lowest value at the desired position due to the attractive term. The trajectories of three satellites are displayed in Fig. 2. We can see that the 1st and 2nd satellite take a detour due to the repulsive potential function in Fig. 2.
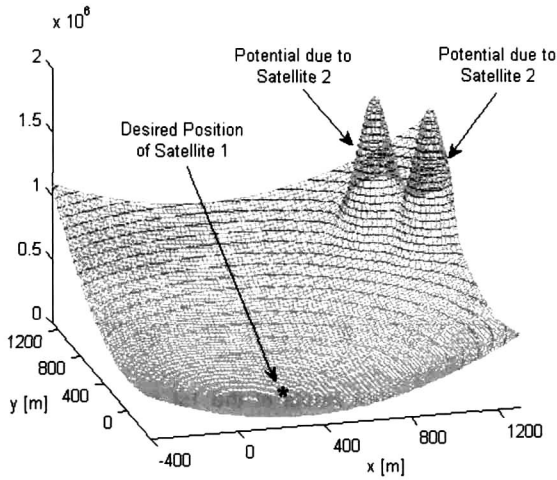
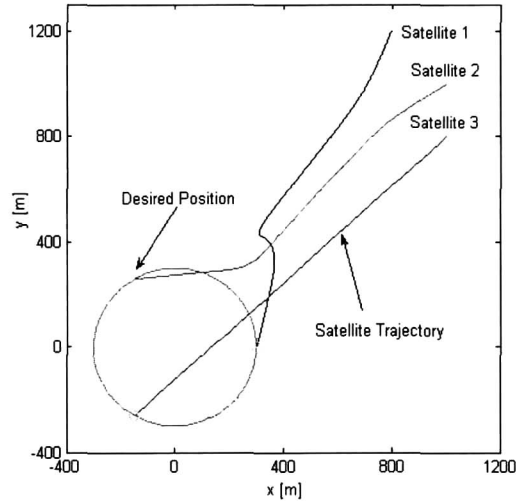Fig. 1. Potential function of the 1st satellite at the initial time

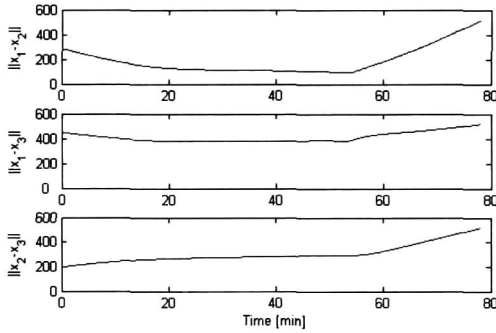

Fig. 2. Satellite trajectories in the x–y plane

Fig. 3. Relative Distances
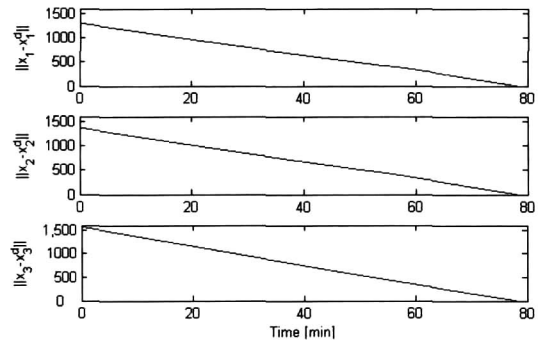(satellite 1 & 2, 1 & 3, and 2 & 3)

Fig. 4. Distances from the desired positions

Fig. 3 and Fig. 4 show that all satellites approach their desired positions simultaneously without collision against each other. The relative distances are larger than about 100 m, and all satellites arrive at their desired positions at the same time (Note that total simulation time is 78.3 minute). The total ΔV for the maneuver of the configuration is 6.4 m/s for the 1st satellite, 6.8 m/s for the 2nd, and 6.2 m/s for the 3rd.

Even though one satellite gets to the desired point, it should stay near the point to make the final configuration until other satellites approach to their desired points. This problem can make the satellite waste fuel near the desired position because the satellite does not stay in the vicinity of the desired point due to the external disturbances and properties of relative dynamics if control inputs are not applied. Thus the second simulation was done using $\kappa(\mathbf{x}) = \alpha$ instead of Eq. 5 to investigate this problem. In this study, the scalar velocity function $\kappa(\mathbf{x})$ is introduced to overcome this problem, which determines the appropriate velocity of a satellite and makes all satellites approach at their desired points simultaneously. Fig. 5(a) shows the control inputs of the 1st satellite using the scalar velocity function defined in Eq. (5), and Fig. 5(b) displays the results from the second simulation. The additional control inputs are not necessary to make the final configuration in Fig. 5(a) because three satellites arrive at their desired positions simultaneously.
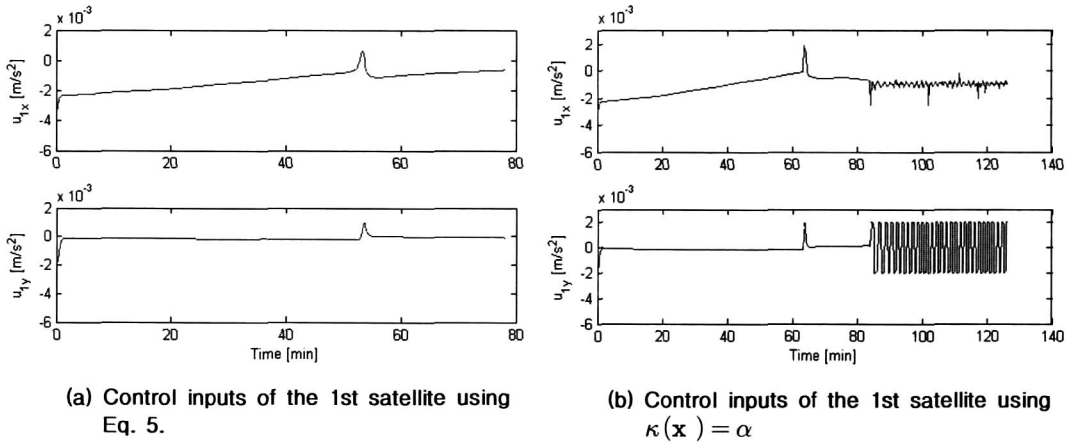
(a) Control inputs of the 1st satellite using Eq. 5.

(b) Control inputs of the 1st satellite using $\kappa(\mathbf{x}) = \alpha$

Fig. 5. Control inputs of satellites

However, vibrations of the control inputs appear after about 80 minute in Fig. 5(b) because the 1st satellite arrives at its desired point faster than other satellites, i.e. there is no function in $\kappa(\mathbf{x}) = \alpha$ to make all satellites approach the desired points simultaneously. It means that control efforts are necessary to make the 1st satellite stay near the desired point under the external disturbances and the relative dynamics until other satellites get close to their goal positions. According to the second simulation, the 1st satellite arrived at the desired position in 84.05 minute, but the configuration was completed in 125.6 minute. This problem increased the total ΔV for the maneuver of the configuration. The total is 11.5 m/s for the 1st satellite, 11.8 m/s for the 2nd, and 11.9 m/s for the 3rd in the second simulation.

Conclusions

It is known that a potential function is a powerful tool for collision avoidance in the collaborative system. Thus, sliding mode controller based on a potential function was designed for collision avoidance in the configuration of SFF. The sliding manifold was constructed to enforce the satellite to move along the gradient of a potential function. The scalar velocity function was introduced in the sliding manifold such that all satellites reach the goal point simultaneously. The controller guaranteed the exact tracking of the gradient of a potential function. Simulation results show that the designed controller is robust because it drives the satellite toward the desired position along the gradient of the potential function under external disturbances.

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