

# Transonic Flutter Suppression of the 2-D Flap Wing with External Store using CFD-based Aeroservoelasticity

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## Abstract

An analysis procedure for the combined problem of control algorithm and aeroelastic system which is based on the computational fluid dynamics(CFD) technique has been developed. The aerodynamic forces in the transonic region are calculated from the transonic small disturbance(TSD) theory. An linear quadratic regulator(LQR) controller is designed to suppress the transonic flutter. The optimal control gain is estimated by solving the discrete-time Riccati equation. The system identification technique rebuilds the CFD-based aeroelastic system in order to form an adequate system matrix which involved in the discrete-time Riccati equation. Finally the controller, that is constructed on the basis of system identification technique, is used to suppress the flutter phenomenon of the airfoil with attached store. This approach, that is, the CFD-based aeroservoelasticity design, can be utilized for the development of effective flutter controller design in the transonic region.

**Key Word** : Transonic small disturbance, System identification, Flutter suppression

## Introduction

Aeroelasticity, which is considered to be important in the aircraft design process, is the field of studying the interaction between the aerodynamic forces and the structural elastic forces. The aeroelastic problems have come into the spotlight recently because of the need for the development of lightweight and flexible aircrafts. These fluid-structure coupled systems, that is, aeroelastic system typically experience buffeting, divergence and flutter etc. Among them, the flutter, the dynamic instability phenomenon, is the most severe and critical problem to the structure. From those self-excited vibration, fatigue can be accumulated on aircraft structure and sudden structural failure can be occurred. Therefore, preventing this catastrophic incident has become an important issue and the aeroelastic problems have to be thoroughly analyzed and prepared.

To increase the boundary of stability of flutter, many researchers, conventionally, have changed the geometrical properties of a wing; for example, mass, stiffness and shape etc. Besides active control technology(ACT) or smart materials such as piezoelectric materials have been applied to suppress the instability actively by changing the aerodynamic characteristics. Added to that, several supplementary external attachments can be loaded on the wing and varied in combination in accordance with aircrafts own missions and working environment. These flight conditions can result in quite different aeroelastic responses. To be adaptable for many cases, passive control is

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not available to cover the broadband of uncertainties. Moreover the ACT which uses, especially, the trailing edge flap as an actuator, is appropriate to follow the requirements of multidisciplinary design process.

However, for the case of cruising in transonic region, the basic assumption of predictable flutter boundary is no longer applied, that is, due to the nonlinear aerodynamic characteristics on the wing, the flutter stability boundary cannot be analyzed linearly. This phenomenon has been occurred by the existence of the shock wave on the wing, in which the aerodynamic characteristics are totally changed. Therefore, the flutter boundary is plotted nonlinearly with respect to the Mach number. This is called as 'transonic dip' phenomenon. For the cases of linear aerodynamic forces, many researchers had performed the analyses and control of flutter in the frequency and time domain by approximating the aerodynamic forces as a function of structural responses. But, to predict the nonlinearity of the transonic flow, Batina et al. [2] approximated the aerodynamic forces in the complex  $s$ -plane. However, due to the advancement in computer performance recently, the previously approximated aerodynamic forces can be simulated with further accuracy by using CFD technique. The CFD technique can reckon the aerodynamic nonlinear effects more precisely than the frequency domain approach does and can examine the influence of actuating motion directly in the time domain.

However, the problem lies in combining the control scheme with the CFD aeroelastic system. The aeroelastic dynamic equations are integrated in the time domain by steps. That makes the external aerodynamic forces term being in the right-hand side of dynamic equations, in which, however, the control force term is already exist, at each steps. To determine the transfer function of aerodynamic force term with respect to the structural responses, Friedmann et al. [3] adopted the system identification technique in which the method transforms the aeroelastic equations into the simple form of state space equation. Djayapertapa et al. [5] used the equivalent control hinge moment(ECHM) concept to imply the external control force into the approximated aeroelastic system. To control the aeroelastic phenomenon, many control schemes have been adapted into the aeroelastic system in many papers. For instance, velocity feedback, output feedback, neural network control, sliding mode control, time delay control etc.

In this research, for the general adaptation of flutter suppression, the trailing edge flap is used as an actuator. The aerodynamic coefficients are calculated from the TSD theory and to reconstruct the aeroelastic system in a manner of simple state space form the auto-regressive exogenous input(ARX) modeling method is employed. Finally the optimal control gain is acquired by solving the discrete-time Riccati equation. The controller synthesized aeroelastic system is examined on the two dimensional wing-store-flap coupled system.

## Theoretical Backgrounds

### Aerodynamic Analysis in the Transonic Region

The aerodynamic governing equation based on the TSD theory has assumptions like inviscid, irrotational and small disturbance flow field. The conservative form of the TSD equations in the computational domain that is using the modified shearing transformation can be written as follows [6, 7]:

$$-\frac{\partial}{\partial t} \left[ \frac{A}{\xi_x} \phi_t + B \phi_{\xi} \right] + \frac{\partial}{\partial \xi} \left[ E \xi_x \phi_{\xi} + F \xi_x^2 \phi_{\xi}^2 + G (\xi_x \phi_{\xi} + \phi_{\eta})^2 + \frac{\xi_y}{\xi_x} (\xi_x \phi_{\xi} + \phi_{\eta}) + H \xi_y \phi_{\xi} (\xi_x \phi_{\xi} + \phi_{\eta}) \right] + \frac{\partial}{\partial \eta} \left[ \frac{1}{\xi_x} (\xi_x \phi_{\xi} + \phi_{\eta}) + H \phi_{\xi} (\xi_x \phi_{\xi} + \phi_{\eta}) \right] + \frac{\partial}{\partial \zeta} \left[ \frac{1}{\xi_x} \phi_{\xi} \right] = 0 \quad (1)$$

where  $\xi$ ,  $\eta$ , and  $\zeta$  represent the axes in the computational domain which is non-dimensionalized by the reference chord length. The coefficients are defined as

$$A = M^2, B = 2M^2, E = 1 - M^2, F = -0.5(\gamma + 1)M^2, G = 0.5(\gamma - 3)M^2, H = -(\gamma - 1)M^2 \quad (2)$$

Equation (1) is solved using the time-accurate approximate factorization(AF) algorithm which consists of a time linearization procedure and Newton iteration technique. An Engquist-Osher(E-O) difference operator has been used in AF algorithm to achieve numerical stability when shock waves exist. Non-reflecting far boundary conditions are used to achieve accurate and efficient calculation. Details can be found in the Ph. D. thesis of Kwon and Kim [6,7].

### Structural Dynamics Equations

In this study, the store attached airfoil having trailing-edge flap is considered. Figure 1 is the schematic diagram of the two-dimensional airfoil-store-flap system. This figure is modified from Gade et al. [4,8]. It doesn't have equipped the trailing-edge flap which acts as a control -input-force generator suppressing the dynamic instabilities.

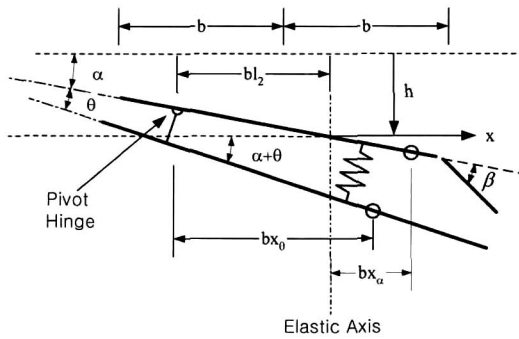


Fig. 1. Schematic diagram of airfoil-flap-store coupled system

This system has 4 degree of freedom(DOF) such as plunge, pitch, flap angle and relative angle of attack(AOA) of store with respect to pitch.

$$[M]\{\ddot{x}(t)\} + [K]\{x(t)\} = \{F(t)\} \quad (3)$$

where,

$$[M] = \begin{bmatrix} m + m_s & S_\alpha + S_\theta - bl_2 m_s & S_\theta & S_\beta \\ S_\alpha + S_\theta - bl_2 m_s & I_\alpha + I_\theta - 2bl_2 S_\theta + (bl_2)^2 m_s & I_\theta - bl_2 S_\theta & I_\beta + b(c_\beta - a_\theta) S_\beta \\ S_\theta & I_\theta - bl_2 S_\theta & I_\theta & 0 \\ S_\beta & I_\beta + b(c_\beta - a_\theta) S_\beta & 0 & I_\beta \end{bmatrix} \quad [K] = \begin{bmatrix} K_\theta & 0 & 0 & 0 \\ 0 & K_\alpha & 0 & 0 \\ 0 & 0 & K_\theta & 0 \\ 0 & 0 & 0 & K_\beta \end{bmatrix} \quad \{F(t)\} = \begin{bmatrix} -L \\ M \\ 0 \\ H \end{bmatrix} \quad (4)$$

Equation (4) represents that the store is coupled with airfoil except for flap. It means that the flap motion is indirectly influenced by the store which is affecting the response of plunge and pitch. The aerodynamic forces induced by the store are neglected in this study.

### Aeroelastic System Reconstruction using System Identification Technique

The CFD-based aeroelastic system, equation (3), can be usually transformed into the state space equation form. The transformed equation in the time domain is solved using numerical scheme such as 5<sup>th</sup>-order Runge-Kutta method. However the modified equation always includes the aerodynamic forces term and it does not work like control force but like disturbance. This term also cannot be modeled to be a deterministic mathematical form due to its nonlinear characteristics in transonic region. Hence, the aeroelastic system should be remodeled. Particularly, one of the simple and the well-known system identification techniques is the auto-regressive moving average(ARMA) model. This method can identify the system, even if the

system is nonlinear, as a linearly superposed form of auto-regressive signals. The rearranged auto-regressive with exogenous input (ARX) model in general form as follows:

$$\{y(t)\} + \sum_{i=1}^{m_a} [A_i] \{y(t-i)\} = \sum_{i=1}^{n_b} \{b_i\} \beta(t-nk-i) \quad (5)$$

Equation (5) is the single-input, multi-output (SIMO) system where  $\{y(t)\}$  and  $\beta(t)$  represent a wing responses and flap input motion, respectively.

$$\{y(t)\} = [\theta(t)]^T \{\phi(t)\} \quad (6)$$

where,

$$[\theta(t)]^T = [-[A_1] \ -[A_2] \ \cdots \ -[A_{m_a}] \ \{b_1\} \ \{b_2\} \ \cdots \ \{b_{n_b}\}] \quad (7)$$

$$\{\phi(t)\} = [\{y(t-1)\}^T, \{y(t-2)\}^T, \dots, \{y(t-na)\}^T, \beta(t-1), \dots, \beta(t-nb)]^T \quad (8)$$

The system is supposed to be a time invariant system because the Mach number and air density is fixed. Also, the controller is turned on and designed at the instance of flutter state so that the updating the parameters online is disregarded. This form is from Friedmann et al. [3].

$$\{x_p(t+1)\} = [A_p] \{x_p(t)\} + \{b_p\} \beta(t-nk) \quad (9)$$

where,

$$[A_p] = \begin{bmatrix} -[A_1] & [I] & [0] & [0] & [0] \\ -[A_2] & [0] & [I] & [0] & [0] \\ -[A_3] & [0] & [0] & [I] & [0] \\ -[A_4] & [0] & [0] & [0] & [I] \\ -[A_5] & [0] & [0] & [0] & [0] \end{bmatrix} \quad \{b_p\} = \begin{Bmatrix} \{b_1\} \\ \{b_2\} \\ \{b_3\} \\ \{b_4\} \\ \{b_5\} \end{Bmatrix} \quad \{x_p(t)\} = \begin{Bmatrix} \{y(t)\} \\ \{h_1(t)\} \\ \{h_2(t)\} \\ \{h_3(t)\} \\ \{h_4(t)\} \end{Bmatrix} \quad (10)$$

Equation (9) shows that the disturbance-like aerodynamic forces term is resolved into the system matrix and control gain matrix indirectly.

## Synthesis of the Aeroelastic Equation and Control Law

All the state values in Equation (9) is composed of the linear superposition of plunge, pitch, flap and store angle auto-regressive signals and all of which are measurable. Therefore linear quadratic regulator (LQR) control technique can be applied to suppress the flutter.

$$J = \sum_{k=0}^{\infty} (\{x_p\}^T [Q] \{x_p\} + r_w \beta^2) \quad (11)$$

To minimize the performance index,  $J$ , in equation (11), the optimal control law is able to be expressed like,

$$\beta = -\{G\}^T \{x_p\} \quad (12)$$

where,

$$\{G\}^T = \frac{\{b_p\}^T [P]_k [A_p]}{r_w + \{b_p\}^T [P]_k \{b_p\}} \quad (13)$$

$$[P]_k = [A_p]^T \left( [P]_{k-1} - \frac{[P]_{k-1} \{b_p\} \{b_p\}^T [P]_{k-1}}{r_w + \{b_p\}^T [P]_{k-1} \{b_p\}} \right) [A_p] + [Q] \quad (14)$$

where the optimal gain vector, (13), can be estimated by solving the discrete-time Riccati equation, (14), iteratively.

In case that the required optimal control input angle is not sufficiently included into the aeroelastic system, the equivalent control hinge moment (ECHM) concept is useful alternatively. This can be found from the paper Djayapertapa et al. [5].

$$x_\beta \ddot{\xi} + [(C_\beta - a_h)x_\beta + r_\beta^2]\ddot{\alpha} + r_\beta^2 \ddot{\beta} + \omega_\alpha^2 r_\beta^2 (\frac{\omega_\beta}{\omega_\alpha})\beta = \omega_\alpha^2 (\frac{U^{*2}}{\pi\mu})C_H + \omega_\alpha^2 r_\beta^2 (\frac{\omega_\beta}{\omega_\alpha})\Delta\beta_c \quad (15)$$

### Results

The two-dimensional airfoil with trailing-edge flap system is validated. Figure 2 shows the steady aerodynamic forces. The results are well agreed with the experimental data in NASA TP-2731 and the result from Yang et al. [1].

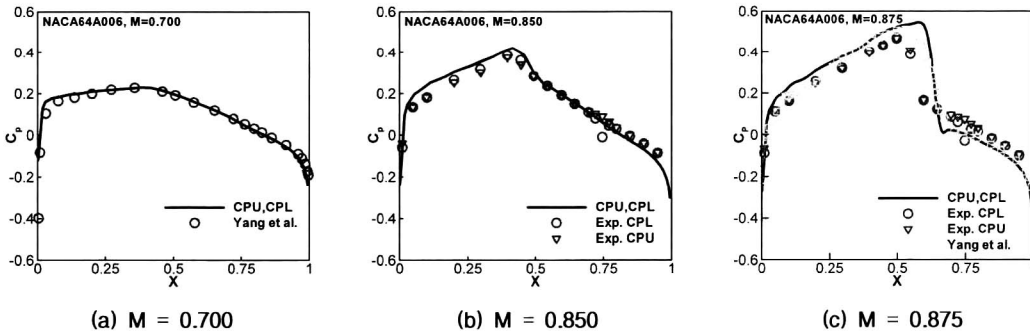


Fig. 2. Steady aerodynamics results for NACA 64A006 airfoil-flap system

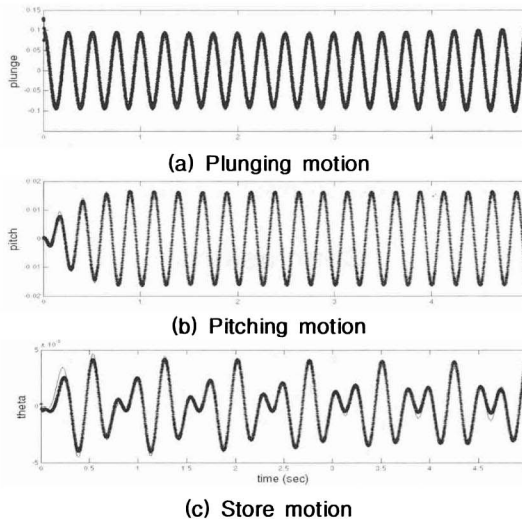


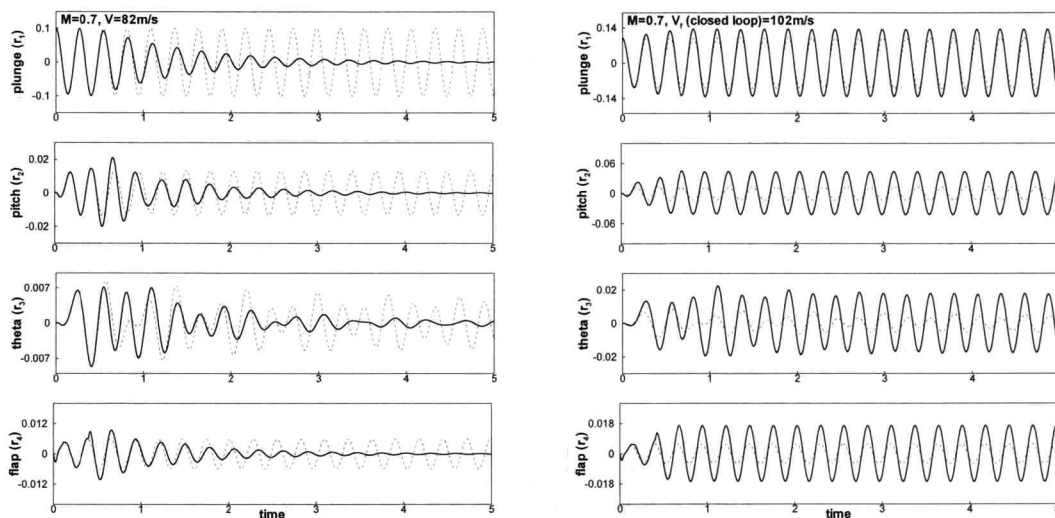
Fig. 3. Aeroelastic system remodeling by system identification technique

Though the predicted results, solid line, shows a slightly higher value than the experimental data, those tendency of above results is already appeared in the RTO technical report[9]. And the slight differences on that plots do not affect determining the flutter point. Hence, the calculated flutter boundary of two-dimensional airfoil with control surface is well agreed with the previously determined flutter points. Therefore the present two-dimensional TSD code is available to be used as the aerodynamic solver.

Figure 3 shows that the reconstructed model using the ARX modeling. As previously mentioned, the training signal is generated from the flutter state responses. Among them, the flap motion is regarded as a control input and the other DOF is considered to be the system responses including the effects of the nonlinear aerodynamic forces. To estimate the parameters in equation

(6) the least-square method is applied. However there is no systematic method for determining the number of time delay signal. In this paper the regression number is arbitrarily chosed.

Figure 4(a) shows that the controller can successfully suppress the fully CFD integrated system. In each figure, the solid lines represent the suppressed results; the dotted lines are the responses of without controller. The controller started its operation at 0.4 second. From the result in figure 4(b), the flutter point is increased about 20%. However if the system identification result is not agreed well with the teaching signal, that is, the system coefficients are not estimated well, the system becomes unstable at once. Hence the adaptive parameter estimation technique is required.



(a)  $M=0.7$ ,  $V=82\text{m/s}$

(b)  $M=0.7$ ,  $V=102\text{m/s}$

Fig. 4. Flutter suppression result for the airfoil–store–flap system

## Conclusions

In this study, a method for analyzing the integrated problem of control and CFD-based aeroelastic system has been developed. This method shows good performance in suppressing the flutter. LQR controller is constructed from the system identification technique. Even though the estimated states for LQR controller are only the displacement and velocity of each DOF, it is capable of controlling the nonlinear aeroelastic system. This simple controller also can be adopted easily for the three-dimensional wing problem that is constructed from the modal coordinate. In addition, to be more active and robust in control, the parameter estimation technique of the system identification method should be modified toward the systematic way.

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