Hybridal Method for the Prediction of Wave Instabilities Inherent in High Energy-Density Combustors (I): Modeling of Nonlinear Cavity Acoustics and its Evolution

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Abstract

This paper targets a direct and quantitative prediction of characteristics of unstable waves in a combustion chamber, which employs the governing equations derived in terms of amplification factors of flow variables. A freshly formulated nonlinear acoustic equation is obtained and the analysis of unsteady waves in a rocket engine is attempted. In the present formalism, perturbation method decomposes the variables into time-averaged part that can be obtained easily and accurately and time-varying part which is assumed to be harmonic. Excluding the use of conventional spatially sinusoidal eigenfunctions, a direct numerical solution of wave equation replaces the initial spatial distribution of standing waves and forms the nonlinear space-averaged terms. Amplification factor is also calculated independently by the time rate of changes of fluctuating variables, and is no longer an explicit function for compulsory representation. Employing only the numerical computation, major assumptions inevitably inherent, and in erroneous manner, in up to date analytical methods could be avoided. With two definitions of amplification factor, 1-D stable wave and 3-D unstable wave are examined, and clearly demonstrated the potentiality of a suggested theoretical-numerical method of combustion instability.

Key Word : Combustion Instability, Acoustic Wave, Rocket Engine

Introduction

Combustion instability has been a major design criterion and, in certain cases, very severe problem since 1940s. Even though combustion instability has been studied by numerous researchers with experimental, analytical, and computational approach, each analysis method has several fatal problems. Experimental method is time-consuming, whereas the analytical approach inevitably accompanies too many assumptions, and computational method requires heavy computational load. Thereby a more accurate and effective analysis method is desired to clarify the driving, sustaining, or damping mechanism, and to predict the occurrence of unstable acoustic waves in a rocket system.

Analytical investigations have concentrated mainly on particular acoustic models with governing equations effectively expressed in terms of combustion response and admittance in the frequency domain. Crocco introduced combustion time-lag and pressure interaction index. Mitchell expanded Crocco's time-lag model with numerical codes. Culick proposed a more generalized acoustic model to investigate the time-varying amplitudes of pressure waves and accomplished the linear and nonlinear analysis. Yang et al. expanded this to study the behavior of a limit cycle.

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Although these approaches gave fast results and basic characteristics about combustion instability, they have many limitations inherently because of oversimplified combustion and combustor dynamics.

Computational analysis based on numerical simulations focuses on the specific driving mechanism of combustion instability and fundamental combustion process models and their contributions to instability. T'ien and Sirignano regarded the time-lag caused by propellant thermal inertia as a primary driving mechanism. Dubois and Habiballah have studied the effects of passive control devices on instability. Liang et al. have also developed a comprehensive model. Litchford and Jeng, and Kim et al. utilized 2-dimensional annular geometry and studied the effects of spray combustion and various operating condition combinations on the stability limit. Yoon and Chung developed the Entropy Controlled Instability to research the effects of turbulence, nonlinearity, and nonisentropic processes. A majority of computational methods mentioned above still employs only the simplified physical models and simple or simplified geometry, and required too heavy computational load.

Present study is concerned with the improved analytical method to overcome the major assumptions and approximations enforced in the conventional approaches and computational methods. Analytical approaches proposed by Culick et al. are based on critical assumptions with uncertainties in the derivation and applications. Whereas, the fatal disadvantages for the computational method are insufficient calculation capability and excessive simplified models. To obtain more accurate numeric, further complex models and massive computation are required.

In this paper, a direct and quantitative estimation method of combustion instability is accomplished by space-averaging of the nonlinear acoustic equations and ODE with mean space variable obtained numerically. They are calculated by preconditioning scheme with spray combustion and turbulent model. General governing equations of combustion instability are obtained in terms of the modal amplification factors from Navier-Stokes system of equations for gas phase. Condensed phase in the chamber flow is excluded in the present formulation. Using the numerically obtained steady values, critical assumption of conventional analysis method can be avoided. Initial value and its temporal changes of each flow variable are calculated based on the modal amplification factors and orthogonal modes. Histories of targeted acoustic variables are filtered by FFT to identify the driving frequencies and acoustic modes of the system.

Direct Prediction Approach of Acoustic Instability

Governing Equations of Combustion Instability

The governing equations of combustion instability are derived in terms of the modal amplification factors for each flow variable, based on the 3-dimensional Navier-Stokes system of equations for gas-phase as follows.

$$\frac{\partial U}{\partial t}+\frac{\partial F}{\partial x_i}+\frac{\partial G}{\partial x_i}=H \tag{1}$$

where

$$U=\left[\rho,\rho v,\rho E,\rho Y_k\right]^T$$

$$F=\left[\rho v_i,\rho v_i v_j+p\delta_{ij},\rho E v_i+p v_i,\rho Y_k v_i\right]^T$$

$$G=\left[0,-\tau_{ij},-\tau_{ij}v_j+q_i,\rho D\frac{\partial Y_k}{\partial x_i}\right]^T$$

$$H=\left[\dot{m}_l,\rho\sum_{k=1}^{n}Y_k f_{ki}+\dot{m}_l v_i,\rho\sum_{k=1}^{n}Y_k f_{ki}v_i+\dot{m}_l C_p T_l,\omega_k\right]^T$$

Here, H is the source vector associated with the transfer processes between two phases. $\rho, v_i, p, Y_i, \dot{m}_l, f_{ij}, C_p$, and ω_k are density, velocity vector, pressure, concentration of specie i, rate of evaporation, body force, specific heat and mass production rate, respectively. τ_{ij} and q_i are the viscous stress tensor and heat flux, and thermal and species mass diffusions are represented by Fourier's and Fick's laws, respectively.

All independent variables in Eq. (1) are decomposed into time-averaged and time-varying parts, and the latter is assumed to be harmonic. The effect of the definitions of the amplification factor on the analysis results will be shown in following section. The two definitions of amplification factor and the equations of the decomposed variable are shown as below respectively.

The first definition and related equations are presented as below.

$$\epsilon_A = \frac{A'}{\overline{A}} \tag{2-a}$$

$$A(t,\vec{x}) = \overline{A}(\vec{x}) + A'(t,\vec{x}) = \overline{A}(\vec{x}) + \epsilon_A \overline{A}(\vec{x}) \tag{2-b}$$

$$\dot{m}_l = \overline{\dot{m}_l} + \dot{m}_l' = \overline{\dot{m}_l}\{1 + \alpha_v{}^* \epsilon_p + \beta_v{}^* \epsilon_v\} \tag{2-c}$$

In Eq. (2) ϵ, A, A', $\alpha_v(\vec{x})$, and $\beta_v(\vec{x})$ are mode amplification factor, fluctuating and time-averaged flow properties, pressure-coupled response and velocity-coupled response, respectively. Specially in this definition, burning rate of condensed matter can be decomposed into two parts as like other variables and a perturbation of burning rate is represented by Priem-Heidmann model.

The second definition and related equations are as follows.

$$\epsilon_{Ak}^{(n)} = \frac{\int \phi_k A'^{(n)} d\Omega}{\int \phi_k \widetilde{A}' d\Omega} \tag{3-a}$$

$$A(t,\vec{x}) = \overline{A}(\vec{x}) + A'(t,\vec{x}) = \overline{A}(\vec{x}) + \sum_i \epsilon_{Ai}(t) \widetilde{A}_i(t,\vec{x}) \tag{3-b}$$

Here ϕ and Ω are probability density of each mode vector and the system volume, respectively. The superscripts in the parenthesis remarks the instant temporal step and the subscripts are the spectral number of eigenvectors. Unlike the former, the burning rate of condensed matter cannot be decomposed yet as like other flow variables, because its perturbation cannot be set simply to be linear sums of products of amplification factors and eigenmodes of other variables. It is the problem that is not solved completely until now. With the introduction of independent modal amplification factors, unlike the Culick's conventional approximate method, the time rates of changes of each independent variable can be calculated respectively and no assumption such as isentropic process is used for their relationships. And because only steady solution is required, no further assumption is necessary for the spatial distribution of flow variables. Therefore this method can be applied to the study of nonhomogeneous and nonlinear combustion instability that is the most realistic problem without loss of generality.

In Eq. (2-a,b) and (3-a,b), A means a flow variable. In Eq. (3-a,b), the modal decomposition is applied. Subscript k is the index of set of eigenmodes and superscript (n) means calculation time step. Note that the amplification factors are defined in Eq. (3-a,b) for flow variables and their modes respectively, unlike the former definition. By using the modal amplification factors, it is expected that the history of flow variables can be described reasonably. With the definition of (2-a) or as like this, the time-varying parts can be represented by only the change of overall magnitude of perturbation, and the frequency of wave motion cannot be acquired. So ϕ_k is introduced as the weighting function for kth mode of flow variables to decompose the overall changes of perturbation with respect to its modes. But note here that ϕ_k is not necessary to be given by the known or compulsory function as like Culick's approximate analysis. In this paper, ϕ_k can be obtained numerically with the numerical acoustic model.

Finally, the governing equations of combustion instability are summarized in terms of amplification factors, introducing the concept of amplification factors defined in Eq. (2-a) or (3-a), multiplying weighting function for each flow variable, and integrating for system volume. The set of governing equations are obtained as like Eq. (4) or (5). In other words, the time rates of changes of each amplification factor are given by the linear or nonlinear combination of all amplification factors of variables. With the steady solution and initial distributions of each variable, the temporal changes of perturbations of each variable can be computed numerically by these sets of governing equations.

$$\begin{aligned} \frac{\partial \epsilon_\rho}{\partial t} &= f_\rho\left[\epsilon_\rho, \epsilon_{v_j}, \epsilon_p\right] \\ \frac{\partial \epsilon_{v_j}}{\partial t} &= f_{v_j}\left[\epsilon_\rho, \epsilon_{v_j}, \epsilon_p\right] \\ \frac{\partial \epsilon_p}{\partial t} &= f_p\left[\epsilon_\rho, \epsilon_{v_j}, \epsilon_p\right] \end{aligned} \tag{4}$$

$$\begin{aligned} \frac{\partial \epsilon_{\rho k}}{\partial t} &= f_{\rho k}\left[\epsilon_{\rho i}, \epsilon_{v_j i}, \epsilon_\pi\right] \\ \frac{\partial \epsilon_{v_j k}}{\partial t} &= f_{v_j k}\left[\epsilon_{\rho i}, \epsilon_{v_j i}, \epsilon_\pi\right] \\ \frac{\partial \epsilon_{pk}}{\partial t} &= f_{pk}\left[\epsilon_{\rho i}, \epsilon_{v_j i}, \epsilon_\pi\right] \end{aligned} \tag{5}$$

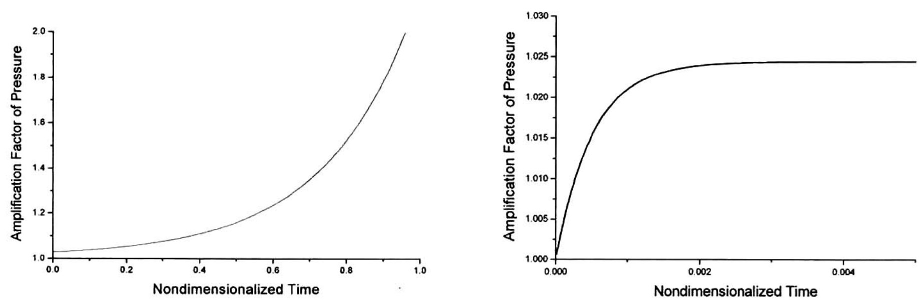
Application

In this paper, the two definitions of amplification factors are considered. With the first definition, the source terms can be decomposed and represented by Priem-Heidmann model. On the other hand, with the second definition, that cannot be modeled completely now and the study for the relationship between source terms and modal fluctuations is required. It is remained for future work. So only when the first definition is used, the combustion processes are considered.

In the first application, consider the axisymmetric combustor of 0.15 (m) radius and 1 (m) length, where n-heptane droplets are chemically reacting with subsonic air flow. Boundary conditions for air at inlet surface consist of pressure 1 (atm), temperature 1000 (K), and velocity 12 (m/s). Repeating the injection condition examined by Yoon, fuel spray characteristics are initialized with the 131 droplet groups of different drop diameters, velocity vectors, and mass flux distributions. The structured grid with 2 blocks is used and each grid consists of 81×10×21 grid points. The steady solution is obtained by applying preconditioning scheme with $k-\epsilon$ 2-equations turbulent model. In the second case, the governing equations are derived from the Euler equations, and combustion-related processes and heat addition are ignored. Homogeneous field is assumed. The distance from one end (closed end) to the other end (open end) is 0.4 (m). The pressure is 1 (atm), temperature is 300 (K), and specific heat ratio is 1.4. The number of grid points is 51 and they are equally spaced. The first three longitudinal modes of each variable are calculated with Galerkin method and Rosenbrock's 4th order ODE solver.

Results and Discussion

The direct prediction of wave motion by newly developing analysis method suggested in this study shows the satisfactory results in Figs. 1~4. And the effects of definitions of



(a)Linear Analysis (b) Nonlinear Analysis

Fig. 1. Time Trace of Pressure Perturbation (Without Modal Decomposition)

amplification factor on analysis result are clearly shown from Fig. 1. The ability of detecting of eigenvalues and eigenmodes in the second case is proved to be quite excellent. Therefore we are able to assure that the suggested method in that case can be applicable to the analysis of combustion instability with the combustion response to each acoustic mode. Figure 1(a) and (b) show the time traces of pressure perturbations in linear and nonlinear analysis. The amplification factor in nonlinear analysis converges to 1.024 rapidly. But the amplification factor in linear analysis diverges infinitely. In both cases, the behavior of amplification factor is not wave motion. So the frequency of resonant acoustic mode can not be known. That is, if the definition of amplification factor as like the Eq. (2-a) is used, we expect that the wave motion of perturbation of each variable cannot be described adequately.

Figure 2 shows the calculated eigenmodes of pressure perturbation. The first three longitudinal mode shapes in Fig. 2 are almost similar to the analytical solution for each

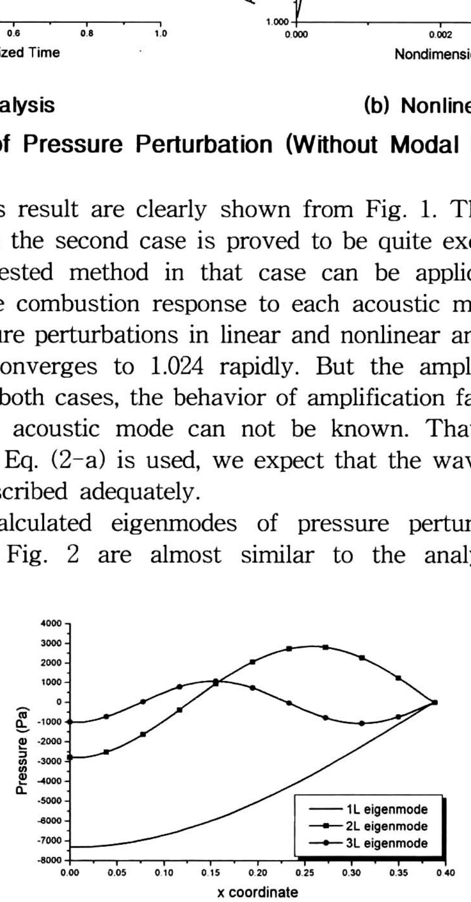
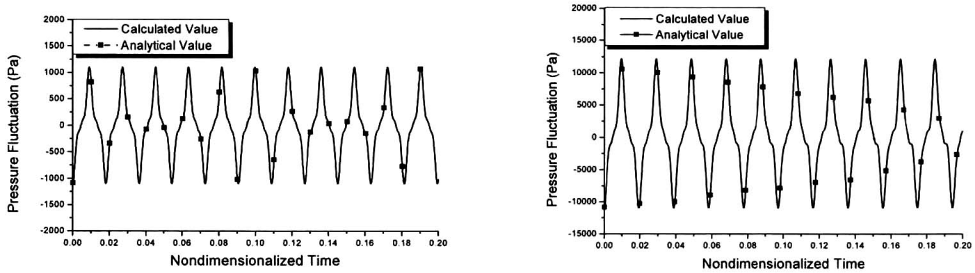


Fig. 2. Calculated Eigenmode of Pressure Perturbation



(a) Case 1 : 1% Initial Pressure Perturbation (b) Case 2 : 10% Initial Pressure Perturbation

Fig. 3. Time Trace of Pressure Perturbation at Fixed Point

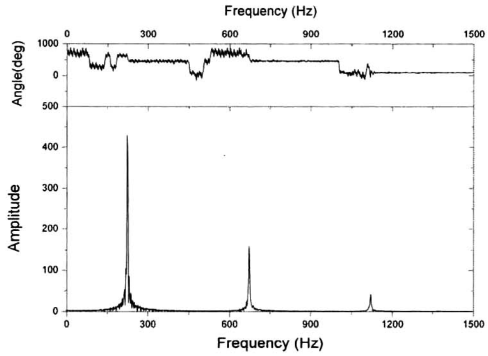


Fig. 4. Identification of Eigenvalues by Calculated Wave Motion

eigenmode. It is noted that the degree of agreement is so high that the symbols on the solid limes are bearly distinguishable. So it is sure that the modal decomposition with Galerkin method is accomplished successfully. Figure 3 also shows the history of pressure perturbations at fixed point near the inlet. The results of case (a) and (b) in Fig. 3 are all sufficiently accurate. The calculated pressure wave motions are almost similar to those of analytical wave solution. Therefore we conclude that the each eigenmode calculated with Galerkin method describes the characteristics of wave in the system with sufficient accuracy. And we assure that second definition of amplification factor in this paper works well in the processes detecting and identifying eigensystem and the definition of them is valid physically.

Figure 4 shows the FFT result to identify of eigenvalues with time trace data of pressure perturbation. The frequencies of first three longitudinal modes are 216, 650, and 1084 (Hz) by classical acoustics formula and 224, 672, and 1121 (Hz) by direct prediction method suggested in this paper respectively. The errors are 3.7, 3.3, and 3.4 (%) respectively and those are negligibly small. Therefore, the capability of newly developing hybrid method is proved in identifying of frequencies of eigensystem.

Conclusions

In this paper, a direct prediction of stable and unstable wave motion in combustion chamber is accomplished by using the freshly suggested theoretical-numerical method of combustion instability. The effects of the definition of amplification factor on the analysis results are showed. The definition of amplification factor with modal decomposition works better than that of the other. With the introduction of modal amplification factors and derivation of governing equations in terms of them, we prove that the wave motion can be analyzed with sufficient accuracy and efficiency. Moreover we are able to assure that the suggested analysis method can be applicable to analysis of combustion instability.

References

1. Vigor Yang and Williams E. Anderson (eds), 1995, "Liquid Rocket Engine Combustion Instability", AIAA.

2. Fred E. C. Culick, 1966, "Acoustic Oscillations in Solid Propellant Rocket Chambers", Astronautica Acta, Vol. 12, No. 2, pp. 113-126.

3. Habiballah, M., Dubois, I., Vingert, L., Lourme, D., and Scherrer, D., 1992, "High Frequency Combustion Instability in Liquid Propellant Rocket Engine - A Review of Studies Carried Out at ONERA for the Ariane Launcher", AIAA Paper 92-0774.

4. Crocco, L., and Cheng, S. I., 1956, "Theory of Combustion Instability in Liquid Propellant Rocket Motors", AGARD Monograph, No. 8, Butterworths, London, Chaps. 1-3.

5. Mitchell, C., 1989, "Stability Design Methodology", TR 89-041, Astronautics Laboratory, Air Force Space Center.

6. Culick, F., and Yang, V., 1995, "Combustion Instabilities in Liquid Rockets", AIAA Progress in Astronautics and Aeronautics, Vol. 169, pp. 3-37.

7. Culick, F., 1971, "Combustion Instabilities in Propulsion Systems", Combustion Science and Technology, Vol. 3, No. 1, pp. 1-16.

8. Culick, F., 1994, "Some Recent Results for Nonlinear Acoustics in Combustion Chambers", AIAA J., Vol. 32, No. 1, pp. 146-169.

9. Yang, V., Kim, S., and Culick, F., 1987, "Third-Order Nonlinear Acoustic Instabilities in Combustion Chambers, Part I : Longitudinal Modes", AIAA Paper 87-0152.

10. T'ien, J., and Sirignano, W., 1971, "Unsteady Thermal Response of the Condensed-Phase Fuel Adjacent to a Reacting Gaseous Boundary Layer," Thirteenth Symposium (International) on Combustion, The Combustion Institute.

11. Dubois, I., and Habiballah, M., 1991, "Numerical Simulation of High Frequency Instability in an Oxygen/Hydrogen Rocket Engine," AIAA paper 91-1860.

12. Liang, P., Fisher, S., and Chang, Y., 1985, "Comprehensive Modelling of a Liquid Rocket Combustion Chamber", AIAA Paper 85-0232.

13. Jeng, S., and Litchford, R., 1992, "Liquid Rocket Spray Combustion Stability Analysis", AIAA Paper 92-3227.

14. Kim, Y., Chen, C., and Ziebarth, J., 1992, "Prediction of High Frequency Combustion Instability in Liquid Propellant Rocket Engines", AIAA Paper 92-0775.

15. Yoon, W.S., and Chung, T.J., 1993, "Nonlinearly Unstable Waves Dominated by Entropy Mode", J. of Acoust. Soc. Am, Vol. 96, No. 2, pp. 1096-1103.

16. F. E. C. Culick, 1968, "A Review of Calculations for Unsteady Burning of a Solid Propellant", AIAA Journal, Vol. 6, pp 2241-2255.

17. Culick, F. E. C., and Yang. V., 1992, "Prediction of the Stability of Unsteady Motions in Solid Propellant Rocket Motors", Nonsteady Burning and Combustion Stability of Solid Propellants, edited by L. Deluca and M. Summerfield, Vol. 143, Progress in Astronautica and Aeronautics, AIAA, Washington, DC, Chap. 8, pp. 719-780.

18. Tong, A.Y., and Sirignano, W.A., 1983, "Analysis of a Vaporizing Droplet with Slip, Internal Circulation, and Unsteady Liquid-Phase and Quasi-Steady Gas-Phase Heat Transfer", ASME-JSME Thermal Engineering Joint Conference, Vol. 2, American Society of Mechanical Engineers, New York, pp. 481-487.

19. Yang, V., and Culick, F.E.C., 1990, "On the Existence and Stability of Limit Cycles for Transverse Acoustic Oscillations in a Cylindrical Combustion Chamber. 1: Standing Modes", Combustion Science and Technology, Vol. 72, Nos. 1-3, pp. 37-65.

20. G.Y. Lee and W.S. Yoon, 2000, "High Frequency Wave Instability and Combustion Responses", AIAA-2000-3296.

21. F. E. C. Culick, 1976, "Nonlinear Behavior of Acoustic Waves in Combustion Chambers-II", Acta Astronautica, Vol. 3, pp. 735-757.

22. W.S. Yoon, "Prediction and Evaluation of the High-Frequency Liquid Rocket Instabilities", Tech. Rept., 2002.