Nonlinear Aeroelastic Analysis of a High-Aspect-Ratio Wing with Large Deflection Effects

Kyung-Seok Kim*, In-Gyu Lim* and In Lee******
Division of Aerospace Engineering, Department of Mechanical Engineering
Korea Advanced Institute of Science and Technology (KAIST) 373-1
Guseong-dong, Yuseong-gu, Daejeon, Korea 305-701

Jae-Han Yoo**
Hyundai Kia Motors Corporate Research & Development Division,
772-1, Jangduk-Dong, Hwaseong-Si, Gyeonggi-Do, Korea 445-706

Abstract

In this study, nonlinear static and dynamic aeroelastic analyses for a high-aspect-ratio wing have been performed. To achieve these aims, the transonic small disturbance (TSD) theory for the aerodynamic analysis and the large deflection beam theory considering a geometrical nonlinearity for the structural analysis are applied, respectively. For the coupling between fluid and structure, the transformation of a displacement from the structural mesh to the aerodynamic grid is performed by a shape function which is used for the finite element and the inverse transformation of force by work equivalent load method. To validate the current method, the present analysis results of a high-aspect-ratio wing are compared with the experimental results. Static deformations in the vertical and torsional directions caused by an angle of attack and gravity loading are compared with experimental results. Also, static and dynamic aeroelastic characteristics are investigated. The comparisons of the flutter speed and frequency between a linear and nonlinear analysis are presented.

Key Word : HALE (High-Altitude, Long-Endurance) Aircraft, Flutter, Nonlinear Aeroelasticity, Large Deflection Beam, Static Aeroelastic Analysis

Introduction

Aeroelastic stability and response of high-altitude, long endurance (HALE) aircraft have been studied in recent years. HALE aircraft is used for a variety of flight missions such as unmanned reconnaissance, long term surveillance, environmental sensing, and communication relay. To make the missions available, high-aspect-ratio wing and weight reduction are needed. Therefore, the wing of HALE has aspect ratio of about 35 and highly flexible structures. Due to these characteristics of the wing, large deflection of wing tip which is about 25% of wing semi-span can occur. Such a large deflection is included in a geometrical structural nonlinearity.

Aeroelastic analyses which are taken account of a geometrical structural nonlinearity have been studied by using the rotor blade of helicopter since 1970s. In the case of rotor blade, the aeroelastic

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* Ph.D. Student

** Research Engineer

*** Ph.D. Student

**** Professor, President of KSAS, Associate Fellow of AIAA, and Member of the Korean Academy of Science and Technology

E-mail : inlee@asdl.kaist.ac.kr Tel : 042-869-3717 Fax : 042-869-3710

stability can be affected by a geometric structural nonlinearity. Generally, a blade is modeled by using a beam to consider a large deformation. In aeroelasticity for a fixed-wing aeroelasticity, aeroelastic characteristics of HALE aircraft have been investigated by using nonlinear beam model. Schoor and Flotow [1] have investigated aeroelastic characteristics using a human-powered aircraft model. The complete aircraft was modeled using a few modes of vibration, including rigid-body modes. In addition, the doublet lattice theory is applied for the aerodynamic model. Pendaries [2] has presented linear aeroelastic flight dynamic analysis results. However, both analyses are linear and did not consider the geometrical nonlinearities induced by large deformations. Patil *et al.* [3-4] have studied static and dynamic aeroelastic characteristics on a high-aspect-ratio wing. For a structural analysis, large deflection effects of a beam are described using Rodrigues parameter which represents the rotation. The analysis is based on a geometrical structural analysis and finite-state unsteady aerodynamics with stall. The results indicate that the flutter speed and frequency are decreased due to a geometric structural nonlinearity. Hall *et al.* [5] have presented the results obtained by using a three-dimensional non-planar aerodynamic theory coupled with a linear structural analysis. The instability of flutter speed was reduced with wing curvature. Theoretical and experimental investigation of flutter and limit cycle oscillations using a nonlinear beam model and ONERA stall model have been performed by Tang and Dowell [6]. Patil and Hodges [7] have developed the aeroelastic analysis by using a nonlinear structural model and three-dimensional aerodynamics. For a static aeroelastic analysis, VLM considering curvature of the wing is applied. On the other hand, DLM is used for a dynamic aeroelastic analysis. A difference between the airloads calculated using the non-planar wing and the loads calculated assuming a planar wing geometry is negligible.

In the above investigations, aerodynamic models are two- or three-dimensional panel method for aeroelastic analyses. However, it is difficult to consider the effect of a camber and a thickness of a wing with those methods. To make it possible, computational fluid dynamics (CFD) techniques were introduced. Among these CFD methods, the transonic small disturbance (TSD) theory is well known as an efficient method for aeroelastic analysis. The TSD theory has easiness to make a computational mesh and efficient computing time. Recently, Yoo *et al* [8] have studied static aeroelastic analysis using TSD equation for aerodynamic analysis and the large deflection beam theory for structural analysis, respectively. The static aeroelastic results are compared with the reference [6]. This research is the extension work of previous study [8]. In this study, aeroelastic analyses of HALE aircraft wing were performed using the TSD and large deflection beam theory for aerodynamic and structural analysis, respectively. For the aeroelastic analyses, a structural solver based on finite element method is directly coupled with a fluid solver. Also, structural mesh and CFD grid are not coincided with each other. Transformation methods of displacements from FEM mesh to CFD grid and forces from CFD grid to FEM mesh are considered.

Theoretical Background

Structural Analysis

To analyze a wing structure of a HALE aircraft using the large deflection beam theory, following assumptions are introduced: there is no deformation on the cross sectional plane. Hence, one-dimensional model is available along the beam axis. The beam can have initial curvature and twist. Initial curvature, however, is small like a practical wing. The strain level remains low even if a large deflection occurs. And the wing is modeled as a cantilever beam.

Using Euler angles, a deformed shape can be expressed with respect to a reference coordinate. The transformation matrix can be written as the follows:

$$T=\begin{bmatrix} \cos\beta\cos\psi & \cos\beta\sin\psi & \sin\beta \\ -\sin\theta\sin\beta\cos\psi-\cos\theta\cos\psi & \cos\theta\cos\psi-\sin\psi\sin\beta\sin\theta & \cos\beta\sin\theta \\ -\cos\theta\sin\beta\cos\psi+\sin\theta\sin\psi & -\sin\theta\cos\psi-\sin\psi\sin\beta\cos\theta & \cos\beta\cos\theta \end{bmatrix} \tag{1}$$

To calculate the strain in the curvilinear coordinate, Green-Lagrange strain tensor is introduced. By the assumption of small initial curvature of the beam, high order terms in Green-Lagrange strain can be neglected. Thus, engineering strains are obtained as follows:

$$\begin{aligned} &\epsilon_{11} = \bar{e}_{11} + x_3\kappa_2 - x_2\kappa_3 + w_1{}',\ \epsilon_{22} = w_{2,2},\ \epsilon_{33} = w_{3,3} \\ &\gamma_{12} = 2\bar{e}_{12} - x_3\kappa_1 + w_{1.2} + w_2{}',\ \gamma_{23} = w_{2,3} + w_{3,2},\ \gamma_{13} = 2\bar{e}_{13} - x_2\kappa_1 + w_{1,3} + w_3{}' \\ &\kappa_i = K_{i-k_i} \end{aligned} \tag{2}$$

where x_1, x_2 and x_3 are curvilinear coordinates. w_1, w_2 and w_3 are the general warping displacements of an arbitrary point on the cross section. The force strains ($\bar{e}_{11}, 2\bar{e}_{12}, 2\bar{e}_{13}$) and moment strains ($\kappa_1, \kappa_2, \kappa_3$) components are given in Jeon *et al.* [9]. Herein, ( )′ means the derivative with respect to x_1 and $( )_{,i}$ means the derivatives with respect to x_i, i = 2,3. Through a quasi-linear approximation, these three-dimensional kinematics are divided into two-dimensional cross-sectional analysis and the one-dimensional global analysis. Using the Hamilton's principle, the equation of motion can be obtained as the follows:

$$\int_{t_1}^{t_2} \sum_{i=1}^{m} (\delta U_i - \delta T_i - \delta W_i)dt = 0 \tag{3}$$

where δU_i, δT_i and δW_i are the variation of strain energy, the variation of kinetic energy and the virtual work done by external forces. The nonlinear finite element equation of motion is obtained in the matrix form,

$$[M(q)]\{\ddot{q}\} + P(q) - P_A(q) = \{0\} \tag{4}$$

where $M(q)$ and $P(q)$ are the mass and the internal elastic force vector. $P_A(q)$ is external forces by aerodynamic forces. Newton-Raphson method combined with the line search method to improve convergence and reliability is applied.

Aerodynamic Analysis

The three-dimensional modified, unsteady transonic small disturbance (TSD) equation may be written in conservation law form as [10]

$$\frac{\partial f_0}{\partial t} + \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z} = 0 \tag{5}$$

where

$$f_0 = -A\phi_t - B\phi_x\ ,\ f_1 = E\phi_x + F\phi_x^2 + G\phi_y^2 + G\phi_z^2\ ,\ f_2 = \phi_y + H\phi_y\ ,\ f_3 = \phi_z \tag{6}$$

The above equations are given in a physical coordinate system (x, y, z, t), and subscript x, y and z denote the derivative of the each direction. ϕ represents the disturbance velocity potential. The coefficients are defined as

$$A = M^2,\ B = 2M^2,\ E = 1 - M^2,\ F = -\frac{1}{2}(\gamma+1)M^2,\ G = \frac{1}{2}(\gamma-3)M^2,\ H = -(\gamma-1)M^2 \tag{7}$$

where M and γ are the freestream Mach number and the specific heat ratio, respectively. The TSD equation is solved using a time-accurate approximate factorization (AF) method. The AF algorithm consists of a Newton linearization procedure coupled with an internal iteration technique. The solution process involves two steps. First, a time linearization step is performed to determine an estimate of the potential field. Second, internal iterations are performed to minimize linearization and factorization errors.

To solve the governing equation numerically, The TSD equation, Eq. (5), may be expressed in computational coordinates as

$$-\frac{\partial}{\partial t}\left[\frac{A}{\xi_x}\phi_\tau + B\phi_\xi\right] + \frac{\partial}{\partial \xi}\left[E\xi_x\phi_\xi + F\xi_x^2\phi_\xi^2 + G\,(\xi_y\phi_\xi + \phi_\eta)^2 + \frac{\xi_y}{\xi_x}(\xi_y\phi_\xi + \phi_\eta) + H\xi_y\phi_\xi(\xi_y\phi_\xi + \phi_\eta)\right]$$
$$+\frac{\partial}{\partial \eta}\left[\frac{1}{\xi_x}(\xi_y\phi_\xi + \phi_\eta) + H\phi_\xi(\xi_y\phi_\xi + \phi_\eta)\right] + \frac{\partial}{\partial \zeta}\left[\frac{1}{\xi_x}\phi_\zeta\right] = 0 \tag{8}$$

where *x, y,* and *z* are the *nondimensional computational coordinate in the x, y,* and *z directions,* respectively. The conditions imposed on the outer boundary of the computational domain are applied to non-reflecting boundary conditions. To calculate the flow field, the TSD equation with a finite-difference scheme based on the approximate factorization (AF) scheme combined with the Engquist-Osher method has been used. Also, the wing flow-tangency boundary condition is given as

$$\phi_z^{\pm} = f_x^{\pm} + f_t \tag{9}$$

which is *imposed at the mean plane of the wing. Here, f is described as the lift surface function* of a cross section of the wing considering angle of attack. The plus and minus superscripts indicate the upper and lower wing surfaces, respectively. Subscripts are differentiation in each direction. In a static aeroelastic analysis, the second term of right hand side is equal to zero. Thus, flow tangency condition depends on the slope of lifting surfaces only. In this study, a HALE aircraft wing is in motion on the flap direction. Hence, the first term of right hand side is not affected by the motions. The TSD equation can be applied for a thin airfoil and small angle of attack. More details of aerodynamic analysis schemes are discussed in reference [10]. According to reference [7], the aerodynamic nonlinearity due to curvature seems to be quite negligible for the static loads, even for the large deflections considered. In aerodynamic analysis, hence, the curvature effects are neglected.

Static and Dynamic Aeroelastic Analyses

To directly couple fluid flow solver with structural solver, the transformation of forces and displacements between finite element mesh and aerodynamic grid should be introduced. For the displacements mapping into CFD grid, a shape function of cubic Lagrangian element is used. To transfer forces on FEM mesh into CFD grid, inversely, work equivalent energy method is applied: The work done by virtual displacements and distributed forces is equal to that of virtual displacements and nodal forces. In the finite element model based on the large deflection beam theory, a global coordinate system differs from a deformed local coordinate system. In the aerodynamic model based on the TSD theory, on the other hand, those two coordinates are coincided with each other. According to Patil *et al.* [7], the deviation of aerodynamic forces by the curvature effects can be negligible. Thus, the curvature effects is ignored although three-dimensional aerodynamic forces are calculated by the TSD equation. Using static aeroelastic analysis, *the deformation of a wing can be obtained under the given static forces which are* gravity and steady aerodynamic forces, herein. Also, steady aerodynamic forces are changed according to the deformed shape. Thus, a converged deformation shape is calculated using the iterative procedure. At the static equilibrium state, mode shapes and natural frequencies are obtained. Using this information, the dynamic aeroelastic analyses are conducted.

Results and Discussion

Structural Analysis under Gravity Loading

Figure 1 shows the analysis model. The planform of the high-aspect-ratio wing has root chord length(*C*) of 0.0508 m, span length(*L*) of 0.4508 m and aspect ratio of about 9. The NACA 0012 airfoil section is selected for lifting surfaces. More details of the model are discussed in Tang

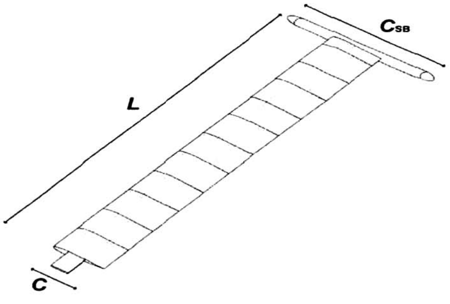
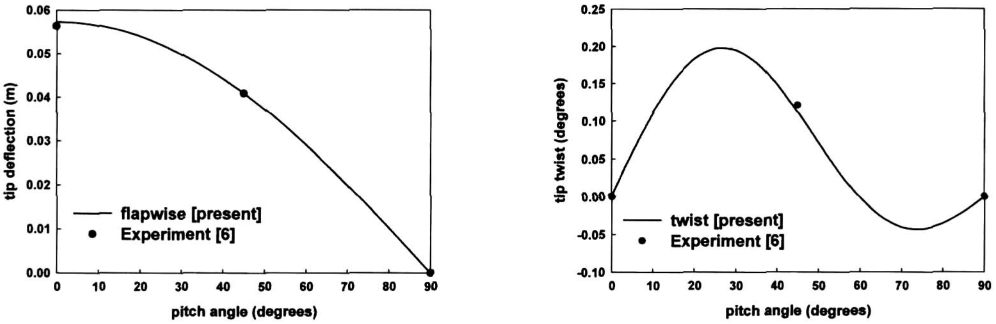


Fig. 1. Analysis wing model



(a) Flap direction (b) Twist angle

Fig. 2. Static displacement of the wing under gravity loading at the wing tip

et al. [6]. Five of four noded cubic elements are used in the structural analysis.

Figure 2 shows the comparisons of static deflections between present results and experimental data. For investigating the static flap bending deflections and twist at tip vs θ_0 under gravity effect only, three pitch angles, θ_0 = 0, 45, 90 deg, are selected. Herein, flap bending refers to bending perpendicular to the wing chord. When pitch angle is equal to zero, the flap deflections is dominant. As pitch angle increases, flap deflections decrease. From the comparisons between the present and experimental results[6], the nonlinearity effects of the large deflection are well described by the present analyses.

Figure 3 illustrates free vibration analysis results of the beam under gravity load. For the experimental result, the first flap and twist modes are presented. In the present analysis, on the other hand, the lowest three modes are calculated. The agreement for the flap and torsion seems to be reasonably good. Twist and flap frequencies tend to decrease as pitch angle increase.

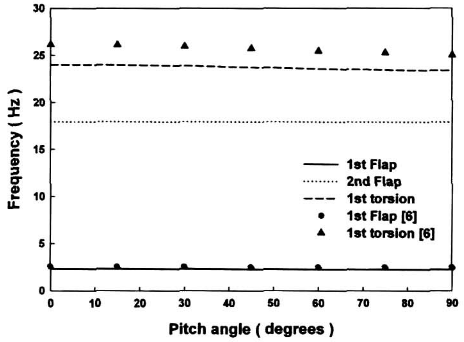
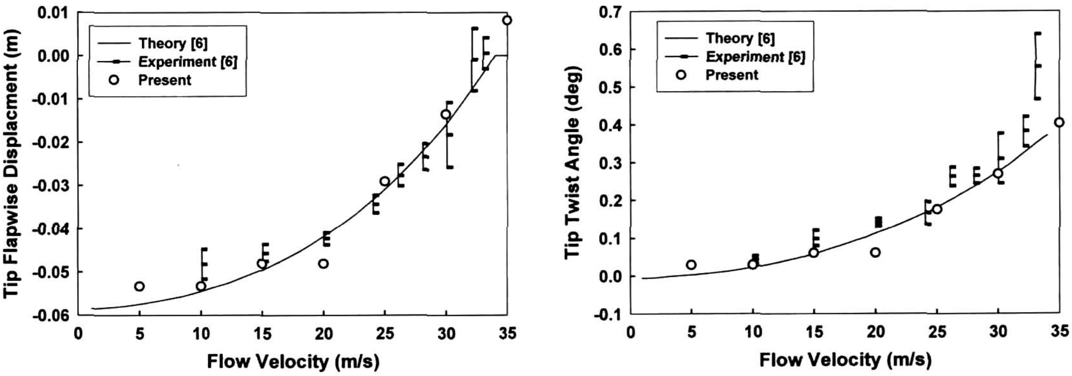


Fig. 3. Natural frequencies of the deformed wing under gravity loading

Static and Dynamic Aeroelastic Analyses

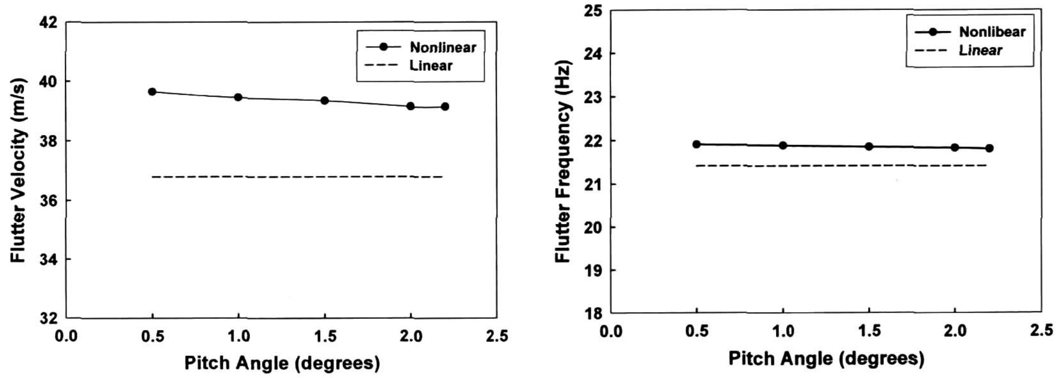
For the aerodynamic analysis, the grids contain 80 x 78 x 40 points in *x, y,* and *z* directions, respectively. Although the experimental model has a tip store, its aerodynamic effect is negligible. Thus, the tip store is ignored in the aerodynamic analysis. When the angle of attack is 1.0 deg, the flap deflection and twist angle at the tip are shown in Figs. 4a-b. In the higher velocity range, the data fluctuation increases due to greater aerodynamic interference. Due to the aerodynamic forces, both the tip deflection and twist angle increase with the increase of flow velocity. The experimental data appear have some scatter due to the turbulent aerodynamic noise, although the noise is small. Both the tip and twist deflection increase with increasing flow velocity, but the tip deflection is always negative until the flow velocity is around 34 m/s. The measure data are acquired before the onset of flutter[6]. At low speed range, there are some differences between experimental result and present analysis one in the tip flapwise displacement. It seems to be curvature effect. Generally, the agreement is good except for some points.

The balance between aerodynamic forces and the gravity force occurs at U = 34 m/s. This means that the aerodynamic forces provide sufficient lift to overcome the effect of gravity. The dynamic flutter velocities and frequencies are shown in Fig. 5. For comparison, the linear flutter boundary is also plotted. In the present analysis case, both the nonlinear flutter velocity and frequency are higher than the linear results. Because of geometrical nonlinearity, the stiffness is increased after deformation.



(a) Flapwise displacement (b) Twist angle

Fig. 4. Static aeroelastic analysis results at θ_0 = 1.0°



(a) Flutter velocity (b) Flutter frequency

Fig. 5. Flutter boundary of the present wing model at various pitch angle

Conclusions

In this study, aeroelastic analyses of a high-aspect ratio wing are performed. To consider a geometrical nonlinearity of the wing, the large deflection beam theory is introduced. Also, the transonic small disturbance theory is used for aerodynamic analyses. Conducting the structural analysis under a gravity effect, the present structural analysis program is validated. Static aeroelastic analyses are achieved using the structural solver based on the large deflection beam theory coupled with the TSD aerodynamic solver. The results using the present program agree well with experimental ones. In the case of present analysis model, the flutter velocity and frequency from nonlinear analyses are higher than those from linear analyses. For more accurate aeroelastic analysis to consider the turbulent aerodynamic effect and lead-lag mode which is important dynamic characteristics for very high-aspect-ratio wing such as a rotor blade, higher level aerodynamic theory such as Navier-Stokes theory should be considered.

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