Hybridal Method for the Prediction of Wave Instabilities Inherent in High Energy-Density Combustors (II): Cumulative Effects of Pressure Coupled Responses on Cavity Acoustics

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Abstract

Theoretical-numerical approach of combustion instability in a specific rocket engine is conducted with parametric response functions. Fluctuating instantaneous burning rate is assumed to be functionally coupled with acoustic pressures and have a finite or time-varying amplitudes and phase lags. Only when the amplitudes and phases of combustion response function are sufficiently large and small respectively, the triggered unstable waves are amplified.

Key Word : Hybrid Model, Acoustic Instability, Combustion Response Function

Introduction

Hybrid method of high frequency combustion instability has been designed and developed to predict and estimate directly the stability quality or stability boundary in a specific rocket engine and examine closely detailed physics involved. Emphasis has been placed on the theoretical consistency and availability for practical studies. Thus, previous works by authors more or less focused on finding a degree of validity, accuracy and applicability of theoretical-numerical approach with simplified geometries and thermodynamic conditions and in a qualitative manner. In those studies, a linear combustion response model by Priem-Heidmann has been adopted which is capable of describing exact in-phase or out-of-phase responses only. So, more rigorous study on the effects of amplitudes and phase lags of response function on the stability limit had been remained as future work.

However, the investigation of combustion response is essential to the study of behaviors and properties of unstable waves triggered by instability and the determination of stability margin and boundary of each propulsion system, because the driving mechanism of high frequency combustion instability is the mutual interaction between acoustic pressure and combustion energy release. The ultimate object of hybrid method of combustion instability is also to examine the characteristics of related phenomena and their feedback processes as like those of other methods. Therefore, it is critical step to establish the proper and accurate combustion response model with sufficiently validated hybrid technique of combustion instability.

The object of this paper is to predict and estimate the effects of amplitudes or phases of combustion response on the characteristics of acoustic waves in a specific liquid rocket engine with the hybrid method of combustion instability and various artificial combustion response models. Artificial combustion response models of sinusoidal function type are added to the previously developed numerical codes. Without the changes of other driving or damping factors such as initial pressure amplitude, injection characteristics, and combustion dynamics, the amplitudes and phases of combustion response are set to be the only independent variables.

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Theoretical-Numerical Analysis of Combustion Instability

Theoretical-Numerical Method

Hybrid method of combustion instability suggested and developed by authors is composed of three major steps : 1) identification of combustor dynamics, 2) formulation of combustion response, and 3) direct calculation of transient unstable acoustic waves. In the first step, the initial conditions, eigenvalues and eigenmodes of perturbed flow variables, for evolving unstable waves are obtained by the computation of numerical acoustic model based on the classical linear wave equations and FEM technique. The spatial distribution of a natural mode of acoustic waves can be fully represented by the linear combinations of these orthogonal eigenmodes. Then, combustion response model is required to couple the fluctuation quantities of flow variables with the source term, and establish the unique governing equation system that is derived in terms of modal amplification factors of each flow variable. Combustion response plays a very important role on the expression of feedback process between the combustor and combustion dynamics as well as the coupling the variables in the derivation procedure. The final step is to trace and estimate the evolution of modal amplification factors in the governing equations with the initial conditions obtained in the first step and the spatial distributions of mean flow variables at arbitrary selected reference time. Hybrid model of combustion instability is an advanced approach in the two points. The geometrical restriction is free due to numerical acoustic models and it requires very low computation load by the coupling between transient amplitude and distribution of acoustic waves and its eigenmodes calculated at arbitrarily selected time, when compared with the previous analytical or numerical methods.

Amplification Factor

Temporal changes of amplitudes of unstable waves triggered by instability can be measured directly and quantitatively by modal amplification factors of each flow variable. The modal amplification factor is defined as like Eq.(1).

$$\epsilon_{Ai}^{(n)} = \frac{\int \phi_{Ai} A'^{(n)} d\Omega}{\int \phi_{Ai} A'^{(1)} d\Omega} \tag{1}$$

where ϵ, A, ϕ, and Ω mean the modal amplification factor, flow variable, weighting function to separate the modal changes from the total changes of acoustic variable, and system volume respectively. A' is the perturbed quantity of flow variables which is spatially and temporally varying. Superscript 1 and n indicate the initial and nth time step, respectively, and subscript i denotes the mode index. By definition, the initial values of all modal amplification factors are set to be 1. Using the definition of modal amplification factor, Eq.(1), the flow variables can be decomposed as follows.

$$A(\vec{x},t) = \overline{A}(\vec{x}) + A'(\vec{x},t)$$

$$A(\vec{x},t) = \overline{A}(\vec{x}) + \sum_{i=1}^{m} \epsilon_{Ai}^{(n)}(t) A'^{(n)}_i(\vec{x},\tilde{t})$$

In above equations, $\vec{x}$, t, $\overline{A}$, m, and $\tilde{t}$ indicate the position vector, time, the mean quantity of flow variable, the total number of modes for each variable, and the reference initial time respectively. The time-averaged flow variable $\overline{A}(\vec{x})$ and the eigenmodes of variable $A_i'^{(n)}(\vec{x},\tilde{t})$ are the values at previous step, thus known. Therefore, only the modal amplification factor $\epsilon_{Ai}^{(n)}(t)$ is unknown variable and all flow variables can be updated after all modal amplification factors are determined. Here, the modal amplification factors describe the temporal changes of acoustic variables, and are unknown. Therefore, in the present study, the governing equations of unstable waves are finally presented in terms of these amplification factors.

Combustion Response Model

Combustion response is the energy source of acoustic waves initiated and sustained by instability and the most important factor that determines if the driving mechanism of combustion instability is working or not. Therefore, the combustion response modeling is indispensable to identify the energy balancing conditions of specific propulsion system. Especially in the hybrid approach, combustion response model is used to represent combustion-related source terms in terms of acoustic variables and make the system of governing equations maintain uniqueness. Also, the description of combustion response requires not only amplitudes but also phases, since the thermal and acoustic waves between two phases cannot completely synchronize due to the thermal inertia of fuel droplet.

In the studies of combustion instability, combustion response model has been adopted in those researches based on the experimental data, theoretical formulation, and numerical simulation. Because combustion response is very complex phenomenon and related physics are not fully understood yet, even some theoretical modeling inevitably depend on the experimental data. Especially in the earlier stability theory, combustion response is obtained from droplet burning theory as open-loop response function. Heidmann and Wieber and Tang and Crocco assumed that all other processes related to combustion are sufficiently fast compared to the droplet vaporization. In other words, it forms the basis in their theory that the burning coincides with vaporization. Dykema was the first to discover a response peak in the proper frequency range on basis of simplified sphericosymmetric droplet burning model. Strahle considered both longitudinal and transverse waves acting upon an overventilated diffusion flame in accordance with the experimental observations of Kumagai and Kimura. For large Reynolds numbers, Strahle and Williams have considered longitudinal sound wave perturbations acting on the leading edge of burning droplets. As the computer technique is developed and the related physics are found out, the numerical simulation becomes powerful tool to investigate the combustion response. Vaporization of liquid droplet at high pressure, especially near critical or supercritical environments, has been studied numerically by Hsieh, Hsiao, Yang et al. and P. Lafon et al. to obtain the numerical correlation of response function. However, the authors believed that the above researches must have theoretical or numerical uncertainty in developing the theories and correlations.

In the previous studies by authors, Priem-Heidmann linear combustion response function is adopted in the numerical code. It is based on the film theory and very rigorous model that includes various thermodynamic properties and droplet combustion characteristics. The mass evaporation terms are expressed in terms of modal amplification factors of flow variables with Priem-Heidmann response modeling as follows.

$$\dot{m}_l = \overline{\dot{m}_l} + \dot{m}_l{}' = \overline{\dot{m}_l}\{1+\alpha_v(\vec{x})p'(t,\vec{x})+\beta_v(\vec{x})u'(t,\vec{x})\} \quad (2)$$
$$= \overline{\dot{m}_l}\{1+\alpha_v(\vec{x})\epsilon_\pi(t)\tilde{p}(\tilde{t},\vec{x})+\beta_v(\vec{x})\epsilon_{ui}(t)\tilde{u}_i(\tilde{t},\vec{x})\}$$

In Eq. (2), the coefficients $\alpha_v(\vec{x})$ and $\beta_v(\vec{x})$ are the amplitudes of pressure-and velocity -coupled response, respectively. However, the above model deals only with exact in-phase or out-of-phase combustion response. Therefore, the details of effects of combustion response on the system behaviors cannot be investigated fully only with this model.

This paper targets to show the effects of amplitudes and phase angles of a response function with hybrid model of combustion instability. So the response modeling of repulsive sinusoidal function type in Eq. (3) is included in numerical module. Because there is not reliable and simple theoretical model of response function for sub- or supercritical conditions, the authors modified the Priem-Heidmann model to reflect the changes of amplitudes and phase angles of perturbation of mass evaporation rate. The modified response model is shown in Eq. (3)

$$\dot{m}_l = \overline{\dot{m}_l}\{1+\alpha_v(\vec{x})p_{amp}(t)\sin(2\pi ft+\Theta)\} \quad (3)$$

where $p_{amp}(t)$, f, and Θ are the time-lagged amplitude which is in accordance with that of acoustic pressure, driving frequency, and phase angle respectively.

Governing Equation of Combustion Instability

Governing equations of combustion instability in hybrid approach are derived from full Navier-Stokes equations with combustion source term, Eq. (3). Formulation is based on three-dimensional Navier-Stokes equation system for reacting 2-phase flows, because the high frequency combustion instability is inherently 3-dimensional and combustion-related phenomena.

$$\begin{aligned}
&\frac{\partial \rho}{\partial t}=-\frac{1}{r}\frac{\partial(\rho r u)}{\partial r}-\frac{1}{r}\frac{\partial(\rho v)}{\partial \theta}-\frac{1}{r}\frac{\partial(\rho w)}{\partial z}+\omega \\
&\rho\frac{\partial u}{\partial t}=-\rho u\frac{\partial u}{\partial r}-\rho\frac{v}{r}\frac{\partial u}{\partial \theta}+\rho\frac{v^2}{r}-\rho w\frac{\partial u}{\partial z}-\frac{\partial p}{\partial r}+\omega\delta u \\
&\rho\frac{\partial v}{\partial t}=-\rho u\frac{\partial v}{\partial r}-\rho\frac{v}{r}\frac{\partial v}{\partial \theta}+\rho\frac{uv}{r}-\rho w\frac{\partial v}{\partial z}-\frac{1}{r}\frac{\partial p}{\partial \theta}+\omega\delta v \\
&\rho\frac{\partial w}{\partial t}=-\rho u\frac{\partial w}{\partial r}-\rho\frac{v}{r}\frac{\partial w}{\partial \theta}-\rho w\frac{\partial w}{\partial z}-\frac{1}{r}\frac{\partial p}{\partial z}+\omega\delta w \\
&\rho C_p\frac{\partial T}{\partial t}=-\rho C_p u\frac{\partial T}{\partial r}-\rho C_p\frac{v}{r}\frac{\partial T}{\partial \theta}-\rho C_p w\frac{\partial T}{\partial \theta}+\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}+\frac{v}{r}\frac{\partial p}{\partial \theta}+w\frac{\partial p}{\partial z}-\omega\Delta h
\end{aligned} \tag{4}$$

where ω, δu, δv, δw, and Δh are mass evaporation rate, radial, tangential, and axial relative velocity and energy release by combustion per unit mass, respectively. Viscous dissipation and heat conduction terms are neglected for simplicity. Here, the pressure replaces the temperature in the energy conservation equation by an equation of state for ideal gas. After substituting decomposed flow variables into Eq. (4), multiplying the weighting function, and integrating over the space, ordinary differential equations for modal amplification factors can be obtained as follows.

$$\begin{aligned}
&\frac{\partial \epsilon_{\rho k}}{\partial t}=f_{\rho k}\left[\epsilon_{\rho i},\epsilon_{Vi},\epsilon_{\pi}\right] \\
&\frac{\partial \epsilon_{Vk}}{\partial t}=f_{Vk}\left[\epsilon_{\rho i},\epsilon_{Vi},\epsilon_{\pi}\right] \\
&\frac{\partial \epsilon_{pk}}{\partial t}=f_{pk}\left[\epsilon_{\rho i},\epsilon_{Vi},\epsilon_{\pi}\right]
\end{aligned} \tag{5}$$

The right hand sides of Eq. (5) are the nonlinear function of modal amplification factors. Solving these coupled ODE, histories of modal amplification factors of each flow variable are calculated and the evolutions of individual fluctuation of flow variables can be predicted. So, stable or unstable state of the system and its growth/decay trend can be estimated directly and quantitatively.

Results and Discussion

In this study, the geometry and thermophysical conditions of a liquid rocket engine are as follows. The total length of engine is 0.452 (m) and the axial distance from injection surface to the nozzle entrance is 0.202 (m). The radius of cylindrical part and nozzle neck are 0.210 and 0.155 (m). Mean pressure and temperature in the chamber are 13 (atm) and 2300 (K) respectively. The propellant combination is LOX/Kerosene and initial droplet radius is 50 (micron). Conventional D^2 -law of droplet vaporization is assumed. Fuel droplet is assumed to be evaporating and react immediately just after it is injected. And the injection and combustion process are assumed to occur one-dimensionally along the axial direction. Only the mass evaporation rate is calculated, and the combustion processes are not treated in detail. Only the linear analysis is conducted due to too heavy computational load required for nonlinear analysis. 10 % initial pressure perturbation triggers acoustic waves in all cases. The driving frequencies of the 1st radial modes are 2791 and 2020 (Hz). And those of the 1st tangential modes are 1341 and 1025 (Hz). The former are calculated by linear

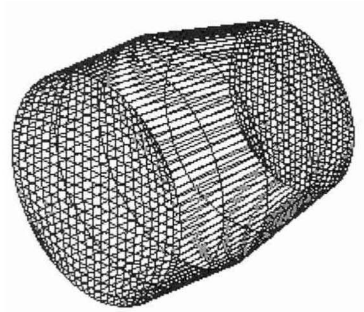
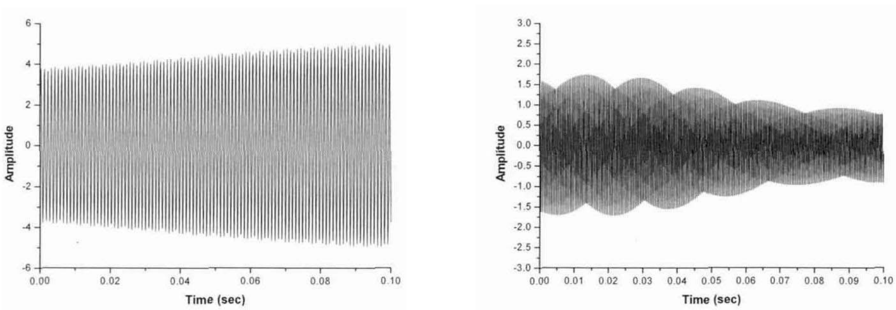


Fig. 1. Grid Shape



(a) 1st Tangential Mode (b) 1st Radial Mode

Fig. 2. Modal Amplification Factors (Phase Angle : 10 (degree))

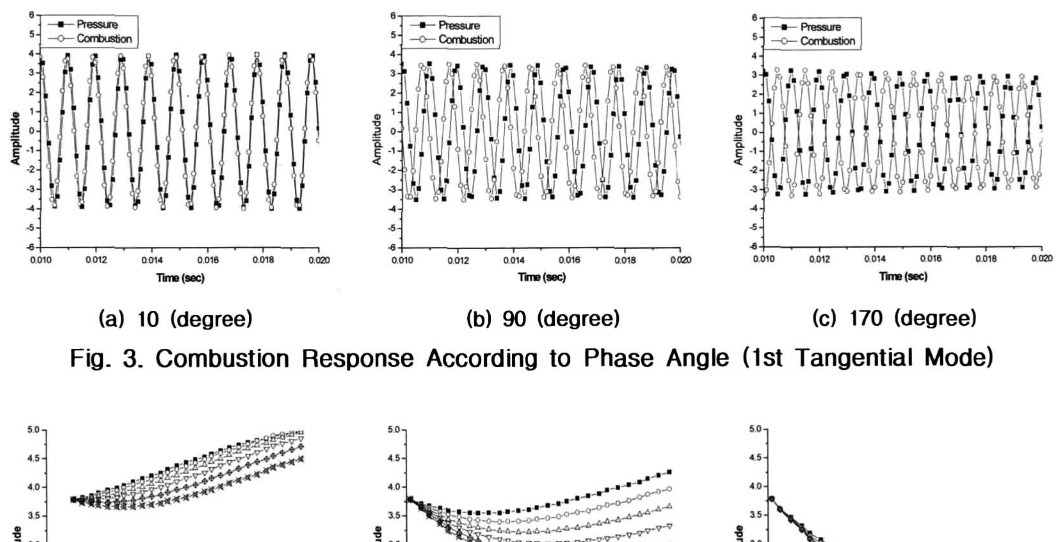
acoustic model with FEM technique and the latter are filtered by FFT after the computation of modal amplification factors for pure acoustic field. The frequencies of instantaneous perturbations of mass evaporation rates are chosen as like the FFT filtering results-2020 (Hz)(1R) and 1025 (Hz)(1T) respectively. The only independent variables in this application are amplitudes and phase angles of combustion response. The amplitudes of combustion response are 50, 100, and 200 (%) of that of Priem-Heidmann model. And the phase angles are changed from 10 to 170 (degree) by 10 (degree) for each mode. The mass evaporation rates are given by the repulsive sine function with constant or time-varying amplitude. The magnitudes of amplitudes of mass evaporation rate are set to be those of time-lagged modal amplification factors automatically in the code.

Figure 1 shows the grid shape used in this application. The numbers of grid points are 21(x-direction), 21(y-direction), and 4(axial direction) respectively. To minimize the numerical errors in integrating energy coefficients for the 1st radial and tangential modes simulation where the number of total allowed grid points are limited due to the heavy computational load, the resolution at x- and y-direction is set to be much higher than z-direction.

Figure 2(a) and (b) depict the time history of simulated transient wave for the 1st tangential and radial modes with modal amplification factors respectively. For both cases, the phase difference between mass evaporation rate and acoustic pressure is set to be 10 (degree). But the 1st tangential pressure wave is amplified, while the 1st radial acoustic wave is damped. In this application, the mass evaporation rate is given by the repulsive sinusoidal function-frequency, amplitude, and phase angles of that are not changed in simulation, while pressure waves are calculated numerically. Therefore, if frequencies of the pressure waves are changed due to the numerical error or physical phenomena in the computation, the phase difference can be also changed from the initially intended quantity. So the resultant acoustic waves can present the unpredictable behaviors and properties in some cases. Figure 2(b) is the result due to these changes of phase angle by some causes. But the authors believe that more detail and massive computations are required so that these phenomena are understood more thoroughly. The relationship between the acoustic pressures and perturbations of fuel mass evaporation rate-combustion response-are shown in Fig. 3. In the computation of the 1st tangential mode, the phase changes mentioned above paragraph are not occurred in Fig. 3(a), (b) or (c). Therefore, the predictable and physically reasonable acoustic waves are obtained as presented in Fig. 4 and 6 respectively.

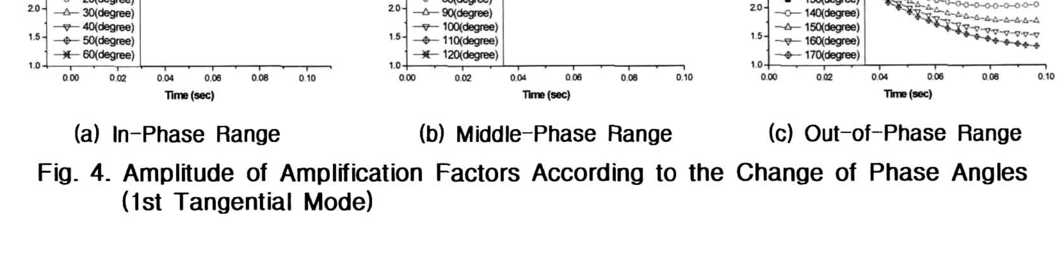
The histories of amplitudes of amplification factors, according to the changes of initially given phase angles, are presented in Fig. 4(1T mode) and 5(1R mode). In these calculations, the amplitude of mass evaporation rate is varied and in accord with the amplitude of pressure perturbation, even though it has a time lag. In Fig. 4, the resultant acoustic waves prove that the re-interpreted Rayleigh Criterion is reasonable and useful. The unstable waves are amplified in the in-phase region (Fig. 4(a)) and damped in the out-of-phase region (Fig. 4(b)) monotonically. The more the phase angle become large, the more the engine system turns out to be stable. While the amplitudes of modal amplification factors are damped at first and amplified gradually in the middle range (Fig.4(c)). It is because the phase differences between pressure perturbations and combustion are changed at the time when the decaying acoustic waves starts to grow. As mentioned above paragraph, the repulsive sinusoidal function type is selected as the combustion model in this study. Therefore, it is certain that the phase differences shift somewhat, even though the confident evidences are not known. In Fig. 5, the 1R modes show all damped motions unlike the 1T modes. Authors guess that the numerical errors in the 1R mode calculation are much more greater than those in the 1T mode simulation as like the differences of driving frequency in two cases-acoustic model and numerical simulation. : 771 (Hz)(1R) and 316 (Hz)(1T). The larger the driving frequency is, the larger the numerical error becomes. Therefore, authors expected that these differences and unreasonable output presented above will be improved in future study with more accurate numerical technique.

Figure 6 and 7 also depict the temporal changes of amplitude of amplification factors as like in Fig. 4 and 5. But, in these cases, the mass evaporation rate is periodically varied with 'fixed'



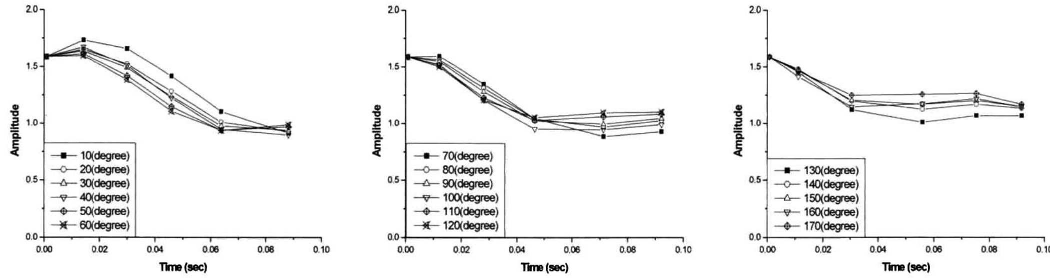
(a) 10 (degree) (b) 90 (degree) (c) 170 (degree)

Fig. 3. Combustion Response According to Phase Angle (1st Tangential Mode)



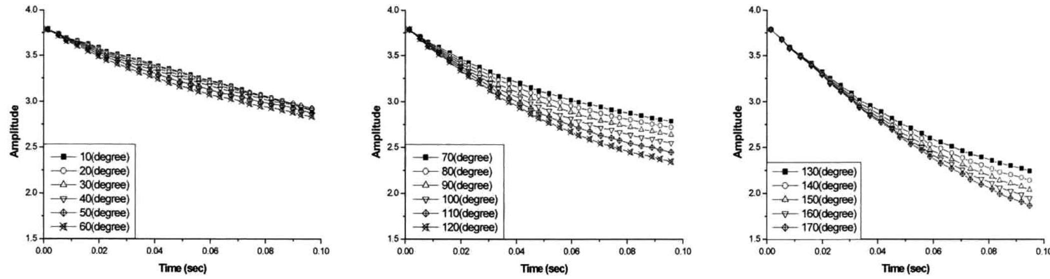
(a) In-Phase Range (b) Middle-Phase Range (c) Out-of-Phase Range

Fig. 4. Amplitude of Amplification Factors According to the Change of Phase Angles (1st Tangential Mode)



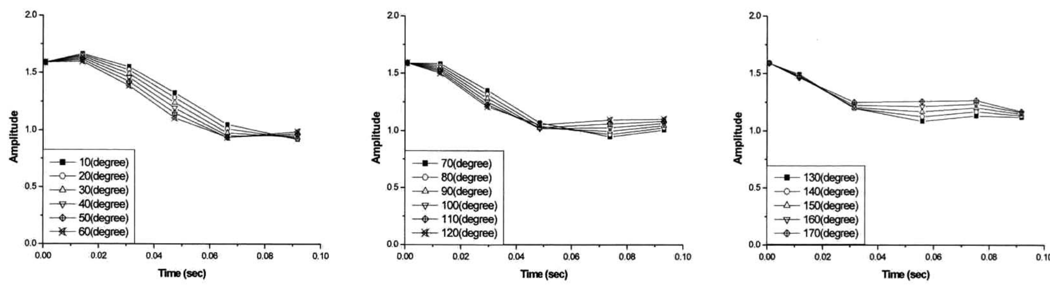
(a) In-Phase Range (b) Middle-Phase Range (c) Out-of-Phase Range

Fig. 5. Amplitude of Amplification Factors According to the Change of Phase Angles (1st Radial Mode)



(a) In-Phase Range (b) Middle-Phase Range (c) Out-of-Phase Range

Fig. 6. Amplitude of Amplification Factor According to the Change of Phase Angles (Limited Amplitude Combustion Response)

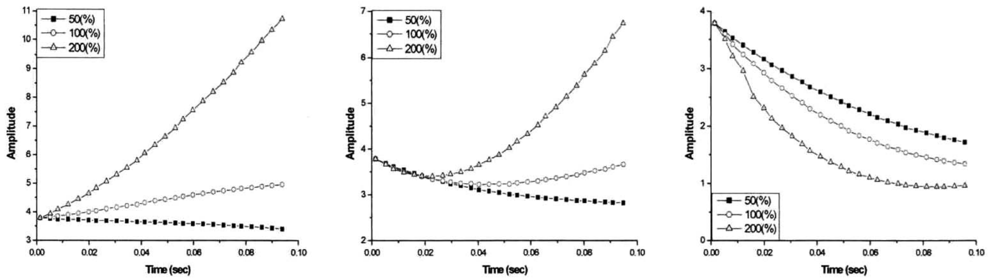


(a) In-Phase Range (b) Middle-Phase Range (c) Out-of-Phase Range

Fig. 7. Amplitude of Amplification Factor According to the Change of Phase Angles (Limited Amplitude Combustion Response)

amplitude. That is, the amount of perturbed mass evaporation does not grow larger or smaller than the initial quantity according to the acoustic energy. Therefore, the amount of energy release by combustion is not changed and is not in accord with the history of acoustic energy so that the feedback process between chamber acoustics and combustion cannot be working. As a result, the system cannot be amplified by any strong pressure perturbation. So in both cases (Fig. 6 and 7), all unstable waves are damped monotonically.

The effects of coefficients in the source terms are presented in Fig. 8. In this numerical experiment, the coefficient of combustion response is independent variables. The quantity calculated in Priem-Heidmann linear response model is considered as the reference value. In Fig. 8, 50, 100, and 200 (%) means the coefficients of response α_v in Eq. (5) are given by $0.5\alpha_{v,ref}$,



(a) 10 (degree) (b) 90 (degree) (c) 170 (degree)

Fig. 8. Amplitude of Amplification Factors According to the Initial Amplitude of Response

$\alpha_{v,ref}$, and $2\alpha_{v,ref}$. That is, the larger α_v is, the more combustion energy release. As shown in Fig. 8(a), (b), and (c), the more sufficient combustion energy release is possible, the more steeply the amplification factors become growing. In the other hand, if the energy release is not sufficient, the acoustic waves are decaying. In Fig. 8, the effects of phase angles of mass evaporation rate on the system behavior are also shown. Their trends are same with the results already mentioned above paragraphs.

Conclusions

In this paper, the hybrid approach of instability with the modified Priem-Heidmann models of repulsive sinusoidal function type is established to investigate the effects of combustion response on the behaviors and properties of the liquid rocket engine system. The effects of amplitudes and phase angles are as follows. 1) Only sufficiently large mass evaporation rate-combustion response can drive the instability. 2) Magnitudes of response make a critical change of balancing conditions between the amplifying and damping acoustic energies. 3) In the in-phase range the unstable waves are amplified, while, in the out-of-phase range the acoustic waves are damped. In the middle range, the instabilities are damped or amplified.

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