Minimum-Time Guidance and Control Law for High Maneuvering Missile

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Abstract

This paper deals with design procedure of online guidance and control law for future missiles that requires agile maneuverability. For the purpose, the missile with high powered side thruster is proposed. The guidance and control law for such missiles is discussed from a point of view of optimal control theory in this paper. Minimum time problem is solved for the approximated system. It is derived that bang-bang control is optimal input from the necessary conditions of optimal solution. Feedback guidance without iterative calculation is useful for actual systems. In this paper, multiple design point method is applied to design feedback gains and feedforward inputs of the guidance and control law. The numerical results show that the proposed guidance and control law has a high-performance for wide-ranging boundary conditions.

Keywords : Guidance and control, Optimization, High Maneuvering

Introduction

For the future missile, it is well known that high maneuvering and high response rate are required. However, conventional steered wing systems lack ability for the future missile to cope when high maneuvering and high response conversion is needed. In addition, the front wing (canard) on the missile model has tendency to be unstable when the control systems are designed[1]. The minimum time guidance and control of the missile systems are studied among many researchers with guidance of point mass model, control of rigid body systems, and etc[2-3]. In addition, there are also cases which mainly concerns on attitude control with linear equation of motion [4-5].

The authors have been studying on guidance and control system design of the missile with the efficient side thruster method for the high maneuvering and high response rate. The side thruster system has an independent gas generator and generates thrust in the perpendicular direction to body axis. In addition, the thrust does not depend on dynamic pressure and is effective even in the low-speed region immediately after launching or in the high altitude region. It is expected that this method is useful for high maneuvering and rapid response in the future missiles. It is assumed that the side thruster device changes the amplitude of thrust continuously.

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In this paper, the new design procedure that uses optimal control theory in order to aim at designed guidance and control system of missile using the side thruster is proposed. The proposed control law is superior in safety and effective in this agile motion. The guidance and control system is required to calculate a optimal trajectory under wider–ranging boundary conditions in short period of time.

Iterative calculation is used in the conventional algorithm to solve optimal control problem [2–3,6–7]. However, another calculation is necessary to derive the trajectory for the different boundary condition. It is not suitable for realistic missiles system. The proposed design procedure is the design system that uses the guidance law by the multiple models which optimizes multiple boundary conditions simultaneously. This feature is calculating the switching function by feedback.

As the previous study of the multiple design point method, following papers are written [8–10]. Miyazawa applies this design procedure to design feedback controller [8–9]. Optimal trajectory is designed in reference [10].

The proposed procedure also has ability to correspond flexibly to the change in the change frequency of bang–bang input by using the tendency that the switching function continuously changes by the initial condition. The designed system is able to calculate the continuing optimal input and trajectory within designated section with different boundary conditions. This new designed procedure is applied to the side thruster on–board missile and the numerical simulation is done. The usefulness of the proposed system is verified.

Missile Model

In this paper, maneuvering in the vertical plane is treated. In this section, the missile model of longitudinal motion is derived. Outline of the missile model and coordinate system are shown in figure 1. The missile model uses the equation of motion of a general missile that flies in the inertial space. In addition, this equation expresses both rotational and translational motion. The velocity and mass of the simulation is set as constant, the effect on the aerodynamic coefficient by the control surface and thrusters are omitted, and gravity force is not taken into account. Under these conditions, the equations of motion are as follows [11].

$$\dot{\gamma} = Z_\alpha(\theta - \gamma) + Z_\delta \delta_r + Z_T T_S \quad (1)$$

$$\dot{q} = M_\alpha(\theta - \gamma) + M_\delta \delta_r + M_T T_S \quad (2)$$

$$\dot{z} = V_m \gamma \quad (3)$$

$$\dot{q} = q \quad (4)$$

Here, the constraints on steering input and thruster input are given as follows.

$$\delta_{rmin} \le \delta_r \le \delta_{rmax} \quad (5)$$

$$T_{Smin} \le T_S \le T_{Smax} \quad (6)$$

Where V_m [m/s] is flight velocity and δ_r [deg] is rear wing steering angle. T_S [N] and L_r [N] shows thrust of the orthogonal direction to body axis and lift. In addition, Z_α, Z_δ [1/s] and M_α, M_δ [1/s^2] are aerodynamic differential coefficients. The state variables are region attitude angle θ [deg], pitch rate q [deg/s], flight–path angle γ [deg], and distance of z [m] axial direction.

Here, the thruster combusts the gas to vertical direction and the valve is used to control the difference between two thrusters. Therefore, this is the non–impulse thruster. It is possible to modify the thrust continuously. In addition, the thruster arrangement is to the front from center of gravity, the rear steering wings are set at the rear from center of gravity.

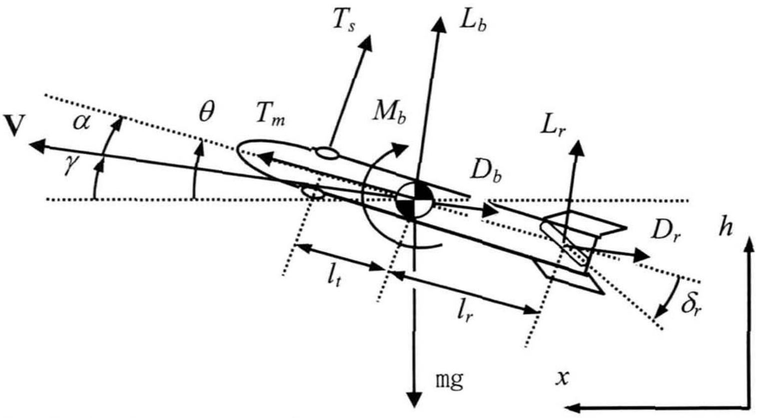

Fig.1. Outline of the missile model and coordinate system

Furthermore, the constraint on the angular velocity of the steering angle is not given in this paper. The horizontal directional distance is not given for they are not used in the optimal control problem used in this paper.

Design Procedure

3.1 Minimum time problem

a) Statement of problem

In this chapter, optimum input is derived by solving the minimum time problem with open loop numerical calculation. Outline of that system is shown in figure 2. In this paper, problem of terminal maneuvering is treated. Block diagram of conventional design system is shown in figure 3. A missile is required to shift in the z direction within the minimum time. The required distance of maneuvering is presented as Z_0. The trajectory for minimum time maneuver is calculated under equation of motions Eqs. (1) ~ (4) with given initial conditions and constraints. The initial condition and the terminal condition are defined as follows [12].

$$\theta(0)=q(0)=\gamma(0)=0, z(0)=Z_0 \tag{7}$$

$$\theta(t_f)=q(t_f)=\gamma(t_f)=0, z(t_f)=0 \tag{8}$$

These constraints define that the missile is placed in parallel position at terminal condition as shown in Fig. 3. However, t_f is terminal time. The criterion function is as follows

$$J=\int_0^{t_f} dt \tag{9}$$

From criterion Eq. (9) and boundary conditions Eqs. (7) and (8), the optimal input for the thrust and the steering angle is derived.

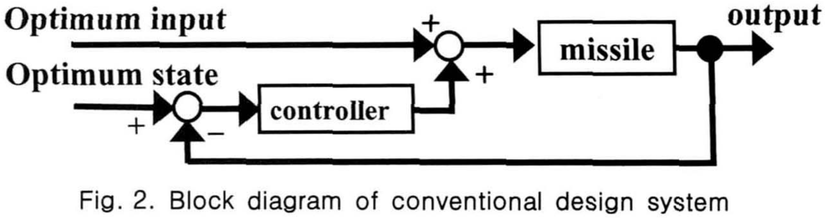

Fig. 2. Block diagram of conventional design system

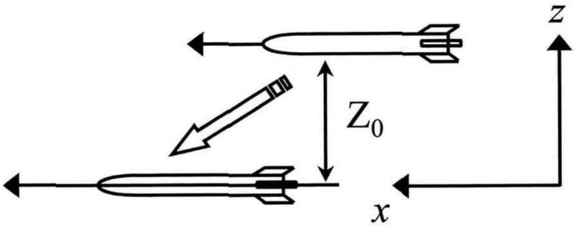

Fig. 3. Outline of minimum time problem

b) Necessary condition of optimization solution

Before numerical calculation, the necessary condition from optimal control theory is shown. When the adjoint variables are λ, the Hamiltonian of the system is as follows with the integrand of the criteria L and state equation f.

$$H = L + \lambda^T f \tag{10}$$

In case of the designated thruster on−board missile model, the Hamiltonian is as follows.

$$\begin{aligned} H &= 1 + \lambda_1 \left(Z_\alpha (\theta - \gamma) + Z_\delta \delta_r + Z_T T_s \right) \\ &+ \lambda_2 \left(M_\alpha (\theta - \gamma) + M_\delta \delta_r + M_T T_s \right) \\ &+ \lambda_3 V_m \gamma + \lambda_4 q \end{aligned} \tag{11}$$

From necessary conditions of the optimal control theory, following two equations are derived.

$$\sigma_1 = \partial H / \partial \delta_r = \lambda_1 Z_\delta + \lambda_2 M_\delta \tag{12}$$

$$\sigma_2 = \partial H / \partial T_s = \lambda_1 Z_T + \lambda_2 M_T \tag{13}$$

The optimum inputs are given as follows.

$$\delta_r = \begin{cases} \delta_{rmax} (\sigma_1 < 0) \\ \delta_{rint} (\sigma_1 = 0) \\ \delta_{rmin} (\sigma_1 > 0) \end{cases}, \; T_s = \begin{cases} T_{smax} (\sigma_2 < 0) \\ T_{sint} (\sigma_2 = 0) \\ T_{smin} (\sigma_2 > 0) \end{cases} \tag{14}$$

δ_{rint} and T_{sint} are singular input which uses intermediate values of input, obtained when the switching function σ_i keeps 0. From differentiate of σ_i given from necessary condition of optimum control theory, σ_i is difficult to keep 0 in finite time. Therefore, existence of singular input is impossible and optimal solution is bang−bang input. From the following section, optimum input is calculated as bang−bang input.

c) Condition of the simulation

Simulation condition is at altitude of 10,000ft, flight velocity of Mach 2, and designed the Z_0 displacement of the missile as 10m.

As the flight velocity in simulation is constant at Mach 2, aerodynamic characteristic does not change.

3.2 Example calculation

The minimum time problem that is shown with the previous section is solved numerically, optimum input is calculated. The side thruster is arranged at 0.5m forward from center of gravity of the missile with total length of 3m. The maximum input for the side thruster and steering angle is limited to 4g and 20 deg due to the quality and flight

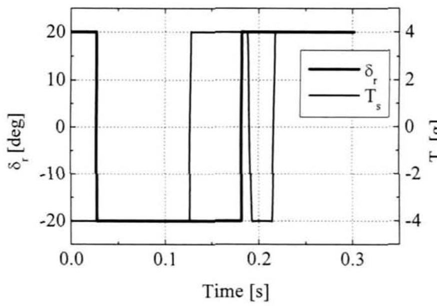

Fig. 4. Optimum input (thrust)
(Design point at 10m)

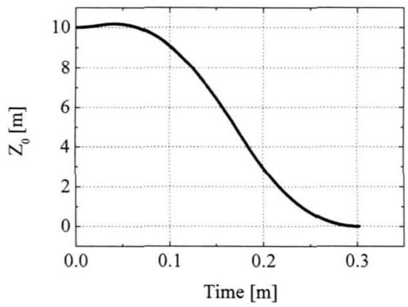

Fig. 5. Z coordinate
(Design point at 10m)

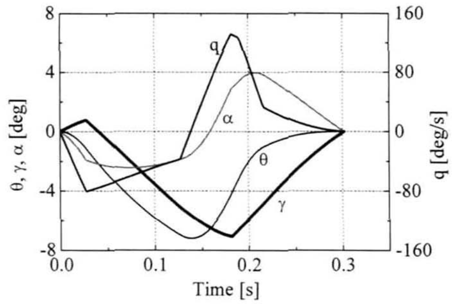

Fig. 6. State variables (Design point at 10m)

characteristics of the hardware. The optimum calculation result is shown in Fig.4−6. In Fig. 4, input becomes bang−bang input, both the steering angle and the thrust. In Fig. 5, transition of the flight−path obtains the satisfying result with little overshoot by use of the side thruster. In Fig.6, state variable are obtained within the satisfying result without errors.

Multiple Design Point Method

4.1 Design procedure

Large amount of calculation is necessary to solve the optimal control problem like the one that is shown in the previous chapter. It is impossible for real−time systems to solve the optimal control problems because of calculation time and reliability of conversion in iterative calculation. The optimal inputs or reference trajectories are required to be provided as functions of time for the different boundary conditions. This paper proposes a new design procedure to provide a guidance law which is suitable for real−time systems. The guidance law calculates the switching function of input variables from the given initial conditions and the varying state variables.

The switching function is formed with the feedback term and the feedforward term that are defined as function of nondimension time. The feedback is necessary in order to cope with terminal error of state variables. However, it is difficult to generate the optimum input only by linear combination of state variables. Therefore, the feedforward term is necessary. Iterative calculation is not necessary the feedback term and feedforward term. It is possible to calculate the switching function optimally without iterative calculation even if the boundary condition changes. The switching function is shown in the following equation.

$$\sigma_i(t)=\sigma_{0i}(t)+k_i x \quad (i=1,2) \tag{15}$$

where k_i is a vector of feedback gains. The subscript i represents the input variable, $i=1$ is the steering angle of rudder and $i=2$ is the thrust of side thruster. It is useful that the feedforward term is common for each boundary condition. The terminal time, however, depends on each boundary condition. Therefore, non-dimensional time is introduced in this paper. The definition is as follows.

$$\tau = t/t_f \tag{16}$$

where t_f is estimated terminal time, which is given as the following equation.

$$t_f = A_1\sqrt{Z_0}+A_2 \tag{17}$$

where A_1 and A_2 are constants and Z_0 is the initial position of the missile in z-direction. The feedforward term is given as follows.

$$\sigma_{0i}(t)=\sigma_{0N}(t/t_f)=\sigma_{0N}(\tau) \tag{18}$$

where $\sigma_{0N}(\tau)$ is the common feedforward term in non-dimensional time. The input variables are given as bang-bang control in the optimal solution. Bang-bang control, however, is not suitable for optimal design procedure because the input variable does not change even if the switching function changes in most regions. Therefore, this paper proposes that the input variables are given as following equation.

$$\begin{pmatrix}\delta_r\\ T_s\end{pmatrix}=\begin{pmatrix}\frac{2}{\pi}\delta_{r\max}\tan^{-1}\sigma_1\\ \frac{2}{\pi}T_{S\max}\tan^{-1}\sigma_2\end{pmatrix} \tag{19}$$

The input variable depends on the switching function in all of the regions. The block diagram of proposed guidance law is shown in Fig.7. The unknown parameters to be designed are feedback gains k_i in Eq. (15) and A_1 and A_2 in Eq. (17). The unknown function to be designed is $\sigma_{0N}(\tau)$ in Eq. (18).

The unknown parameters and function should be designed under optimization technique. Multiple-model technique is used in this paper. In this paper, the multiple models mean the multiple boundary conditions. The multiple optimal control problems with the same guidance law and the different boundary conditions are solved simultaneously. The solution provides the optimal parameters k_i , A_1 and A_2 , and the optimal function $\sigma_{0N}(\tau)$.

An example using two models is shown in this chapter. The state variables are $x_{(1)}$ and $x_{(2)}$. The numbers of subscript represent the number of each boundary condition. The state differential equations are given in the following equation.

$$\frac{d}{d\tau}\begin{pmatrix}x_{(1)}\\ x_{(2)}\end{pmatrix}=\frac{d}{d\tau}\begin{pmatrix}(\gamma_{(1)},q_{(1)},z_{(1)},\theta_{(1)})^T\\ (\gamma_{(2)},q_{(2)},z_{(2)},\theta_{(2)})^T\end{pmatrix}=\begin{pmatrix}t_{f(1)}\cdot f(x_{(1)},u_{(1)})\\ t_{f(2)}\cdot f(x_{(2)},u_{(2)})\end{pmatrix} \tag{20}$$

The independent variable is the non-dimensional time defined in Eq. (16). The initial and terminal conditions are given as follows.

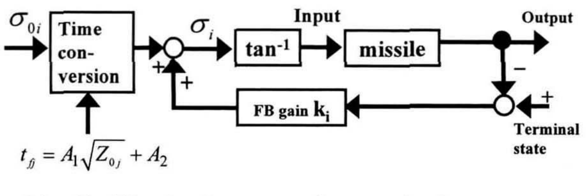

Fig. 7. Block diagram of new design system

$$\begin{pmatrix} \left(\gamma_{(1)}, q_{(1)}, z_{(1)}, \theta_{(1)}\right)^T \\ \left(\gamma_{(2)}, q_{(2)}, z_{(2)}, \theta_{(2)}\right)^T \end{pmatrix}_{\tau=0} = \begin{pmatrix} \left(0,0,Z_{01},0\right)^T \\ \left(0,0,Z_{02},0\right)^T \end{pmatrix} \tag{21}$$

$$\begin{pmatrix} \left(\gamma_{(1)}, z_{(1)}, \theta_{(1)}\right)^T \\ \left(\gamma_{(2)}, z_{(2)}, \theta_{(2)}\right)^T \end{pmatrix}_{\tau=1} = \begin{pmatrix} \left(0,0,0\right)^T \\ \left(0,0,0\right)^T \end{pmatrix} \tag{22}$$

The pitching rate q is not specified at the terminal. The input variables are given using Eqs. (15)−(19). The criterion to be minimized is given as the summation of terminal time and terminal error of q.

$$J = \sum_{j=1}^{2} \left(t_{f(j)} + \left[q_{(j)}^2 \right]_{\tau=1} \right) \tag{23}$$

The design procedure is defined as an optimal control problem. The optimal parameters and function are given in the optimal solution.

4.2 Example calculation

In this section, the control system that was designed in the previous chapter is used to verify various basic characteristics. Figs. 8−10 shows the results from the initial position of 10[m] by the guidance and control law which is designed at 5[m] and 10[m] through simultaneous optimum calculation. Fig. 8 shows that thrust input becomes bang−bang input in deformed shape. However, steering angle becomes almost bang−bang input. Fig. 9 shows time history of z trajectory. The same result by optimum offline calculation with single design point is shown in Fig. 5. The terminal time is a little longer in Fig. 9 than in Fig. 5. However, two lines in Figs. 5 and 9 are similar. State variable fluctuation is small enough compared to the whole scale in Fig. 10. Although there is a little error in pitch rate at the terminal, the error is small enough and obtains satisfying results. From the results of optimization with multiple conditions, the design procedure is possible to obtain almost similar result with optimally calculated results as a single design point by iterative numerical calculation. The results show that deigned online optimal control was able to control the missile with littlie deterioration. However, when Z_0 exceeds design points as in Fig. 11, the design procedure is difficult for on−board missile since terminal error becomes too large. Z_f is terminal error in z coordinate. From these results, the optimal computation of the wide region maneuver becomes possible using Eq. (15), which is the real−time calculation without iteration. For the proposed guidance and control law, iterative calculation is only once and off−line in the design procedure. It is great advantage calculation time for a real−time system.

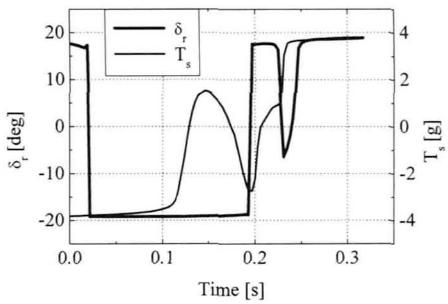

Fig. 8. Input(Z_0=10m)
(Design points are 5m and 10m)

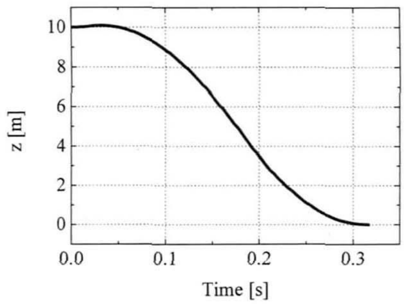

Fig. 9. Z coordinate
(Z_0=10m)

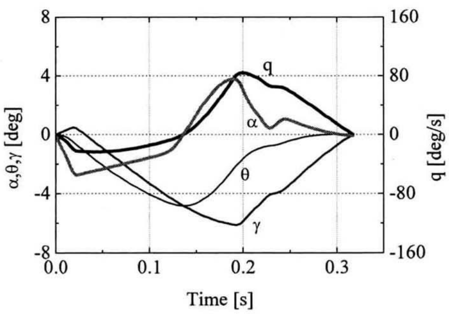

Fig. 10. State variables (Z_0=10m)

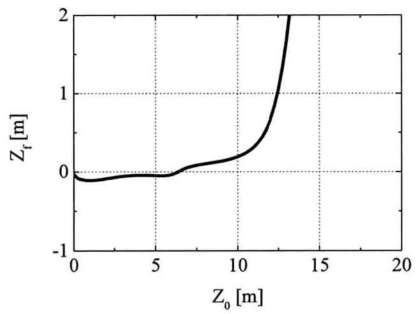

Fig. 11. Terminal error in z coordinate (Design points 5m and 10m)

New Guidance and Control Law

In the previous chapter, the good result was obtained by the method. However, the following point is necessary in the realistic missile guidance and control system. The point is the simultaneous optimization whose terminal error is small in the wide region.

In this chapter, the guidance law for realistic missile guidance and control system is proposed by improving the method of the previous chapter. The guidance law is verified by the following law.

1) The guidance law of the continual coefficient change which connects design section and connecting section
2) The guidance law by continuous design section

Details of the guidance law are as follows.

1) Guidance law Ⅰ

Outline of the guidance law is shown in Fig. 12. The procedure connects design section and connecting section which makes one guidance law.

Here, d is interval of the design points. Design section is between two design points, and connecting section is between two design sections. n represents the number of design section. n_{max} is the maximum number of n . D_{max} is the maximum design point which is equal to $(n_{max}+1)d$. D_{max} is optional value and is defined 20[m] in the following simulation. The optimum calculation is calculated from d [m] to D_{max} [m].

Optimal calculation for every d [m] interval, and k_i in Eq. (15) are defined as follows.

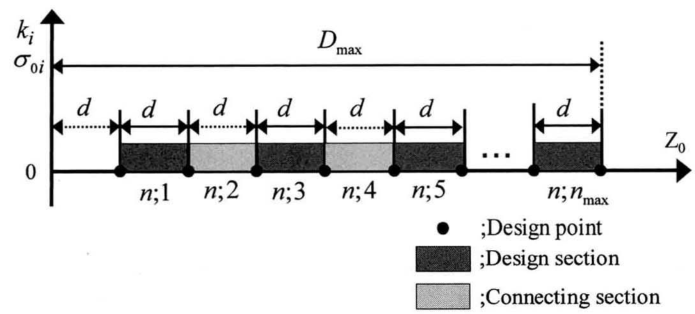

Fig. 12. Outline of Guidance law Ⅰ

$$k_i = \begin{cases} k_{i(nd,(n+1)d)} & (nd \le Z_0 \le (n+1)d \quad n;odd) \\ a \times k_{i((n-1)d,nd)} + b \times k_{i((n+1)d,(n+2)d)} \\ \qquad (nd \le Z_0 \le (n+1)d \quad n;even) \end{cases} \tag{24}$$

$$a = \frac{(n+1)d - Z_0}{d}, \quad b = \frac{Z_0 - nd}{d}$$

where k_i is $k_{i(d,2d)}$ when Z_0 is smaller than d and k_i is $k_{i(n_{\max}d,D_{\max})}$ when Z_0 is greater than $D_{\max}$.

σ_{0i} are similarly defined as follows.

$$\sigma_{0i}(t) = \begin{cases} \sigma_{0i(nd,(n+1)d)}(t) & (nd \le Z_0 \le (n+1)d \quad n;odd) \\ a \times \sigma_{0i((n-1)d,nd)}(t) + b \times \sigma_{0i((n+1)d,(n+2)d)}(t) \\ \qquad (nd \le Z_0 \le (n+1)d \quad n;even) \end{cases} \tag{25}$$

$$a = \frac{(n+1)d - Z_0}{d}, \quad b = \frac{Z_0 - nd}{d}$$

$\sigma_{0i}(t)$ is $\sigma_{0i(d,2d)}(t)$ when Z_0 is smaller than d and $\sigma_{0i}(t)$ is $\sigma_{0i(n_{\max}d,D_{\max})}(t)$ when Z_0 is greater than $D_{\max}$.

t_f are defined as follows.

$$t_f = \begin{cases} A_1\sqrt{Z_0} & (Z_0 \le d) \\ A_1\sqrt{Z_0} + A_2 & (d < Z_0 \le D_{\max}, Z_0 > D_{\max}) \end{cases} \tag{26}$$

$$A_i = \begin{cases} A_{i(nd,(n+1)d)} & (nd \le Z_0 \le (n+1)d \quad n;odd) \\ a \times A_{i((n-1)d,nd)} + b \times A_{i((n+1)d,(n+2)d)} \\ \qquad (nd \le Z_0 \le (n+1)d \quad n;evev) \end{cases} \quad a = \frac{(n+1)d - Z_0}{d}, \quad b = \frac{Z_0 - nd}{d}$$

When Z_0 is smaller than d, A_2 in Eq. (17) is not used because the terminal time is 0 in case of initial error is 0. A_i is $A_{i(d,2d)}$ when Z_0 is smaller than d and A_i is $A_{i(n_{\max}d,D_{\max})}$ when Z_0 is greater than $D_{\max}$.

At first, relationship of feedback gain of thrust to state variable z is shown in the Fig. 13. Feedback gain is changed continuously. Terminal error in z coordinate is shown in Fig. 14. From the result, optimum calculation of the wide region became possible. However, terminal errors appears with connecting section. The terminal state variables are shown in Fig.15. Error has appeared largely in connecting sections. The connecting sections is the section whose change is large.

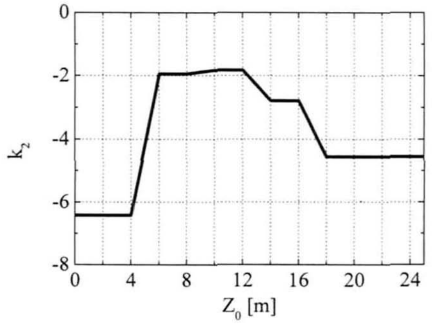

Fig. 13. Feedback gain from state variable z to thrust(Guidance law using Eq. (24) ~ (26), d =2)

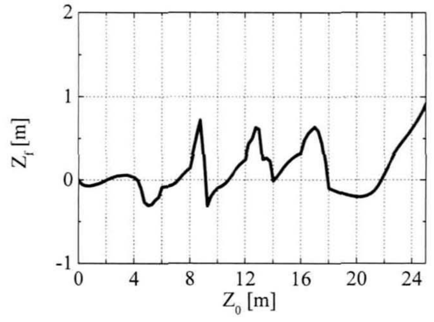

Fig. 14. Terminal error in z coordinate (Guidance law using Eq. (24) ~ ~ (26), d =2)

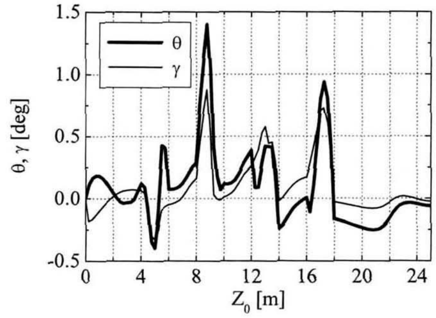

Fig. 15. Terminal state variable θ, γ (Guidance law using Eq. (24) ~ (26), d =2)

2) Guidance law II

The guidance law I was designed to satisfy the rapid wide range calculation. The result has shown that the guidance law has some problems with the terminal errors. Therefore, the guidance law with continuous design section is designed. Outline of the guidance law is shown in Fig.16. The procedure connects design section and design section and makes one guidance law.

k_i of Eq. (15) are defined as follows.

$$k_i = k_{i(nd,(n+1)d)} \quad (nd \le Z_0 \le (n+1)d \quad n;1 \le n \le n_{\max}) \tag{27}$$

k_i is $k_{i(d,2d)}$ when Z_0 is smaller than d and k_i is $k_{i(n_{\max}d,D_{\max})}$ when Z_0 is greater than $D_{\max}$.

σ_{0i} are similarly defined as follows.

$$\sigma_{0i}(t) = \sigma_{0i(nd,(n+1)d)}(t) \quad (nd \le Z_0 \le (n+1)d \quad n;1 \le n \le n_{\max}) \tag{28}$$

$\sigma_{0i}(t)$ is $\sigma_{0i(d,2d)}(t)$ when Z_0 is smaller than d and $\sigma_{0i}(t)$ is $\sigma_{0i(n_{\max}d,D_{\max})}(t)$ when Z_0 is greater than $D_{\max}$.

t_f are defined as follows.

$$t_f = \begin{cases} A_1\sqrt{Z_0} & (Z_0 \le d) \\ A_1\sqrt{Z_0} + A_2 & (d < Z_0 < D_{\max}, Z_0 > D_{\max}) \end{cases} \tag{29}$$

$$A_i = \{A_{i(nd,(n+1)d)} \quad (nd \le Z_0 \le (n+1)d \quad n;1 \le n \le n_{\max})$$

When Z_0 smaller than d, A_2 in Eq. (17) is not used because the terminal time is 0 in case of initial error is 0. A_i is $A_{i(d,2d)}$ when Z_0 is smaller than d and A_i is $A_{i(n_{\max}d,D_{\max})}$ when Z_0 is greater than $D_{\max}$.

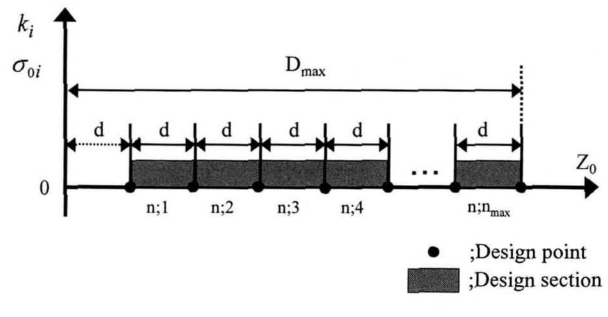

Fig. 16. Outline of the guidance law II

Relationship of feedback gain of thrust to state variable z are shown in the Fig. 17. The feedback gain has not changed continuously. Terminal error in z coordinate is shown in Fig. 18. Since error is small in the wide region, it is possible to obtain the wide region guidance system by making design section continually. State variables are shown in Fig. 19. Error is small in all regions. Therefore, the guidance law of continuous design section is effective.

The interval of d is changed to check the ability of the control law for various calculating conditions. The results are shown in Fig. 20. When the interval of d becomes wide, terminal error becomes large. Therefore, interval of d is suitable at 2m.

Next, in terminal time, the optimum calculation by present guidance law and single point optimum calculation are compared. The results are shown in Fig. 21. The newly designed online guidance law using continuous design point method shows similar result with the offline optimum numerical calculation. From the result, the optimum calculation in wide region is possible according to a simple calculation without the time consuming iterative calculation.

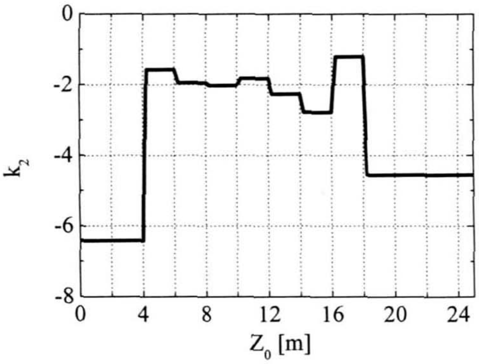

Fig. 17. Feedback gain from state variable z to thrust (Guidance law using Eqs. (27) ~ (29), d =2)

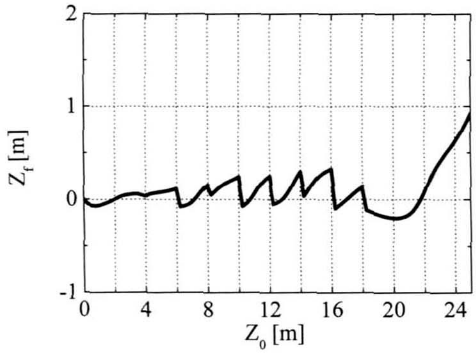

Fig. 18. Terminal error in z coordinate (Guidance law using Eqs. (27) ~ (29), d =2)

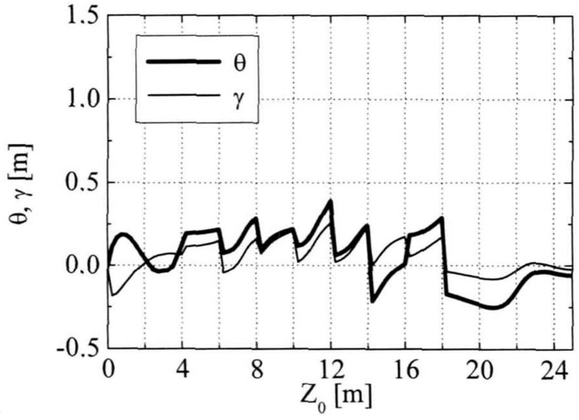

Fig. 19. Terminal state variable θ, γ (Guidance law using Eqs. (27) ~ (29), d =2)

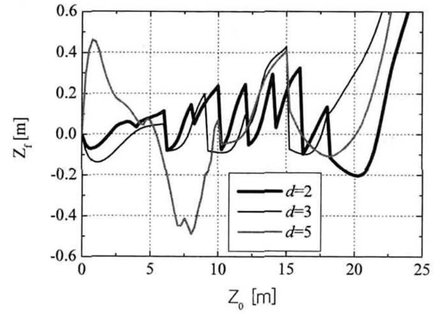

Fig. 20. Terminal error in z coordinate (Guidance law using Eqs. (27) ~ (29))

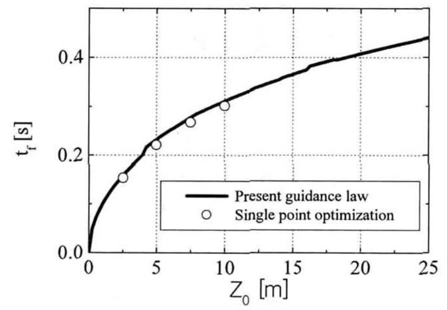

Fig. 21. Terminal time

Conculsion

In this paper, new guidance and control law for the future missile is introduced. More realistic high maneuver construction of missile guidance and control system became possible by using new procedure for the missile which uses the side thruster. Error minute being by using the guidance law which continues design section, wider-ranging guidance and control system was produced. Minimum time maneuver was obtained with almost same result as the result from optimal numerical calculation as a single unit. From these results, the optimum calculation in a wide region was possible according to a simple calculation without the iterative calculation.

References

1. Yoshimasa Ochi : Design of a Normal Acceleration and Angle of Attack Control System for a Missile Having Front and Rear Control Surfaces, Journal of the Japan Society of Aeronautical and Space Sciences, Vol. 51 No.598, 2003, pp. 621-627 (in Japanese).

2. Fumiaki Imado and Takeshi Kuroda : Optimal Midcourse Guidance for Medium-Range Air-to-Air Missiles, Proceedings of AIAA Guidance, Navigation and Control Conference, Minneapolis, Aug. 1988, pp. 100-108.

3. Fumiaki Imado and Takeshi Kuroda : Optimal Thrust Control of a Missile with a Pulse Motor, Journal of Guidance, Control, and Dynamics, Vol.14 No.21, 1991, pp. 377-382.

4. Kevin A. Wise : Bank-to-Turn Missile Autopilot Design using Loop Transfer Recovery, Journal of Guidance, Control, and Dynamics, Vol.13 No.1, 1990, pp. 145-152.

5. Christian Tournes, Yuri Shtessel and Ilya Shkolnikov : Autopilot for Missiles Steered by Aerodynamic Lift and Divert Thrusters using Nonlinear Dynamic Sliding Manifolds, Proceedings of AIAA Guidance, Navigation, and Control Conference, San Francisco, Aug. 2005, AIAA 2005-6382.

6. Arkadi Lichtsinder : Minimum-Time Maneuvers of Thrust-Vectored Aircraft, Journal of Guidance, Control, and Dynamics, Vol.21 No.2, 1998, pp. 244-250.

7. Curtis P. Mracek and D. Brett Ridgely : Missile Longitudinal Autopilots: Connections Between Optimal Control and Classical Topologies, Proceedings of AIAA Guidance, Navigation, and Control Conference, San Francisco, Aug. 2005, AIAA 2005-6381.

8. Yoshikazu Miyazawa : Robust Flight-Path Control System Design with Multiple-Delay Model Approach, Journal of Guidance, Control, and Dynamics, Vol.16 No.2, 1993, pp. 241-246.

9. Yoshikazu Miyazawa : Design with Multiple-Delay-Model and Multiple-Design-Point Approach, Journal of Guidance, Control, and Dynamics, Vol.18 No.2, 1995, pp. 508-515.

10. Seiya Ueno : Minimum-Time Turns of Aircraft by Feedback Guidance and Control Law, Journal of the Japan Society for Aeronautical and Space Sciences, Vol. 45 No. 516, 1997, pp. 29-35 (in Japanese).

11. Hirofumi Eguchi : Designed Guidance and Control System, Tokyo Denki University Publ., 2005, pp. 153-161 (in Japanese).

12. Arthur E. Bryson Jr. and Yu-Chi Ho : Applied Optimal Control, John Wiley & Sons, 1975, pp. 108-117.