Equivalent Plate Modeling of the Wing-Box Structure with Control Surface

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Abstract

In this study, the equivalent plate model is developed using a finite element method(FEM) based on the first order shear deformation theory(FSDT). The substructure synthesis method is used to consider the control surface. For the verification of the equivalent model, the results of free vibration analysis are compared with the ones of 3D wing structure modeled by using the MSC/NASTRAN.

Key Word : Equivalent plate, Wing structure, Control surface

Introduction

In the early design stage, it is difficult to build an exact analytical model for the real structures. So, we need to choose an approximate model using a finite element method to know the structural characteristics. However, the finite element modeling of a real 3D structure is a difficult and time-consuming task. Thus equivalent modeling methods are commonly used in the practical design process [1].

Generally, there are two kinds of equivalent modeling methods such as the equivalent beam modeling and equivalent plate modeling. The equivalent beam modeling is quite effective for the high aspect ratio and isotropic material wings. However, for composite materials and low aspect ratio wings, beam models can be inadequate to get reasonable solutions. The sensitivity of composite beams to root boundary conditions, warping effects and the cordwise bending of low aspect ratio wings are some of the reasons for limitations to apply this model[2]. Thus, equivalent plate modeling is more effective than the equivalent beam modeling for predicting the characteristics of structures.

There are some plate theories which can be applied to analyze the equivalent plate. One is classical plate theory(CPT). It is based on the Kirchhoff-Love hypothesis, that is a straight line normal to the plate middle surface remains straight and normal during the deformation process. This theory works well for truly thin plates. But for thick plates, they tend to overestimate the stiffness of the plate because the effects of through-the-thickness shear deformation are ignored[3,4]. The other is the first order shear deformation theory(FSDT), based on the Reissner-Mindlin model which assumes a uniform transverse shear strain. The FSDT is the most widely used theory for thick plates owing to its simplicity and its low requirement for computation capacity. For more accurate results or more realistic local distributions of the transverse strain and stress, one should use the higher order shear deformation theory(HSDT)[5] or more advanced theory[6].

In this study, the equivalent plate model is developed using a finite element method(FEM)

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based on the first order shear deformation theory(FSDT). The mass and stiffness matrices of spar, rib, and skin are calculated by using equivalent integration. Substructure synthesis method is used to assemble mass and stiffness matrices of equivalent wing and the plate control surface. For the verification of the equivalent model, the results of free vibration analysis are compared with the ones of 3D wing model using MSC/NASTRAN.

Assumptions and Formulations

Constitutive Matrices

Generally, wing structures are composed of skin, ribs and spars, as can be seen in Fig. 1. Figure 2 shows the cross section of a wing where spar and rib have the same shape of cross section which is composed of cap and web. Each component of the wing structure has different values of constitutive matrix.

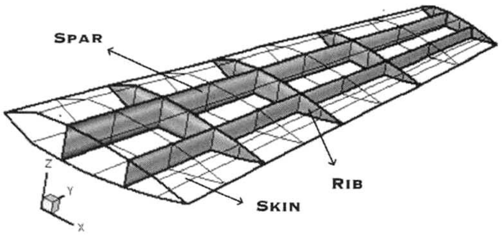


Fig. 1. 3D wing components

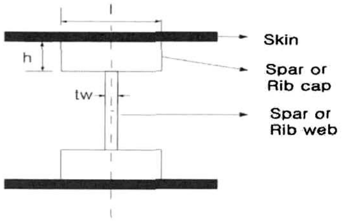


Fig. 2. Cross section of wing

In the case of skin, the structure can be represented by shell structures. Therefore the constitutive matrix of skin, $[D]_{skin}$ can be expressed as follows,

$$[D]_{skin}=\begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 & 0 & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 & 0 & 0 \\ 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & kG & 0 \\ 0 & 0 & 0 & 0 & kG \end{bmatrix} \tag{1}$$

where, E is Young's modulus, ν is Poisson's ratio, G is shear modulus and k is shear correction factor which is 5/6 for thick plate. but for thin skin, the kG terms should be neglected.

In the case of spar and rib, they are composed of cap and web. The cap structures are assumed as a 1D bar. However, in the case of web structure, the shear deformation in transverse direction should be considered. As a result, the constitutive matrices of cap and web can be expressed as Eqs.(2) and (3), respectively.

$$[D]_{cap}=diag[E\ 0\ 0\ 0\ 0] \tag{2}$$

$$[D]_{web}=diag[E\ 0\ 0\ 0\ G] \tag{3}$$

Equivalent Integration

The mass and stiffness matrices are expressed as an integral form. To get the equivalent mass and stiffness matrices of 3D wing structure, equivalent integration is applied for each component. Total mass and stiffness matrices are calculated from the summation of each matrix of wing components.

$$[K]_{total} = [K]_{skin} + [K]_{spar} + [K]_{rib} \tag{4}$$

$$[M]_{total} = [M]_{skin} + [M]_{spar} + [M]_{rib} \tag{5}$$

The detail information can be seen in the reference [7].

Equivalent integration of skin

Figure 3 shows the skin component of the wing. As can be seen, two skin elements, upper and lower skins, belong to an equivalent element. Therefore, the equivalent mass and stiffness matrices of skin can be calculated from the summation of upper and lower skin. Figure 4 illustrates an element of the upper skin.

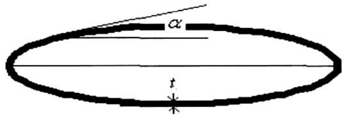


Fig. 3. Skin of the wing

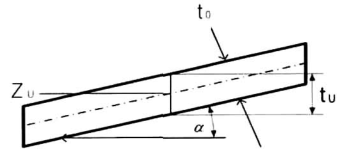


Fig. 4. Skin element

Eq.(6) is the integration for calculating the equivalent mass or stiffness matrix, where the coordinates (x, y) is changed to (ξ, η) to compute the integration numerically using Gaussian quadrature method.

$$\iiint_V F(x, y, z)\,dV = \int_{-1}^{1}\int_{-1}^{1}\left(\int_{Z_L-\frac{1}{2}t_L}^{Z_L+\frac{1}{2}t_L} F\cdot|J|\,dz + \int_{Z_U-\frac{1}{2}t_U}^{Z_U+\frac{1}{2}t_U} F\cdot|J|\,dz\right) d\xi\, d\eta \tag{6}$$

where, the subscript L means lower skin, subscript U means upper skin, Z_U, Z_L, $|J|$ are the z values of the mid point of the upper and lower skin elements, Jacobian owing to the transformation of the coordinates, respectively. $t_{L,U}$ is the thickness of the skin in z direction. From Fig. 4, the thickness of upper and lower skins can be calculated as follows,

$$t_{L,U} = t_0\sqrt{(1+\tan^2\alpha_{L,U})} \tag{7}$$

Equivalent integration of spar

The spar is composed of cap and web. Each of the components can be integrated by using Eqs. (8) and (9). Figure 5 shows an equivalent element of a wing with spar and rib position related with the corresponding 3D wing model. And the half of the cross section of the wing is described in Fig. 6.

The total equivalent mass and stiffness matrices of spar cap can be calculated from the summation of the upper and lower cap.

$$\iiint_V F(x,y,z)\,dV = \int_{-1}^{1}\int_{-1}^{1}\left(\frac{l}{c}\right)\left(\int_{Z_l+\frac{1}{2}t_l}^{Z_l+\frac{1}{2}t_l+h} + \int_{Z_l-\frac{1}{2}t_l-h}^{Z_l-\frac{1}{2}t_l}\right) F\left\{x\left(\left(\frac{l}{c}\right)\xi+\xi_s(\eta),\eta\right), y\left(\left(\frac{l}{c}\right)\xi+\xi_s(\eta),\eta\right), z\right\}|J|\,dz\,d\xi\,d\eta \tag{8}$$

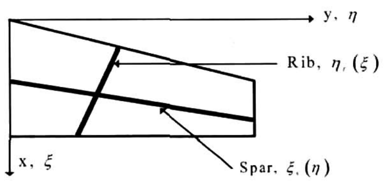


Fig. 5. An equivalent element of the wing

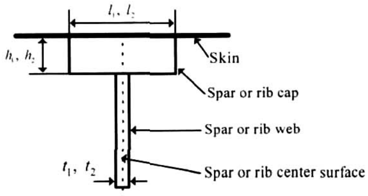


Fig. 6. Cross section of spar

Spar web can be integrated by using similar method of the cap. But there is one web in the equivalent element. Thus it is calculated as follows,

$$\iiint_V F(x,y,z)dV = \int_{-1}^{1}\int_{-1}^{1}\left(\frac{t_w}{c}\right)\int_{Z_L+\frac{1}{2}t_L+h}^{Z_U-\frac{1}{2}t_U-h} F\left\{x((\frac{t_w}{c})\xi+\xi_s(\eta),\eta),y((\frac{t_w}{c})\xi+\xi_s(\eta),\eta),z\right\}|J|dzd\xi d\eta \qquad (9)$$

where $\xi_s(\eta)$ is the position function of the spar, t_w is thickness of the spar web and c is chord length of the element which is calculated from Eq.(10).

$$c=\frac{1}{2}c_0(1-\eta)+\frac{1}{2}c_1(1+\eta) \qquad (10)$$

where c_0 is chord length of root side and c_1 is chord length of tip side.

Equivalent integration of rib

The cross section shape of the rib is the same as the spar's. Therefore, the equivalent integration is similar to the case of spar's. But the direction of the rib is in ξ direction which means that the position function of the rib is function of ξ .

The equivalent mass and stiffness matrices of rip cap are calculated by Eq. (11).

$$\iiint_V F(x,y,z)dV = \int_{-1}^{1}\int_{-1}^{1}\left(\frac{l}{s}\right)\left(\int_{Z_L+\frac{1}{2}t_L}^{Z_L+\frac{1}{2}t_L+h} + \int_{Z_U-\frac{1}{2}t_U-h}^{Z_U-\frac{1}{2}t_U}\right)F\left\{x(\xi,(\frac{l}{s})\eta+\eta_r(\xi)),y(\xi,(\frac{l}{s})\eta+\eta_r(\xi)),z\right\}\cdot|J|dzd\eta d\xi \qquad (11)$$

where s is a spanwise length in an element and $\eta_r(\xi)$ is a position function of rib.

The equivalent mass and stiffness matrices of rib web are calculated by Eq. (12).

$$\iiint_V F(x,y,z)dV = \int_{-1}^{1}\int_{-1}^{1}\left(\frac{t_w}{s}\right)\int_{Z_L+\frac{1}{2}t_L+h}^{Z_U-\frac{1}{2}t_U-h} F\left\{x(\xi,(\frac{t_w}{s})\eta+\eta_r(\xi)),y(\xi,(\frac{t_w}{s})\eta+\eta_r(\xi)),z\right\}\cdot|J|dzd\eta d\xi \qquad (12)$$

After getting the stiffness and mass matrices of equivalent model and plate control surface, separately, the substructure synthesis method is applied to assemble these two matrices [8].

Results

A low aspect ratio wing model which has 6 spars and 9 ribs is used to validate the present developed code. The geometry of the model is described in Fig.7 and Fig.8. The material properties are given as follows: Young's modulus is 72GPa , density is $2700kg/m^3$, Poisson's ratio is 0.33 , Young's modulus of control surface is 220GPa, density and Poisson's ratio of control surface are $2700kg/m^3$, 0.33 respectively. And the results of free vibration analysis are compared with the ones of 3D wing model using MSC/NASTRAN. In the 3D wing model, CQUAD4 elements are used for skin and web. CBAR elements are used for cap. The total number of elements used in this model is 430. In the case of equivalent 2D model, 80 elements which are 9-node quadrilateral elements are used. In these models, torsional spring is not considered between the wing and control surface. For this reason, rigid body mode is obtained. As can be seen in Table 1, the percent errors of the natural frequencies are lower than 5%. The 8th mode of the 3D model is coupled with bending , inplane and control surface bending mode. But in the case of equivalent 2D model, inplane mode appears. But the higher mode is relatively less important than the lower mode, especially in the preliminary design stage. Except the 8th mode, the equivalent plate wing model can predict well the frequencies and mode shapes of 3D wing model using MSC/NASTRAN.

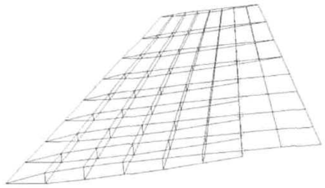


Fig. 7. Wing model

- 7 spar & 9 rib
- Half Span : 2.125 m
- Root Chord : 2.201 m
- Tip Chord : 0.680 m
- Sweep angle at 1/4 chord : 24°
- Control surface thickness : 0.012 m

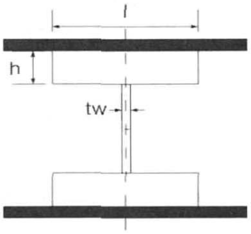
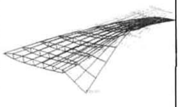
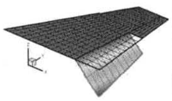
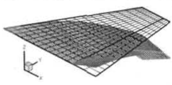
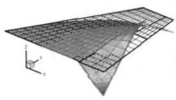
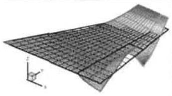
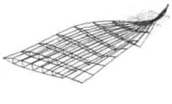
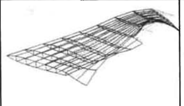
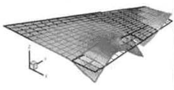
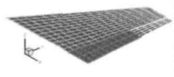
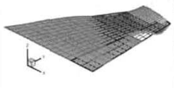


Fig. 8. Cross section of the wing

- Skin thickness : 0.002 m
- l = 0.002 m
- h = 0.0005 m
- tw = 0.001 m

Table 1. Comparison between equivalent plate model and 3D wing model using MSC/NASTRAN

	Rigid body mode	1st mode	2nd mode	3rd mode	4th mode
3Dwingmodel (NASTRAN)	 0Hz	 21.00Hz	 54.85Hz	 70.37Hz	 88.39Hz
Equivalent model	 0Hz	 20.16Hz	 54.92Hz	 71.79Hz	 88.72Hz
Error	0%	4.00%	0.12%	2.01%	0.37%

	5th mode	6st mode	7th mode	8th mode	9th mode
3Dwingmodel (NASTRAN)	 130.28Hz	 146.59Hz	 177.03Hz	 224.90Hz	 235.55Hz
Equivalent model	 126.17Hz	 153.68Hz	 169.89Hz	 193.59Hz	 242.16Hz
Error	3.15%	4.84%	4.03%	Inplanemode	2.81%

Conclusions

Equivalent plate modeling code is developed. The accuracy of present developed code is verified by comparing the results with those of 3D wing model using MSC/NASTRAN. It is found that the present code is efficient to predict the natural frequencies and mode shapes of 3D wing model, using small number of elements. Moreover the design of wing can be easily modified with some changes of parameters. Therefore, the equivalent modeling developed in this research can be efficiently used in the early aircraft design stage.

Acknowledgement

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