

Guidance Synthesis to Control Impact Angle and Time

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Abstract

A new guidance synthesis for anti-ship missiles to control impact angle and impact time is proposed in this paper. The flight vehicle is assumed as a 1st order lag system to consider more practical system. The proposed guidance synthesis enhances the survivability of anti-ship missiles because multiple anti-ship missiles with the proposed synthesis can hit the target simultaneously. The control input to satisfy constraints of zero miss distance and impact angle, and the feedforward bias control input to control impact time constitute the guidance law. The former is from trajectory shaping guidance, the latter is from neural network. And particle swarm optimization method is introduced to furnish reference input and output for learning in neural network. The performance of the proposed synthesis in the accuracy of impact time and angle is validated by numerical examples.

Key Word : Impact angle, Impact time, Optimal guidance, Particle swarm optimization, Neural network

Introduction

Modern Battleships have equipped anti-air defense missile systems and powerful Close-In Weapon Systems (CIWS) against accurate intelligent anti-ship missiles. These defensive weapons reduce the survivability of anti-ship missiles. Especially, the numerous projectiles of CIWS guns increase the hit probability of an anti-ship missile. Therefore, anti-ship missile designers make an effort to develop evasive maneuvers for the anti-ship missile against the defensive weapon systems, especially against CIWS.

CIWS is a naval shipboard weapon system for detecting and destroying incoming anti-ship missiles and enemy aircraft at a short range. In general, CIWS covers some fan-shaped zone limited in range and azimuth. A salvo attack strategy is applied to take advantage of the vulnerability of CIWS in multi-target engagements. Here, salvo attack means that several missiles hit the same target simultaneously. The salvo attack strategy increases the kill probability of anti-ship missiles remarkably. To apply the salvo attack, each missile is able to home to the target along the predetermined impact angle at a given impact time.

There have been a lot of studies on impact angle control laws. Generalized formulation of energy minimization optimal guidance laws for constant speed missiles with an arbitrary system order was proposed by Ryoo et al. [1]. Kim and Grider [2] proposed an optimal impact angle control guidance law in the vertical plane for re-entry vehicle. It is hard to find studies on impact time control. The Impact-Time-Control Guidance (ITCG) for anti-ship missiles was proposed by Jeon et al. [3], recently. Tahk et al. [4] proposed a method to estimate time-to-go precisely. While the studies on impact angle control are abundant and the studies on impact time are few, there are rare studies on proper

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guidance laws to satisfy impact angle and time together. Lee et al. [5] proposed the Impact-Time-and-Angle-Control-Guidance (ITACG) using a virtual state.

In this paper, we proposed a guidance synthesis to control both impact time and angle. First, we obtain the optimal guidance command to satisfy only impact angle constraint under the assumptions that the anti-ship missile is a 1st order lag system and the missile has constant velocity. Then, we introduce the bias control input to control impact time and make database of the bias input according to various flight conditions using an optimization method. Particle Swarm Optimization (PSO) is considered as optimization method and the cost function of PSO is defined to satisfy terminal constraints for zero miss distance, impact time, and angle. Neural network is considered for the application of the adaptive feedforward bias input in real time. Back Propagation Network (BPN) method is used for neural network. Actually, time lag should be considered because real system is lag system. Unexpected effects can be caused when the lag is not considered in the guidance. However, the biased-PNG form of the lag system for impact angle control is so intricate that it is very difficult or impossible to find the analytic bias input for impact time control. Thus, the method proposed in our study provides a practical and robust guidance law to control the impact time and angle. The performance of proposed method is verified by numerical example.

This paper is organized as follows. In the next section, kinematics for anti-ship missiles is first derived. Then, the guidance synthesis using optimal guidance law and neural network is developed. Simulation for salvo attack of two anti-ship missiles is presented to verify the performance of the proposed method. Conclusions are provided in the final section.

Kinematics for Anti-ship Missile

Consider a planar homing guidance geometry for a stationary target shown in Figure 1. Here, $M(t_0)$, $M(t)$; and $M(t_f)$ denote the position of the missile at each time. V , γ , and γ_f represent the missile velocity, the flight path angle, and the predetermined impact angle, respectively. And, the subscript 0 and f denote respectively the initial and terminal time. I-frame is an inertial reference frame to define the position and velocity of the missile and target. G-frame is defined as a guidance frame: its center is fixed to the target and the rotation angle of G-frame with respect to I-frame is defined by the impact angle γ_f .

The equations of motion in the inertial reference frame are given by

$$\begin{aligned}\dot{X} &= V \cos \gamma \\ \dot{Y} &= V \sin \gamma \\ \dot{\gamma} &= a / V\end{aligned}\quad (1)$$

where X , Y are the position of the missile in I-frame and a is the missile's acceleration normal to the velocity vector to change γ .

The boundary conditions are as follows:

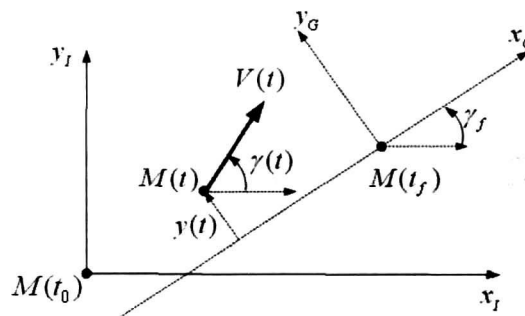


Fig. 1. Homing guidance geometry

$$X(t_0) = X_0, Y(t_0) = Y_0, \gamma(t_0) = \gamma_0 \quad (2)$$

$$X(t_f) = X_f, Y(t_f) = Y_f, \gamma(t_f) = \gamma_f, t_f = t_d \quad (3)$$

where t_d is the designated time-to-go, which is called impact time in this paper.

The error from linearization around the target in G-frame is smaller than the error in I-frame. Therefore, it is proper to describe the motion of the missile in G-frame when we linearize an equation of motion. The relationship of the positions between I-frame and G-frame is obtained as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos \gamma_f & \sin \gamma_f \\ -\sin \gamma_f & \cos \gamma_f \end{bmatrix} \begin{bmatrix} X - X_f \\ Y - Y_f \end{bmatrix} \quad (4)$$

where $[x, y]^T$ is the missile's position in G-frame.

From the equation (1) and (4), the equations of motion of the missile in G-frame are obtained as

$$\begin{aligned} \dot{x} &= V \cos \sigma \\ \dot{y} &= V \sin \sigma \\ \sigma &= a/V \end{aligned} \quad (5)$$

where $\sigma \equiv \gamma - \gamma_f$. Under the assumption that V is constant and σ is small, we can linearize equation (5) as

$$\begin{aligned} \dot{x} &\approx V \\ \dot{y} &\approx V \sigma \\ \sigma &= a/V \end{aligned} \quad (6)$$

Since there is lag in the real system, lag should be considered for real application of a guidance command to the system. If lag is not considered, the performance of the guidance is degraded and the system can be unstable. A 1st order lag system will approximate a real missile, i.e.,

$$\dot{a} = \frac{1}{\tau}(u - a) \quad (7)$$

where u, τ denote the guidance command and the time constant, respectively.

Equations (6) and (7) can be rewritten using the independent parameter, downrange x . Under the assumption that the missile is a 1st order lag system and σ is small, the equations are obtained as

$$z'(x) = Fz(x) + Gu(x) \quad (8)$$

Here, the prime(') means the derivative with respect to x . The boundary conditions become

$$z(x_0) = z_0, z(x_f) = z_f \quad (9)$$

And the state vector, the control input, and matrices are respectively given by

$$\begin{aligned} z &= [y(x) \quad \sigma(x) \quad a(x)]^T, \\ F &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1/V^2 \\ 0 & 0 & -1/\tau V \end{bmatrix} \\ G &= \begin{bmatrix} 0 \\ 0 \\ 1/\tau V \end{bmatrix} \end{aligned} \quad (10)$$

Analytical Guidance Law for Impact Angle Constraint

In this section, the guidance law to control both impact angle and homing constraint to the

target is derived. It is very difficult to solve the optimal control problem satisfied homing, impact angle, and impact time constraints because there is no degree of freedom to control impact time in the optimal homing problem.

To take account the optimal homing problem which satisfy impact angle constraint, let us consider the following optimal control problem: Find control input $u(x)$ which minimizes

$$J = \frac{1}{2} \int_{x_o}^{x_f} u(x)^2 dx \quad (11)$$

subject to equation (8) with equality constraints to satisfy homing and impact angle as

$$y(x_f) = y_f, \quad \sigma(x_f) = \sigma_f \quad (12)$$

Then, the Hamiltonian of the problem is given by

$$H = \frac{1}{2} u^2 + \lambda_y \sigma + \lambda_\sigma \frac{a}{V} + \lambda_a \frac{V}{\tau} (u - a) \quad (13)$$

Costates equations are as follows:

$$(\lambda')^T = [\lambda'_y \quad \lambda'_\sigma \quad \lambda'_a]^T = -F^T \lambda^T \quad (14)$$

And, the terminal conditions of the costate equations are

$$\lambda_{yf} = v_y, \quad \lambda_{\sigma f} = v_\sigma, \quad \lambda_{af} = 0 \quad (15)$$

From the necessary condition for optimality, the optimal control is determined by solving $\partial H / \partial u = 0$. Therefore, the optimal control can be obtained as

$$u^* = \Delta^{-1} \{ C_y y + C_\sigma \sigma + C_{y_f} y_f + C_{\sigma_f} \sigma_f + C_a a \} \quad (16)$$

where

$$C_y = 2 \frac{V}{\tau} \left\{ \left(3x_{go} \frac{V^2}{\tau^2} + 6 \frac{V^3}{\tau} \right) \exp\left(-2 \frac{x_{go}}{V\tau}\right) + \left(3x_{go}^2 \frac{V}{\tau^3} + 6x_{go} \frac{V^2}{\tau^2} - 12 \frac{V^3}{\tau} \right) \exp\left(-\frac{x_{go}}{V\tau}\right) + \left(3x_{go}^3 \frac{V}{\tau^3} - 9x_{go} \frac{V^2}{\tau^2} + 6 \frac{V^3}{\tau} \right) \right\} \quad (17)$$

$$C_\sigma = 2 \frac{V}{\tau} \left\{ \left(3x_{go}^2 \frac{V^2}{\tau^2} + 9x_{go} \frac{V^3}{\tau} + 6V^4 \right) \exp\left(-2 \frac{x_{go}}{V\tau}\right) + \left(x_{go}^3 \frac{V}{\tau^3} + 3x_{go}^2 \frac{V^2}{\tau^2} - 12x_{go} \frac{V^3}{\tau} - 12V^4 \right) \exp\left(-\frac{x_{go}}{V\tau}\right) + \left(2x_{go}^3 \frac{V}{\tau^3} - 6x_{go}^2 \frac{V^2}{\tau^2} + 3x_{go} \frac{V^3}{\tau} + 6V^4 \right) \right\} \quad (18)$$

$$C_{y_f} = 2 \frac{V}{\tau} \left\{ \left(-3x_{go} \frac{V^2}{\tau^2} - 6 \frac{V^3}{\tau} \right) \exp\left(-2 \frac{x_{go}}{V\tau}\right) - \left(3x_{go}^2 \frac{V}{\tau^3} + 6x_{go} \frac{V^2}{\tau^2} - 12 \frac{V^3}{\tau} \right) \exp\left(-\frac{x_{go}}{V\tau}\right) - \left(3x_{go}^3 \frac{V}{\tau^3} - 9x_{go} \frac{V^2}{\tau^2} + 6 \frac{V^3}{\tau} \right) \right\} \quad (19)$$

$$C_{\sigma_f} = 2 \frac{V}{\tau} \left\{ \left(-3x_{go}^2 \frac{V^2}{\tau^2} - 6V^4 \right) \exp\left(-2 \frac{x_{go}}{V\tau}\right) + \left(2x_{go}^3 \frac{V}{\tau^3} + 3x_{go}^2 \frac{V^2}{\tau^2} + 12V^4 \right) \exp\left(-\frac{x_{go}}{V\tau}\right) + \left(x_{go}^3 \frac{V}{\tau^3} - 3x_{go}^2 \frac{V^2}{\tau^2} + 3x_{go} \frac{V^3}{\tau} - 6V^4 \right) \right\} \quad (20)$$

$$C_a = 2 \frac{V}{\tau} \left\{ \left(2x_{go}^3 \frac{1}{\tau^2} + 9x_{go}^2 \frac{V}{\tau} + 12x_{go} V^2 \right) \exp\left(-2 \frac{x_{go}}{V\tau}\right) + \left(2x_{go}^3 \frac{1}{\tau^2} - 24x_{go} V^2 \right) \exp\left(-\frac{x_{go}}{V\tau}\right) + \left(2x_{go}^3 \frac{1}{\tau^2} - 9x_{go}^2 \frac{V}{\tau} + 12x_{go} V^2 \right) \right\} \quad (21)$$

$$\Delta = \left(2x_{go}^3 \frac{V}{\tau^3} + 12x_{go}^2 \frac{V^2}{\tau^2} + 24x_{go} \frac{V^3}{\tau} + 12V^4 \right) \exp\left(-2 \frac{x_{go}}{V\tau}\right) + \left(4x_{go}^3 \frac{V}{\tau^3} + 12x_{go}^2 \frac{V^2}{\tau^2} - 24x_{go} \frac{V^3}{\tau} - 24V^4 \right) \exp\left(-\frac{x_{go}}{V\tau}\right) - \left(x_{go}^4 \frac{1}{\tau^4} - 6x_{go}^3 \frac{V}{\tau^3} + 12x_{go}^2 \frac{V^2}{\tau^2} - 12V^4 \right) \quad (22)$$

where $x_{go} = x_f - x$.

The solution to the optimal control problem given in (16)~(22) satisfies the constraints to control the impact angle and to guarantee homing to the target. However, it cannot control the impact time.

Reference Input and Output for Learning

To apply neural network, we must have the reference input and output for learning. In this paper, an optimization method is introduced to make the reference input and output. Since we want that the proposed guidance law satisfies the impact angle and impact time constraints, the cost function to find the adaptive u_b is considered in the optimization procedure

$$\tilde{J} = \frac{q_t}{2} (t_d - t_f)^2 + \frac{1}{2} [y(x_f) - y_f \sigma(x_f) - \sigma_f] Q \begin{bmatrix} y(x_f) - y_f \\ \sigma(x_f) - \sigma_f \end{bmatrix} \quad (23)$$

where $q_t = 5^2$ is the weighting factor of the impact time constraint, and $Q = \text{diag}.[1 \ 1]$ is the weighting matrix of the state vector.

PSO is introduced for the optimization algorithm in this paper. PSO is the type of probabilistic search algorithm like as evolutionary algorithm (EA) [7]. However, reproduction and mutation to produce the next generation are not needed in the PSO algorithm. PSO initially produces a random population distributed throughout the search space and generates the next population using current population's cost. Each population knows the best solution of the current group and the best solution of the whole optimization process until the current step. The group best solution and historical best solution attracts the motion of the group. Since it always remembers the best position it is an elitist optimization process. The algorithm of PSO is as follows [7]:

Algorithm

1. Initialize a population of particles with random positions and velocities.
2. For each particle, evaluate the desired optimization fitness function.
3. Compare particle's fitness evaluation with particle's overall previous best, pbest(pid). Exchange the particle's fitness value and location with pbest if it is better.
4. Compare particle's fitness evaluation with population's overall previous best, gbest(pgd). Exchange particle's fitness value and location with gbest if it is better.
5. Change the velocity and position of the particle according to the following equations [paper of shi].

$$\begin{aligned} v_{id}(k+1) &= wv_{id}(k) + c_1 r_1 (p_{id} - x_{id}(k)) + c_2 r_2 (p_{gd} - x_{id}(k)) \\ x_{id}(k+1) &= x_{id}(k) + v_{id}(k+1) \end{aligned}$$

6. Go to the step 2 and repeat until criterion is met.

where x_{id} is the position of particle i , v_{id} is the corresponding velocity vector, r_1 and r_2 are independent random numbers between 0 and 1, and w is called inertia weight, and c_1 , c_2 are called cognitive and social parameter, respectively.

Adaptive Guidance Law Using Neural network

The guidance command u_c applied to the missile to satisfy constraints for homing, impact angle, and impact time is derived as

$$u_c = u^* + u_b = \Delta^{-1} (C_y y + C_\sigma \sigma + C_{y_f} y_f + C_{\sigma_f} \sigma_f + C_a a) + k \quad (24)$$

where $u_b = k$. Here, the constant control input u_b , defined the bias control input in this paper for

convenience, is introduced to the control the impact time.

In this section, neural network is considered to application of u_b . Moreover, the structure and the detailed algorithm of neural network are introduced.

Neural Network

The varying flight condition of the missile in homing phase requires an adaptive u_b . However, classical optimization methods are not applicable to practical real time problems due to their computation load. The optimal bias control input u_b may be applied to the missile by gain scheduling, fuzzy law or neural network in real time. In this paper, neural network is introduced for implementation of the adaptive u_b .

Back propagation network (BPN)[6] is considered as neural network. The neural network structure is taken 5-layer network and the number of hidden nodes is seven, since the gain of the system has high nonlinearity. In this paper, The inputs of BPN are x , y , α , and t_{go} and the output is u_b , i.e.,

$$z^0 = X_{\in} = [x \ y \ \sigma t_{go}], \quad z^N = y = u_b \quad (25)$$

In our BPN algorithm, The relationship between input and output is given by

$$\begin{aligned} net^i &= W^i z^i + \beta^i \\ z^i &= f(net^i), \quad i = 1, \dots, N \end{aligned} \quad (26)$$

where W^i denotes the weight matrix and β^j is the connection to the bias node. The activation function vectors are as follows :

$$f(net^i) = \frac{1}{1 + e^{-net^i}} \quad (27)$$

And, weight modification algorithm to train the network weights is given by

$$\Delta W^i = \eta \delta_i z^{i-1} \quad (28)$$

$$\Delta \beta^i = \eta \delta_i \quad (29)$$

where η is the learning rate, and the distance from the output layer to the hidden layer δ_i are derived as

$$\delta_N = (y - z^N) f'(z^N) \quad (30)$$

$$\delta_i = \delta_{i+1} W^{i+1} f'(z^i) \quad (31)$$

Here, $N=4$ because 5-layer neural network is adapted in this paper and

$$f'(z^j) = z^j(1 - z^j) \quad (32)$$

Then, W^i , β^i in the training process for learning are updated as

$$W^i = W^i + \Delta W^i \quad (33)$$

$$\beta^i = \beta^i + \Delta \beta^i \quad (34)$$

Numerical Examples

To verify the proposed guidance synthesis, the homing problem of two anti-ship missiles is considered in this section. The simulation condition of two missiles is represented in Table 1. As

shown in Table 1, the initial states and the initial time of two anti-ship missiles are differently given to check that anti-ship missile can hit the target simultaneously. Missile 2 starts position 20 sec later than Missile 1.

If the optimal guidance command with impact angle constraint is only applied to the anti-ship missiles, they can hit the target with the given impact angles. However, t_f of each the missile is calculated as 103.1 sec and 103.7 sec, respectively. Hence the homing missiles guided by u^* cannot control impact time.

The simulation results are depicted in Figures 2 - 5. Terminal time and terminal states are shown in Table 2. From these results, the proposed guidance synthesis can intercept the target with given impact angle and time simultaneously, although they start at different points and times. The

Table 1. Simulation condition

	t_0 (sec)	$(X, Y)(t_0)$ (km)	$\gamma(t_0)$ (deg)	X_f (km)	Y_f (km)	γ_f (deg)	t_d (sec)
Missile 1	0	(0, 0)	40	10	0	0	140
Missile 2	20	(0, 2)	20	10	0	0	140

Table 2. Terminal time and terminal state

	$X(t_f)$ (km)	$Y(t_f)$ (km)	$\gamma(t_f)$ (deg)	t_f (sec)
Missile 1	10	0	-0.003	140.2
Missile 2	10	0	0.132	140.2

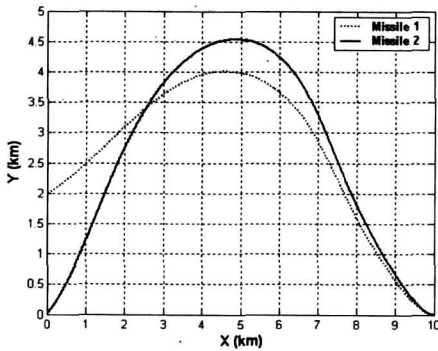


Fig. 2. Trajectory of the guidance synthesis

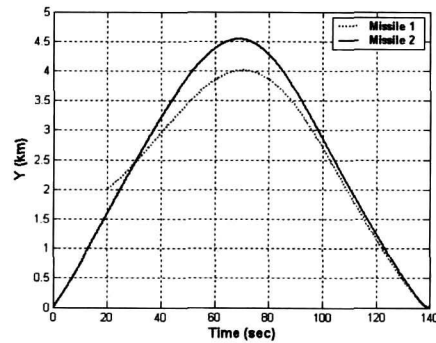


Fig. 3. Cross range profile

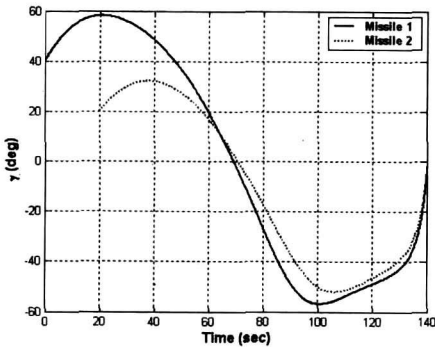


Fig. 4. Flight path angle time history

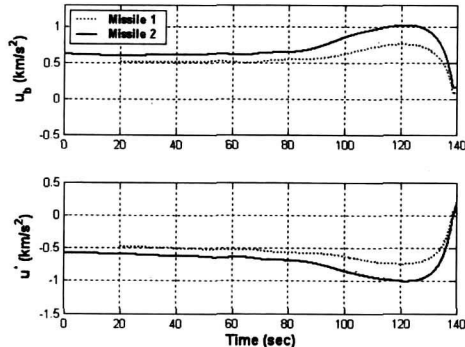


Fig. 5. Guidance command profile

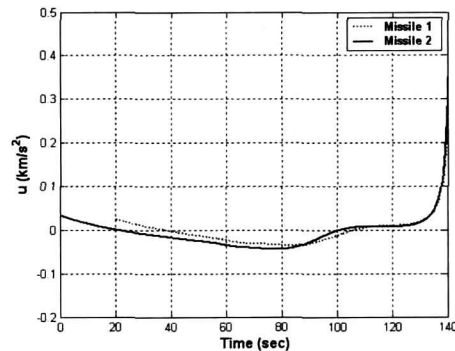


Fig. 6. Guidance command of the proposed guidance synthesis

change of the downrange and cross range of two missile is smooth, but flight path angles of both anti-ship missiles are rapidly changed around the target as shown in Figs. 3 and 4. This results can be expected from the Fig. 5 and 6. As shown in Fig. 5 and 6, the control input of the proposed guidance synthesis tries to satisfy the flight time constraint during the initial flight, while it tries to meet the impact angle constraint as the missile approaches the target.

Conclusions

In this paper, the homing problem of anti-ship missile is addressed. To increase the kill probability of missile against ship-borne defense missile systems, new guidance synthesis is proposed. The Biased PNG(BPN) and neural network constitute the proposed guidance synthesis to satisfy impact angle, impact time, and homing constraints. Particle swarm optimization is considered to make reference input and output for learning in neural network. The performance of the proposed guidance synthesis is verified via numerical simulations. This guidance synthesis achieves practical and efficient salvo attack for anti-ship missiles. It may also find applications in cooperative flight mission of UAVs

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