Optimal design of a piezoelectric passive damper for vibrating plates

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Abstract

In this paper, an efficient piezoelectric passive damper is newly devised to suppress the multi-mode vibration of plates. To construct the passive damper, the piezoelectric materials are utilized as energy transformer, which can transform the mechanical energy to electrical energy. To dissipate the electrical energy transformed from mechanical energy, multiple resonant shunted piezoelectric circuits are applied. The dynamic governing equations of a coupled electro-mechanical piezoelectric with multiple piezoelectric patches and multiple resonant shunted circuits is derived and solved for the one edge clamped plate. The equations of motion of the piezoelectrics and shunted circuits as well as the plate are discretized by finite element method to estimate more exactly the effectiveness of the piezoelectric passive damper. The method to find the optimal location of a piezoelectric is presented to maximize effectiveness for desired modes. The electro-mechanical coupling term becomes important parameter to select the optimal location.

Key Word : Piezoelectric, passive damping, multiple modes, finite element method, location optimization

Introduction

In recent, piezoelectric passive control together with active control has been studied for the purpose of suppressing the vibration motion of structures. The characteristics of the piezoelectric material are to transform mechanical vibration energy to electrical energy. In other words, the piezoelectric can be used as energy transformer. The transformed electrical energy can be dissipated into the heat energy by shunt circuits. Using these features, vibration energy can be dissipated, and consequently vibration can be suppressed. Hagood and von Flotow[1] have presented the passive control using the piezoelectric materials. They introduced the simple shunting circuit consisting of a resistor and an inductance, which make an electrical resonance. As it is tuned optimally to vibration mode desired to suppress, the structural vibration decreased effectively. In that paper, an analysis has been performed by deriving the effective mechanical impedance for the piezoelectric element shunted by an arbitrary circuit. Hollkamp[2] expanded the theory so that a single piezoelectric element can be used to suppress multiple modes. Davis and Lesieutre[3] developed a method for predicting the damping performance in beams with resistively shunted piezoceramics based on a variation of the modal strain energy approach. Tsai and Wang[4] proposed active-passive hybrid piezoelectric network concept for one-dimensional structures with uni-axial loading and showed that the shunt circuit can not only provide passive damping, but also enhance the active action authority if tuned correctly. Hollkamp and Gordon[5] applied piezoelectric passive damper to a two-dimensional planar problem

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by simple mathematical description. The peak electrical energy was utilized to determine the location of piezoelectric patch. However, before applying this determination procedure, the stress field over the entire structure should be evaluated because the peak electrical energy is described in terms of stress field. Saravanos[6] carried out an analysis of composite plates with multiple resistively shunted piezoelectric layers by using Ritz method which is difficult to apply to the complex structures.

Although the previous work has provided valuable insight, most of the previous work on the piezoelectric passive damper has been limited to one-dimensional structures, especially to cantilevered beams and trusses. Moreover, it is difficult to apply the proposed analysis model to complex structures, because very simple analysis model has been adopted. Therefore, more general methods are required to efficiently utilize this piezoelectric passive damping technologies in practical structures. In this work, the dynamic governing equations for a plate with piezoelectric patches connected to multiple resonant shunted circuits are derived to precisely describe the electro-mechanical coupling behavior of this electro-mechanical system. In the dynamic governing equations, the dynamic equilibrium equations for a plate with piezoelectric patches and the dynamic governing equations for multiple resonant shunted circuits are coupled electro-mechanically with each other. These coupling terms between displacements of plate and charges of piezoelectric materials clearly explain how to decrease vibrating responses of host structures with shunted piezoelectric circuits. To provide a general modeling methodology for complex structures and multiple resonant shunted piezoelectric circuits, finite element methods are employed to approximate the governing equations for multiple resonant shunted circuits as well as a plate with piezoelectric patches.

To maximize the effectiveness of the piezoelectric passive damper in suppressing the concerning modes, a method, how to determine the optimal locations of piezoelectric patches, is proposed, where the electro-mechanical coupling terms play an important role to optimally locate the piezoelectric patches in host structures.

Equations of motion

The constitutive equation

This section describes governing equations of motion for piezoelectric structures with electric circuits. The mechanical and electrical behaviors of a piezoelectric material are described. The linear piezoelectric constitutive equations of a piezoelectric material can be expressed as follows

$$\mathbf{E} = \boldsymbol{\beta}^S \mathbf{D} - \mathbf{h}\boldsymbol{\varepsilon}, \quad \boldsymbol{\sigma} = -\mathbf{h}^t \mathbf{D} + \mathbf{c}^D \boldsymbol{\varepsilon} \tag{1}$$

where $\mathbf{E}$ is the electric field vector, $\mathbf{D}$ is the electric displacement vector, σ is the stress vector, ε is the strain vector, and β^S, $\mathbf{h}$, and $\mathbf{c}^D$ represent the piezoelectric constant coefficients(piezoelectric dielectric matrix, stress constant matrix, and stiffness matrix, respectively). The superscripts S and D represent the constant strain field and the constant electric displacement field, respectively. The superscript t denotes transposition of matrix.

For a piezoceramic thin plate polarized in the thickness direction, the material is isotropic in the other two in plane directions. Under plane stress state, the coupling term of electric and elastic equations has the form of

$$\mathbf{h}_3 = \begin{bmatrix} h_{31} & h_{31} & 0 \end{bmatrix} \tag{2}$$

where

$$h_{31} = \frac{d_{31} E^D}{\varepsilon_3^T (1-\nu)} \tag{3}$$

By the relation (2), the constitutive equations for isotropic materials under plane stress state is reduced to the following form

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \frac{E^D}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} - \begin{bmatrix} h_{31} \\ h_{31} \\ 0 \end{bmatrix} D_3 \tag{4}$$

$$E_3 = \beta_3^S D_3 - h_{31}(\varepsilon_x + \varepsilon_y) \tag{5}$$

The governing equations of motion

Dynamic governing equations for a plate equipped with the shunted piezoelectric circuit are derived, based on the Kirchhoff plate theory and Hamilton's principle. In the Kirchhoff plate theory, the displacement field (u, v, w) is assumed to be as shown below

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z(\partial w / \partial x) \\ v(x,y,z,t) &= v_0(x,y,t) - z(\partial w / \partial y) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{6}$$

where (u, v, w) denotes the displacements of the point (x, y, z) along the x, y, z directions, and (u_0 , v_0 , w_0) represent displacements of a point on the midplane (x, y, 0) at time t. The linear strains due to the displacements are written as follows

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \partial u / \partial x \\ \partial v / \partial y \\ \partial u / \partial y + \partial v / \partial x \end{Bmatrix} = \begin{Bmatrix} \partial u_0 / \partial x \\ \partial v_0 / \partial y \\ \partial u_0 / \partial y + \partial v_0 / \partial x \end{Bmatrix} - z \begin{Bmatrix} \partial^2 w / \partial x^2 \\ \partial^2 w / \partial y^2 \\ 2\partial^2 w / \partial xy \end{Bmatrix} \tag{7}$$

The equations of motion of the piezoelectric materials, connected to shunt circuit consisting of resistor and inductor in series, can be derived through the Hamilton's principle.

$$\int_{t_1}^{t_2} [\delta(T - V + W)]dt = 0 \tag{8}$$

where

$$T = \frac{1}{2}\int_{V_S} \rho_S \dot{\mathbf{u}}^T \dot{\mathbf{u}} dV + \frac{1}{2}\int_{V_P} \rho_P \dot{\mathbf{u}}^T \dot{\mathbf{u}} dV \tag{9}$$

$$V = \frac{1}{2}\int_{V_S} \boldsymbol{\varepsilon}^T \boldsymbol{\sigma} dV + \frac{1}{2}\int_{V_P} \boldsymbol{\varepsilon}^T \boldsymbol{\sigma} dV + \frac{1}{2}\int_{V_P} D_3 E_3 dV \tag{10}$$

where T denotes the kinetic energy and V is the sum of strain energy and electrical energy. Subscripts s and p denote the host structure and piezoelectric material, respectively. The virtual work due to applied force and electric potential is described as

$$\delta W = -(L\ddot{Q} + R\dot{Q} - V_a)\delta Q + \int_S \delta \mathbf{u}^T \mathbf{f} dS \tag{11}$$

In equations, **u** is the mechanical displacement vector, V_a is the applied voltage, **f** is applied force vector. L and R is inductance and resistance in the shunted piezoelectric circuit, respectively. Q is the total charge on the electrode of the piezoelectric material due to deformation. Substituting equation (9), (10) and (11) into equation (8) and integrating by parts, the equations of motion of the shunted piezoelectric plate can be expressed as the following weak form

$$\int_{V_S} \rho_S (\delta \mathbf{u})^T \ddot{\mathbf{u}} dV + \int_{V_P} \rho_P (\delta \mathbf{u})^T \ddot{\mathbf{u}} dV + \int_{V_S} (\delta\boldsymbol{\varepsilon})^T \mathbf{c}_S \boldsymbol{\varepsilon} dV + \int_{V_P} (\delta\boldsymbol{\varepsilon})^T \mathbf{c}_P^D \boldsymbol{\varepsilon} dV - \int_{V_P} (\delta\boldsymbol{\varepsilon})^T \mathbf{h}_3^T D_3 dV = \int_S (\delta \mathbf{u})^T \mathbf{f} dS \tag{12}$$

$$L\delta Q\ddot{Q} + R\delta Q\dot{Q} + \int_{V_P}\delta D_3\beta_3^S D_3 dV - \int_{V_P}\delta D_3\mathbf{h}_3\boldsymbol{\varepsilon} dV = V_a\delta Q \quad (13)$$

Finite element model

Finite element model for single electrical mode

Four node conforming plate elements based on the Kirchhoff plate theory are used in the modeling. There are two in-plane degrees of freedom (u, v) and four bending degrees of freedom (w, $w_{,x}$, $w_{,y}$, $w_{,xy}$) at each node.

$$\mathbf{U}_m^e = \{u_0^1 \quad v_0^1 \quad u_0^2 \quad v_0^2 \quad u_0^3 \quad v_0^3 \quad u_0^4 \quad v_0^4\}^T$$

$$\mathbf{U}_b^e = \{w_0^1 \quad w_{,x}^1 \quad w_{,y}^1 \quad w_{,xy}^1 \quad \cdots \quad w_0^4 \quad w_{,x}^4 \quad w_{,y}^4 \quad w_{,xy}^4\}^T$$

The in-plane displacements (u, v) are interpolated by linear functions, and transverse displacement w is interpolated by hermite cubic polynomials. The displacements (u, v, w) can be expressed by nodal degrees of freedom as follows

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \mathbf{F}\mathbf{U}_m^e + z\mathbf{G}\mathbf{U}_b^e \qquad w = \mathbf{W}\mathbf{U}_b^e$$

Equivalently, it can be written in simplified matrix form

$$\mathbf{u} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{bmatrix} \mathbf{F} & z\mathbf{G} \\ \mathbf{0} & \mathbf{W} \end{bmatrix} \begin{Bmatrix} \mathbf{U}_m^e \\ \mathbf{U}_b^e \end{Bmatrix} = \mathbf{N}\mathbf{U}^e \quad (14)$$

where

$$\mathbf{U}^e = \begin{Bmatrix} \mathbf{U}_m^e \\ \mathbf{U}_b^e \end{Bmatrix}$$

From the displacement-strain relation and equation (14), the strains in an element are given by

$$\boldsymbol{\varepsilon} = \mathbf{R}\mathbf{U}_m^e + z\mathbf{S}\mathbf{U}_b^e = \mathbf{B}\mathbf{U}^e \quad (15)$$

The electrical displacement D_3 is the generated charge per area A_P of a piezoelectric material

$$D_3 = \frac{Q}{A_P} \quad (16)$$

By using equation (14), (15), and (16), the following element matrices are obtained.

$$\mathbf{M}_S^e = \int_{V_S}\rho_S\mathbf{N}^T\mathbf{N}dV \quad , \quad \mathbf{M}_P^e = \int_{V_P}\rho_P\mathbf{N}^T\mathbf{N}dV$$

$$\mathbf{K}_S^e = \int_{V_S}\mathbf{B}^T\mathbf{c}_S^D\mathbf{B}dV \quad , \quad \mathbf{K}_P^e = \int_{V_P}\mathbf{B}^T\mathbf{c}_P^D\mathbf{B}dV$$

$$\mathbf{H}^e = \int_{V_P}\frac{\mathbf{B}^T\mathbf{h}_3^T}{A_P}dV \quad , \quad \mathbf{F}^e = \int_S\mathbf{N}^T\mathbf{f}ds$$

where $\mathbf{M}_S^e$ and $\mathbf{M}_P^e$ denote the mass matrices of host structure and piezoelectric material, respectively. $\mathbf{M}_S^e$, and $\mathbf{M}_P^e$ denote the stiffness matrices of host structure and piezoelectric material, respectively.

$\mathbf{H}^e$ is the coupling matrix, $C_P{}^S$ is inherent capacitance of piezoelectric, L and R is inductance and resistance, $\mathbf{F}^e$ is forcing matrix , and V_a is the applied voltage.

After assembling the element matrices, we obtain the following discretized governing equations for electro-mechanically coupled structures.

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & L \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{U}} \\ \ddot{Q} \end{Bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & R \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{U}} \\ \dot{Q} \end{Bmatrix} + \begin{bmatrix} \mathbf{K} & -\mathbf{H} \\ -\mathbf{H}^T & 1/C_P^S \end{bmatrix} \begin{Bmatrix} \mathbf{U} \\ Q \end{Bmatrix} = \begin{Bmatrix} \mathbf{F} \\ V_a \end{Bmatrix} \tag{17}$$

The first row of equation (17) represents the mechanical behavior of the piezoelectric materials whose actuating force is $\mathbf{H}Q$. The mechanical vibration of the piezoelectric material causes the electrical charges on both of the electrodes of the piezoelectric material, and this charge induces the actuating force that suppresses the vibration of structure. And the second row of equation (17) describes the electric circuit which consists of L-R-C elements in series and clearly shows that mechanical deformation induces the voltage difference $\mathbf{H}^T\mathbf{U}$ across the electrodes of piezoelectric materials. It is noted that the electrodes of piezoelectric material as a capacitance can be resonant with properly selected inductor and the accumulated energy is dissipated through the resistor.

Finite element model for multiple electrical modes

Even though Eq. (17) is derived for only one piezoelectric patch connected to one register-inductor element circuit, it can be easily extended to multi piezoelectric patches with multi resonant circuits. Actually, even single piezoelectric patch can suppress the multiple modes only by increasing the number of electrical resonances.

In this work, a plate with two piezoelectric patches is considered. Each piezoelectric patch is connected to resonant circuit designed to produce two types of electrical resonance. It is illustrated in Fig. 1. In this case, four vibrational modes can be suppressed theoretically, because each piezoelectric patch is connected to resonant circuit which can be produce two types of electrical resonance.

The dynamic governing equations of this case can be written in the form of

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{L} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{U}} \\ \ddot{\mathbf{Q}} \end{Bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{U}} \\ \dot{\mathbf{Q}} \end{Bmatrix} + \begin{bmatrix} \mathbf{K} & -\mathbf{H} \\ -\mathbf{H}^T & \mathbf{C}_P^{S-1} \end{bmatrix} \begin{Bmatrix} \mathbf{U} \\ \mathbf{Q} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F} \\ \mathbf{0} \end{Bmatrix} \tag{18}$$

where $\mathbf{L}$, $\mathbf{R}$, and $\mathbf{C}_P{}^{S-1}$ are inductance, resistance, and admittance matrices which are related to shunted circuits.

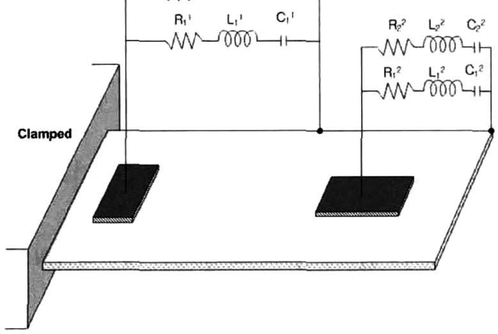

Fig. 1. Plate with two piezoelectric patches

$$\mathbf{L}=\begin{bmatrix} L_1^1 & 0 & 0 & 0 \\ 0 & L_2^1 & 0 & 0 \\ 0 & 0 & L_1^2 & 0 \\ 0 & 0 & 0 & L_2^2 \end{bmatrix},\quad \mathbf{R}=\begin{bmatrix} R_1^1 & 0 & 0 & 0 \\ 0 & R_2^1 & 0 & 0 \\ 0 & 0 & R_1^2 & 0 \\ 0 & 0 & 0 & R_2^2 \end{bmatrix}$$

$$\mathbf{C}_P^{S-1}=\begin{bmatrix} 1/C_1^1+1/C_{P1}^S & 1/C_{P1}^S & 0 & 0 \\ 1/C_{P1}^S & 1/C_2^1+1/C_{P1}^S & 0 & 0 \\ 0 & 0 & 1/C_1^2+1/C_{P2}^S & 1/C_{P2}^S \\ 0 & 0 & 1/C_{P2}^S & 1/C_2^2+1/C_{P2}^S \end{bmatrix}$$

$$\mathbf{H}=[\mathbf{H}_1 \quad \mathbf{H}_1 \quad \mathbf{H}_2 \quad \mathbf{H}_2],\quad \mathbf{Q}=[Q_1^1 \quad Q_2^1 \quad Q_1^2 \quad Q_2^2]^T$$

where L_i^j , R_i^j , C_i^j , and Q_i^j ,are inductor, resistor, capacitor, and charge which are located in the i resonant circuit of the jth piezoelectric, respectively. $\mathbf{C}_{Pj}^S$ is the jth piezoelectric inherent capacitance. $\mathbf{H}_j$ represents coupling term between host structure and piezoelectric at the location of jth piezoelectric patch. The optimal location of jth piezoelectric patch to suppress the specific modes depends on the coupling term $\mathbf{H}_j$.

To analysis damped vibration characteristics without applied force, equation (18) is reduced by using modal reduction technique, and modal damping is introduced. equation(18) is

$$\tilde{\mathbf{M}}\ddot{\mathbf{x}}+\tilde{\mathbf{C}}\dot{\mathbf{x}}+\tilde{\mathbf{K}}\mathbf{x}=\mathbf{0} \tag{19}$$

where

$$\tilde{\mathbf{M}}=\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{L} \end{bmatrix},\quad \tilde{\mathbf{C}}=\begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{bmatrix},\quad \tilde{\mathbf{K}}=\begin{bmatrix} \mathbf{\Lambda} & -\mathbf{\Phi}^T\mathbf{H} \\ -\mathbf{H}^T\mathbf{\Phi} & \mathbf{C}_P^{S-1} \end{bmatrix},\quad \mathbf{x}=\begin{Bmatrix} \eta \\ \mathbf{Q} \end{Bmatrix}$$

where $\mathbf{U}=\mathbf{\Phi}\eta$, $\mathbf{\Phi}^T\mathbf{M}\mathbf{\Phi}=\mathbf{I}$,and $\mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi}=\Lambda$

Changing the equation (19) into state space form of the first order differential equations, it is rewritten as

$$\begin{Bmatrix} \dot{\mathbf{x}} \\ \ddot{\mathbf{x}} \end{Bmatrix}=\begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\tilde{\mathbf{M}}^{-1}\tilde{\mathbf{K}} & -\tilde{\mathbf{M}}^{-1}\tilde{\mathbf{C}} \end{bmatrix}\begin{Bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{Bmatrix}=\mathbf{P}\begin{Bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{Bmatrix} \tag{20}$$

The characteristics of plate with passive shunted piezoelectrics can be determined by carrying out the eigenvalue analysis of the modal matrix **P**.

Optimal location of a piezoelectric patch

As actuating force **HQ** is increased, the damping performance induced by piezoelectric patch is enhanced. Additionally, larger voltage source $\mathbf{H}^T\mathbf{U}$ increases the dissipation of electric energy through the register in the viewpoint of electric circuit. In other words, if the electro-mechanical coupling matrix **H**is selected to increase the actuating forces corresponding to the modes which are to be suppressed, the damping performance is enhanced naturally.

Note that the electro-mechanical coupling matrix **H**depends on the location of piezoelectric patch as well as the piezoelectric material property h_{31}. Therefore, in order to suppress the multiple modes by a piezoelectric patch, it is desirable to place the piezoelectric patch at the location to maximize the minimum modal actuating force. Modal actuating force means the force that is corresponding to the concerning mode. Based on this observation, the location of piezoelectric patch is determined to satisfy the following condition.

$$\text{Maximize} \quad J=\min(|\mathbf{\Phi}_S^T\mathbf{H}_j|) \tag{21}$$

where matrix $\mathbf{\Phi}_S$ is column matrix composed of eigenvectors of corresponding modes. One can determine the optimal location of a piezoelectric patch for the modes to suppress with this criterion.

Application

Cantilevered plate, of which dimensions are 100 mm width, 150 mm length, and 1 mm thickness, are made of the aluminum. The two piezoceramics patches with the dimensions of 25 mm width, 50 mm length, and 0.5 mm thickness, are bonded to the plate as shown in Fig. 2. The piezoceramics are poled perpendicular to the plane of the host structure and the plate is grounded. The material properties of the plate and the piezoceramics are presented in Table 1. Impact hammer test was carried out to observe the characteristics of the piezoelectric passive damper. Excitation force from the impact hammer and the signal from the strain gages are processed at the FFT analyser. We choose the first and the second modes to suppress. For the first mode that is bending mode, the optimization location of a piezoelectric is near roots centered and for the second mode that is twisting mode, it is near root away from the center. The frequencies for the

Table 1. The material properties of plate and piezoceramics

Plate	Young's modulus Density	E=71.3 G ρ=2906 kg/m^3
Piezoceramics	Young's modulus (short) Dielectric constant Coupling coefficient Transverse d constant Capacitance Density	E=59 G h_{31}=8.606×10^8 N/C k_{31}=0.32 d_{31}=260e-12 m/V C_P^S = 77.7 nF ρ=7400 kg/m^3

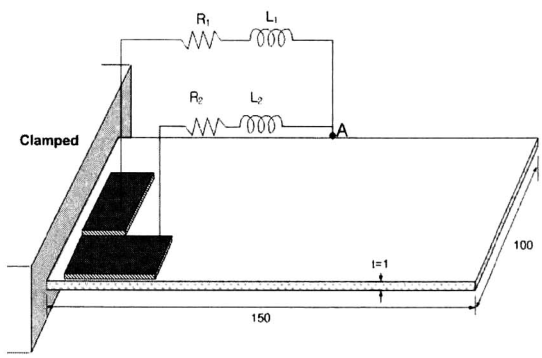

Fig. 2. Clamped plate with piezoelectric passive damper

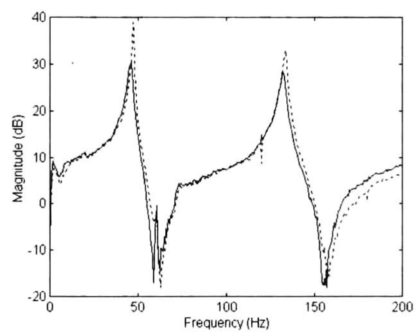

Fig. 3. Open circuit and optimal circuit transfer function, dotted line: open, dashed line: optimal values

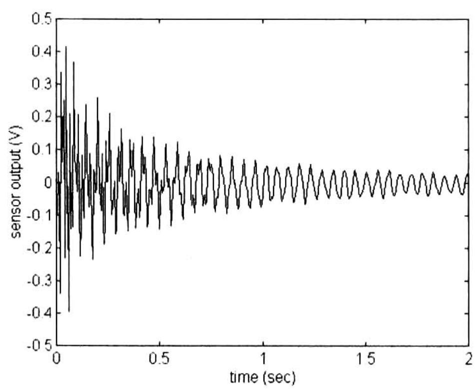

Fig. 4. Time history in the open circuit

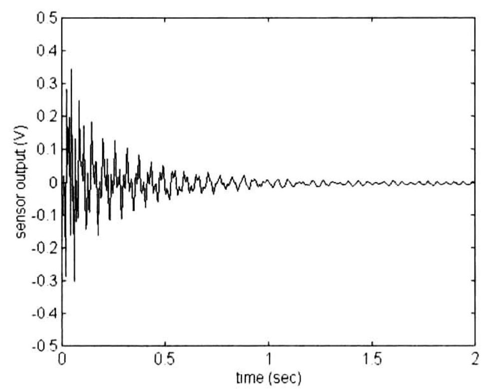

Fig. 5. Time history in the optimum circuits

first mode and the second mode are 47 Hz and 133.5 Hz. One piezoelectric patch with only one resonant circuit is adopted in the experiment. Optimal resistance and inductance are determined from the pole placement methodl. In the experiment, for the bending mode, L=138.4 H and R=13.89 kΩ. And for the twisting mode, L=18.321 H and R=2.687 kΩ. Fig.3 is frequency response plot for the first and the second modes. Dotted line represents the case of the open shunt circuit and dashed line denotes the case with the optimum resistance and inductance. Fig. 4 and Fig. 5 are the time history of sensor outputs in time domain. The first mode and the second mode are reduced by 9 dB and 5 dB, respectively.

Conclusions

In this paper, we investigate the vibrating characteristics of a cantilevered plate with piezoelectric passive damper. The dynamic governing equations for a plate with piezoelectric patches connected to multiple resonant shunted circuits are derived. The finite element methods are employed to approximate the governing equations for multiple resonant shunted circuits as well as a plate with piezoelectric patches to estimate more exactly the effectiveness of the piezoelectric passive damper. As for as location optimization of a piezoelectric patch concerned, the location to maximize the minimum modal actuating force can be optimized position. One piezoelectric patch with only one resonant circuit is adopted in the experiment. The first and second modes were chosen as the target modes for the experiment and the magnitudes in the frequency response were reduced by 9 dB and 5 dB, respectively.

References

1. N. W. Hagood and E. F. von Flotow, "Damping of Structural Vibrations with Piezoelectric Materials and Passive Electrical Networks", *Journal of Sound and Vibration*, Vol. 146, No. 2, pp. 243-268, 1991.

2. J. J. Hollkamp, "Multimode Passive Vibration Suppression with Piezoelectric Materials and Resonant Shunts", *Journal of intelligent Material Systems and Structures*, Vol. 5, pp. 49-57, January 1994.

3. C. L. Davis and G. A. Lesieutre, "A modal strain energy approach to the prediction of resistively shunted piezoceramic damping", *Journal of Sound and Vibration*, Vol. 184, pp. 129-139, 1995.

4. M. S. Tsai and K. W. Wang, "On the structural damping characteristics of active piezoelectric actuators with passive shunt", *Journal of Sound and Vibration*, Vol. 221, No. 1, pp. 1-22, 1999.

5. J. J. Hollkamp and R. W. Gordon, "An experimental comparison of piezoelectric and constrained layer damping", Proceedings of SPIE, Vol. 2445, pp. 123-133, 1995.

6. D. A. Saravanos, "Damped vibration of composite plates with passive piezoelectric-resistor elements", *Journal of Sound and Vibration*, Vol. 221, No. 5, pp. 867-885, 1999.