Design of Optimal Controllers for Spacecraft Formation Flying Based on the Decentralized Approach

Jonghee Bae* and Youdan Kim**
Department of Mechanical and Aerospace Engineering
Seoul National University, Seoul, Korea 151-742

Abstract

Formation controller for multiple spacecrafts is designed based on a decentralized approach. The objective of the proposed controller is to make each spacecraft fly to the desired waypoints, while keeping the formation shape of multiple spacecrafts. To design the decentralized formation controller, the output feedback linearization technique using error functions for goal convergence and formation keeping is utilized for spacecraft dynamics. The primary contribution of this paper is to proposed optimal controller for formation flying based on the decentralized approach. To design the optimal controller, eigenvalue assignment technique is used. To verify the effectiveness of the proposed controller, numerical simulations are performed for three-dimensional waypoint-passing missions of multiple spacecrafts.

Key Word : Spacecraft, Formation flying, Decentralized method, Optimal controller, Eigenvalue assignment technique

Introduction

Formation control for multiple spacecrafts has been an extensive research area over the decades. The multiple spacecrafts formation flying has several merits including the improved performance, reduced cost, reconfigurability, system robustness, and instrument resolution compared to a large single spacecraft [1-2].

Generally, there are two approaches to multiple spacecrafts formation method: centralized and decentralized method [3]. In the centralized method, there is a leader which works as a manager to offer a reference trajectory toward the goal, and the followers track the position and orientation with respect to the leader. The centralized method is easy to implement; however, the whole system may be collapsed when the leader has some problems. Moreover, this method requires full state information for communication with each other, and therefore there may be a delay in reaction to an unexpected situation [3]. In the decentralized method, on the other hand, each spacecraft has the desired objectives such as collision avoidance, goal seeking, and formation keeping. This approach can be implemented with less communication, and the formation objectives can be easily changed depending on the circumstances. However, it is difficult to analyze the stability of the formation control scheme [4-6].

The objective of this study is to propose an optimal controller for spacecraft formation flying based on the decentralized approach. It is assumed that each spacecraft has own objective to move to the given waypoints, and to keep the formation pattern. To design the formation controller, output feedback linearization is used and it is assumed that each spacecraft can communicate each other to share the information of position and attitude. The optimization of gain matrices is also performed to obtain better efficiency of spacecraft formation flying.

* Ph.D Candidate
** Professor
E-mail : ydkim@snu.ac.kr Tel : +82-2-880-7398 FAX : +82-2-887-2662

This paper is organized as follows. In second section, the output feedback linearization of spacecraft dynamics is derived. In third section, the decentralized formation controller and control gain optimization method are proposed. Numerical simulation and analysis to verify the performance of the proposed controller are described in fourth section. Finally, conclusions are presented in final section.

Spacecraft Dynamics

Consider the equations of motion of a spacecraft.

$$\dot{x} = \begin{bmatrix} v \\ g_r \\ \dot{q} \\ g_q \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ g_{u1} & 0 \\ 0 & 0 \\ 0 & g_{u2} \end{bmatrix} u \tag{1}$$

with

$$g_r = \begin{bmatrix} 2\dot{\theta}\dot{y} + \ddot{\theta}y + \dot{\theta}^2 x - \dfrac{\mu x}{\left\| xi + (y+R)j + zk \right\|^3} \\ -2\dot{\theta}\dot{x} - \ddot{\theta}x + \dot{\theta}^2 y - \mu\left(\dfrac{y+R}{\left\| xi + (y+R)j + zk \right\|^3} - \dfrac{R}{\left\| Rj \right\|^3} \right) \\ -\dfrac{\mu z}{\left\| xi + (y+R)j + zk \right\|^3} \end{bmatrix} \tag{2}$$

$$g_q = \begin{bmatrix} g_{q1} \\ g_{q2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -h\times\hat{q} + h\bar{q} - w\times\dot{\hat{q}} + w\dot{\bar{q}} \\ -h^T\hat{q} - w^T\dot{\hat{q}} \end{bmatrix} \tag{3}$$

$$h \triangleq -J^{-1}\left[w\times Jw \right]$$

$$g_{u1} = \frac{1}{m} I_{3\times 3}$$

$$g_{u2} = \begin{bmatrix} g_{u21} \\ g_{u22} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \hat{q}^{\times} + \bar{q}I_{3\times 3} \\ -\hat{q}^T \end{bmatrix} J^{-1} \tag{4}$$

where the state vector $x=[r\ v\ q\ \dot{q}]^T$ denotes the position $r=[x\ y\ z]^T \in R^{3\times 1}$, velocity $v \in R^{3\times 1}$, quaternion $q=[\hat{q}\ \bar{q}]^T \in R^{4\times 1}$, and quaternion rate of the spacecraft $\dot{q} \in R^{4\times 1}$. The control input vector $u=[f\ \tau]^T$ denotes the control force and torque, respectively. J, m, w, and μ are the moment of inertia, mass, angular velocity, and gravitational coefficient, respectively.

The output of the system is given by

$$Y = \begin{bmatrix} r \\ q \end{bmatrix} \tag{5}$$

Differentiating Eq. (5) with respect to time gives

$$\dot{Y} = \begin{bmatrix} \dot{r} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} v \\ \dot{q} \end{bmatrix}$$

Differentiating twice with respect to time yields

$$\ddot{Y} = \begin{bmatrix} g_r \\ g_{q1} \end{bmatrix} + \begin{bmatrix} g_{u1} & 0 \\ 0 & g_{u21} \end{bmatrix} \begin{bmatrix} f \\ \tau \end{bmatrix} \triangleq P + Gu \tag{6}$$

Note that the system is output feedback linearizable if the following condition is satisfied.

$$\det G = \det(g_{u1})\det(g_{u21}) = \frac{1}{2m}(\hat{q}^{\times} + \bar{q} I_{3\times 3}) J^{-1} \neq 0 \tag{7}$$

The above condition can be simplified as $\bar{q} \neq 0$ [7].

Let us defined the map $\Psi : R^{14} \mapsto R^{14}$ to be a diffeomorphism as

$$\zeta = \psi(x) \equiv \begin{bmatrix} r \\ q \\ v \\ \dot{q} \end{bmatrix} \tag{8}$$

$$x = \psi^{-1}(\zeta) = \begin{bmatrix} \zeta_1 \\ \zeta_3 \\ \zeta_2 \\ \zeta_4 \end{bmatrix} \tag{9}$$

Using Eq. (8) and Eq. (9), Eq. (1) can be transformed to

$$\begin{bmatrix} \dot{\zeta}_1 \\ \dot{\zeta}_2 \end{bmatrix} = \begin{bmatrix} \zeta_3 \\ \zeta_4 \end{bmatrix}$$
$$\begin{bmatrix} \dot{\zeta}_3 \\ \dot{\zeta}_4 \end{bmatrix} = \begin{bmatrix} g_r \\ g_q \end{bmatrix} + \begin{bmatrix} g_{u1} & 0 \\ 0 & g_{u2} \end{bmatrix} u \tag{10}$$

The control input using output feedback linearization can be obtained with a new pseudo control input ν as

$$u = G^{-1}\left(v - P\right) \tag{11}$$

Finally, the linearized system can be expressed as

$$\ddot{Y} = v \tag{12}$$

Decentralized Formation Flying and Optimal Controller.

Error Function

In this study, the decentralized formation controller is adapted, which has been studied for formation maneuvers of ground robots [6].

Let us consider two competing objectives to design decentralized formation controller. The first objective is to move the spacecrafts to the final goals. The second objective is to maintain the formation shape during the maneuver. Two error functions are considered to combine these objectives. The error function E_g for the first objective is the total error between the current position of the spacecraft and the desired goal.

$$E_g = \sum_{i=1}^{N} \tilde{Y}_i^T K_g \tilde{Y}_i \tag{13}$$

where N is the total number of spacecraft, Y_i is the position and quaternion of the i-th spacecraft, $\widetilde{Y_i} = Y_i - Y_i^d$, and K_g is symmetric positive definite matrix.

The error function E_f for the second objective is given by

$$E_f = \sum_{i=<N>} (\tilde{Y}_i - \tilde{Y}_{i+1})^T K_f (\tilde{Y}_i - \tilde{Y}_{i+1}) \tag{14}$$

where K_f is symmetric positive semi-definite matrix, and the notation $i=<N>$ is used to indicate a ring summation.

The total error function for the formation flying problem is defined as the sum of Eq. (13) and Eq. (14) as

$$\begin{aligned} E(t) &= E_g + E_f \\ &= \sum_{i=<N>} \left[\tilde{Y}_i^T K_g \tilde{Y}_i + (\tilde{Y}_i - \tilde{Y}_{i+1})^T K_f (\tilde{Y}_i - \tilde{Y}_{i+1}) \right] \end{aligned} \tag{15}$$

The objective of decentralized formation control is to make the total error function $E(t)$ converge to zero asymptotically. To simplify the equation, using the property of Toeplitz matrix C and Kronecker product [8], Eq. (15) can be written as

$$E = \frac{1}{2} \tilde{Y}^T (I_N \otimes K_g + C \otimes K_f) \tilde{Y} \tag{16}$$

where

$$\tilde{Y} = \begin{bmatrix} \tilde{Y}_1^T & \cdots & \tilde{Y}_N^T \end{bmatrix}^T \tag{17}$$

$$C = \begin{bmatrix} 2 & -1 & 0 & \cdots & 0 & -1 \\ -1 & 2 & -1 & 0 & \cdots & 0 \\ 0 & -1 & 2 & -1 & \cdots & 0 \\ \vdots & & & & & \vdots \\ 0 & \cdots & 0 & -1 & 2 & -1 \\ -1 & 0 & \cdots & 0 & -1 & 2 \end{bmatrix} \in \Re^{N \times N} \; C > 0 \qquad C \otimes J = \begin{bmatrix} 2J & -J & \cdots & -J \\ -J & 2J & \cdots & 0 \\ \vdots & & & \vdots \\ -J & \cdots & -J & 2J \end{bmatrix} \tag{18}$$

Decentralized Control Design

In this section, the coupled dynamics of formation controller is derived. Consider the following Lyapunov function candidate.

$$\begin{aligned} V &= E + \frac{1}{2} \sum_{i=1}^{N} \dot{\tilde{y}}_i^T \dot{\tilde{y}}_i \\ &= \frac{1}{2} \tilde{Y}^T (I_N \otimes K_g + C \otimes K_f) \tilde{Y} + \frac{1}{2} \dot{\tilde{Y}}^T \dot{\tilde{Y}} \end{aligned} \tag{19}$$

Differentiating Eq. (19) with respect to time gives

$$\begin{aligned} \dot{V} &= \dot{\tilde{Y}}^T (I_N \otimes K_g + C \otimes K_f) \tilde{Y} + \dot{\tilde{Y}}^T \ddot{\tilde{Y}} \\ &= \dot{\tilde{Y}}^T \left\{ (I_N \otimes K_g + C \otimes K_f) \tilde{Y} - \ddot{\tilde{Y}}^d + \nu \right\} \end{aligned} \tag{20}$$

where $\dot{Y} = [\dot{Y}_1^T \;\cdots\; \dot{Y}_N^T]^T$, and $\nu = [\nu_1^T \;\cdots \nu_N^T]^T$.

Let us propose a controller as

$$v = \ddot{Y}_d - (I_N \otimes K_g + C \otimes K_f)\tilde{Y} - (I_N \otimes D_g + C \otimes D_f)\dot{\tilde{Y}} \tag{21}$$

where $D_g = D_g^T > 0$, and $D_f = D_f^T > 0$. Substitution of Eq. (21) into Eq. (20) yields

$$\dot{V} = -\dot{\tilde{Y}}^T (I_N \otimes D_g + C \otimes D_f)\dot{\tilde{Y}} \tag{22}$$

Note that the Kronecker product of two positive definite matrices is positive definite. Therefore, Eq. (22) is negative semi-definite. Moreover, the total error function $E(t)$ converges to zero asymptotically by LaSalle's invariance principle, because the set $\{(\tilde{Y}, \dot{Y}) | \dot{V} = 0\}$ contains no trajectory excluding $\tilde{Y} = \dot{Y} = 0$ [7]. Therefore, the formation flying control law of the i-th spacecraft is given by

$$\begin{aligned} v_i &= \ddot{Y}_i^d - K_g \tilde{Y}_i - D_g \dot{\tilde{Y}}_i \\ &\quad - K_f(\tilde{Y}_i - \tilde{Y}_{i+1}) - K_f(\tilde{Y}_i - \tilde{Y}_{i-1}) - D_f(\dot{\tilde{Y}}_i - \dot{\tilde{Y}}_{i+1}) - D_f(\dot{\tilde{Y}}_i - \dot{\tilde{Y}}_{i-1}) \end{aligned} \tag{23}$$

Optimal Controller

Total error function is defined as a function of the weight matrices K_g and K_f that are related to two competing objectives: goal convergence and formation keeping. Thus, weighting matrices should be selected considering the performance of spacecraft formation flying such as oscillations of each spacecraft's trajectory, the goal convergence time, and the consumption of the control input. Now, let us consider an optimization problem.

Substituting Eq. (23) to Eq. (12) gives the following equation.

$$\ddot{\tilde{Y}}_i + \left\{ -D_f \dot{\tilde{Y}}_{i-1} + (D_g + 2D_f)\dot{\tilde{Y}}_i - D_f \dot{\tilde{Y}}_{i+1} \right\} + \left\{ -K_f \tilde{Y}_{i-1} + (K_g + 2K_f)\tilde{Y}_i - K_f \tilde{Y}_{i+1} \right\} = 0 \tag{24}$$

The above equation can be expressed as the general mechanical second order equation

$$\ddot{\tilde{Y}} + C\dot{\tilde{Y}} + K\tilde{Y} = 0 \tag{25}$$

where

$$\ddot{\tilde{Y}} = \begin{bmatrix} \ddot{\tilde{Y}}_{i-1} \\ \ddot{\tilde{Y}}_i \\ \ddot{\tilde{Y}}_{i+1} \end{bmatrix} \tag{26}$$

$$C = \begin{bmatrix} D_g + 2D_f & -D_f & -D_f \\ -D_f & D_g + 2D_f & -D_f \\ -D_f & -D_f & D_g + 2D_f \end{bmatrix} \qquad K = \begin{bmatrix} K_g + 2K_f & -K_f & -K_f \\ -K_f & K_g + 2K_f & -K_f \\ -K_f & -K_f & K_g + 2K_f \end{bmatrix} \tag{27}$$

Note that Eq. (25) that the gain matrices K_g and K_f behave like the stiffness matrix, while D_g and D_f act like the damping matrix of the mechanical second order system.

In this study, it is assumed that K_g, K_f, D_g, and D_f are diagonal matrices.

$$\begin{aligned} K_g &= k_g I_3 \qquad & K_f &= k_f I_3 \\ D_g &= d_g I_3 \qquad & D_f &= d_f I_3 \end{aligned} \tag{28}$$

where $k_g > 0$, $k_f \geq 0$, $d_g > 0$, and $d_f \geq 0$ are scalar variables. Then, the x-coordinate position error equation can be expressed as

$$\begin{bmatrix} \ddot{\tilde{x}}_{i-1} \\ \ddot{\tilde{x}}_i \\ \ddot{\tilde{x}}_{i+1} \end{bmatrix} + \begin{bmatrix} d_g + 2d_f & -d_f & -d_f \\ -d_f & d_g + 2d_f & -d_f \\ -d_f & -d_f & d_g + 2d_f \end{bmatrix} \begin{bmatrix} \dot{\tilde{x}}_{i-1} \\ \dot{\tilde{x}}_i \\ \dot{\tilde{x}}_{i+1} \end{bmatrix} + \begin{bmatrix} k_g + 2k_f & -k_f & -k_f \\ -k_f & k_g + 2k_f & -k_f \\ -k_f & -k_f & k_g + 2k_f \end{bmatrix} \begin{bmatrix} \tilde{x}_{i-1} \\ \tilde{x}_i \\ \tilde{x}_{i+1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{29}$$

Similarly, the y- and z-coordinate position error equations can be obtained.

Let us consider the augmented state as follows

$$X = \begin{bmatrix} \tilde{x} \\ \dot{\tilde{x}} \end{bmatrix} \tag{30}$$

With the augmented states, Eq. (29) becomes

$$\begin{aligned} \dot{X} &= AX + Bu \\ u &= -K_a X \end{aligned} \tag{31}$$

where

$$A = \begin{bmatrix} 0_{3\times3} & I_3 \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix} \quad B = \begin{bmatrix} 0_{3\times3} \\ I_3 \end{bmatrix} \quad K_a = \begin{bmatrix} K_d & C_d \end{bmatrix} \tag{32}$$

To design the feedback gain matrix K_a, eigenvalue assignment approach is adopted. In this approach, the desired closed-loop poles are selected considering the reasonable overshoot and rise time.

To sole the closed-loop eigenvalue problem, the gain parameter vector p is given by

$$p = \begin{bmatrix} k_g & k_f & d_g & d_f \end{bmatrix}^T \tag{33}$$

Let us consider the right and left eigenvalue problems of the closed-loop system.

$$\begin{aligned} \text{right}: \quad & \lambda_i \phi_i = (A - BK_a)\phi_i \\ \text{left}: \quad & \lambda_i \varphi_i = (A - BK_a)^T \varphi_i \end{aligned} \tag{34}$$

The objective is to find the parameter vector p that satisfies the design objectives: (i) to assign some of the eigenvalues to a suitable region of the complex plane, and (ii) to maximize a robustness of the closed-loop system. Therefore, the problem of finding the parameter vector p is stated as a nonlinear parameter optimization problem in which the closed-loop eigenvalues become the desired eigenvalues. In this study, the gain parameter vector is obtained using minimum norm correction algorithm with the following constraint [9].

$$f(p) = \lambda(p) - \lambda_d = 0 \tag{35}$$

where $\lambda(p) = \lambda(A - BK_a(p))$, and λ_d is the desired eigenvalues.

Numerical Simulation and Performance Analysis.

In this section, numerical simulation is performed to evaluate the decentralized formation flying control with control matrices described in the previous section. A three-spacecraft formation flying problem is considered. However, it can be easily extended to multiple spacecrafts formation flying. The mission is that each spacecraft should fly passing through two waypoints.

The formation patterns are chosen as follows.

$$P_0 = \left\{ \begin{bmatrix} -30 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 30 \\ 0 \end{bmatrix}, \begin{bmatrix} 30 \\ 0 \\ 0 \end{bmatrix} \right\},\ P_1 = \left\{ \begin{bmatrix} 40 \\ 60 \\ 70 \end{bmatrix}, \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}, \begin{bmatrix} 80 \\ 60 \\ 70 \end{bmatrix} \right\},\ P_2 = \left\{ \begin{bmatrix} 130 \\ 140 \\ 20 \end{bmatrix}, \begin{bmatrix} 150 \\ 190 \\ 20 \end{bmatrix}, \begin{bmatrix} 170 \\ 140 \\ 20 \end{bmatrix} \right\}$$

where the unit of position is meter. The next desired target waypoints are updated when the spacecrafts are passing by the current desired waypoints.

The natural frequency and damping ratio of desired eigenvalues are selected as $\xi = 0.707$ and $\omega_n = 1.0$ for fast response and little oscillation. Then, the desired eigenvalues are

$$\lambda_d = \begin{bmatrix} 0.707 + j0.707 & 0.707 - j0.707 \end{bmatrix}^T$$

The optimized gain matrices using eigenvalue assignment method are obtained as

$$K_g = 0.9829 I_3$$
$$K_f = 1.0018 I_3$$
$$D_g = 1.4054 I_3$$
$$D_f = 0.0029 I_3$$

Figure 1 shows how three spacecrafts fly passing through the desired waypoints in three-dimensional space. Each spacecraft moves to the first waypoint and changes to the next waypoint as soon as it arrives. The formation shape becomes similar, and each spacecraft converges to the final waypoint. Moreover, the trajectories of spacecrafts are pretty smooth passing through the waypoints. Figures 2 and 3 illustrate the histories of position and quaternion, respectively, and Fig. 4 shows the control input history. As shown in Fig. 4, the control inputs are fluctuated much when the spacecraft change its direction toward the next waypoint. Fig. 5 shows the distance between spacecrafts. It describes that each spacecraft moves to the final goal without any collision. Table 1 summarizes the convergence time and the consumption of control inputs.

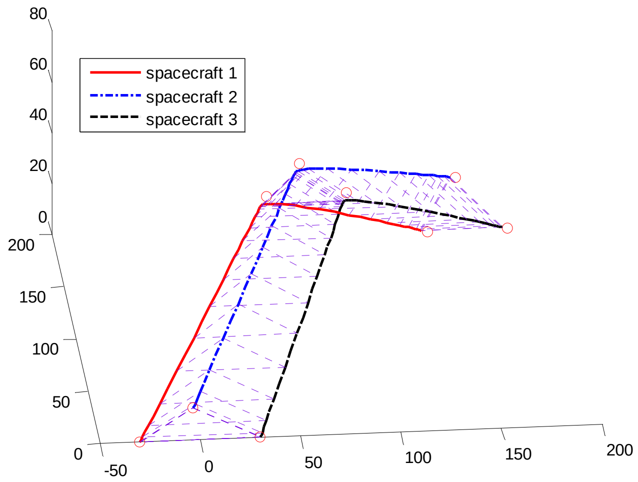

Fig. 1. Three-dimensional Trajectories

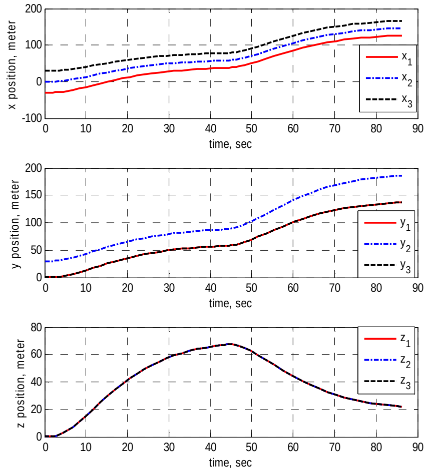

Fig. 2. Position History

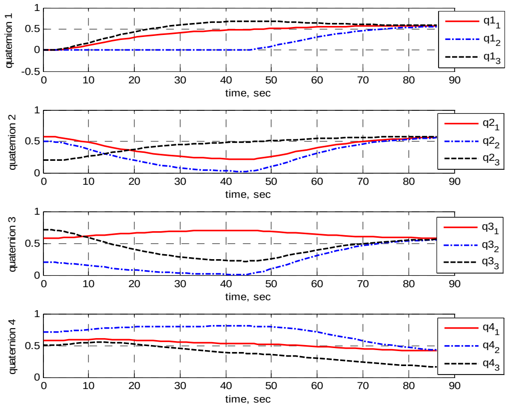

Fig. 3. Quaternion History

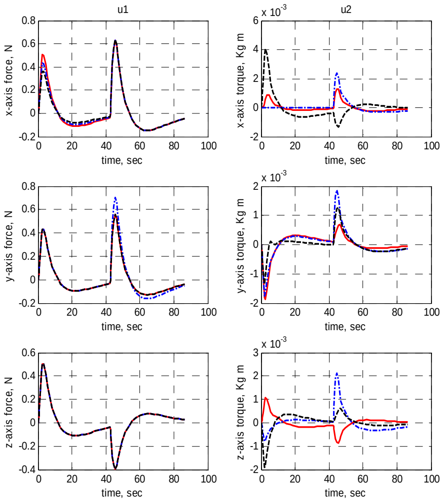

Fig. 4. Control Input History

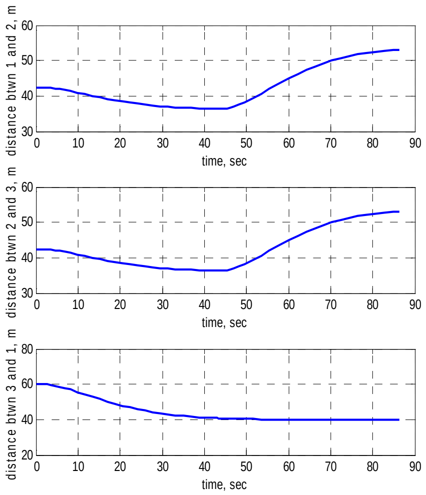

Fig. 5. Distance between Spacecrafts

Table 1. Spacecraft Formation Flying Performance

Convergence time to final waypints (s)	86.2339
Consumption of control force (N)	49.4755
Consumption of control torque (kg m)	0.1331

Conclusions

In this paper, decentralized spacecraft formation flying method is proposed. The output feedback linearization is applied for six degree-of-freedom spacecraft dynamics model. The goal convergence and formation keeping error functions are adapted for decentralized formation flying. To improve the performance of formation flying, optimal controller is designed based on eigenvalue assignment technique. Numerical simulation results indicate that the proposed control system can maintain the formation shape in the efficient way. The proposed method can be applied not only for the satellite formation flying but also for the multiple vehicle formation operation such as ground vehicles, mobile robots, and aerial robots.

Acknowledgement

This research has been supported by KARI under the KOMPSAT-3 Development Program that is funded by the MEST (Ministry of Education, Science, and Technology) of the Republic of Korea.

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