A Numerical Approach for Station Keeping of Geostationary Satellite Using Hybrid Propagator and Optimization Technique

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Abstract

In this paper, a method of station keeping strategy using relative orbital motion and numerical optimization technique is presented for geostationary satellite. Relative position vector with respect to an ideal geostationary orbit is generated using high precision orbit propagation, and compressed in terms of polynomial and trigonometric function. Then, this relative orbit model is combined with optimization scheme to propose a very efficient and flexible method of station keeping planning. Proper selection of objective and constraint functions for optimization can yield a variety of station keeping methods improved over the classical ones. Nonlinear simulation results have been shown to support such concept.

Key Word : Geostationary Satellite, Station Keeping, Orbit Propagation, Optimization Technique

Introduction

The geostationary satellites are useful for communication and observation mission because they appear to remain stationary with respect to a fixed point on the rotating Earth. However, orbital perturbations such as geo-potential perturbation, Sun and Moon gravity forces, and solar pressure cause the satellite to drift. It is essential to control the satellite within station keeping boundary as accurately as possible. Moreover, as the number of the operational satellites in geostationary orbit is on the increase, the requirement for station keeping control is getting stringent due to not only the limited longitudinal resource but also the possibility of frequency interference with the neighboring countries[1].

The precise orbit propagation means that the fully nonlinear equations of orbital motion are numerically integrated to produce the position and velocity vectors at an arbitrary time[2]. For this, highly accurate numerical integration algorithms and precise models of the perturbing forces must be employed to assure the prediction accuracy. Alternatively, a closed-form solution is also available if the equations of motion can be reformulated in a suitable form for analytical integration and the mathematical models of the perturbing forces are simplified or approximated[3, 4]. Hybrid orbit compression model can be used with the advantage of accuracy and simplicity[5,

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6]. This model is to obtain the solution of the relative motion with respect to a known reference orbit. For example, a circular orbit is selected as a reference orbit and the motion in the perturbed orbit is described in terms of a set of linear differential equations called the Clohessy - Wiltshire (CW) equations[7].

The geostationary orbit is defined by constant semi-major axis, zero eccentricity and zero inclination. These orbital elements, however, tend to deviate from ideal values because of various perturbing forces, such as the oblateness of the Earth and the gravitational force of the Sun and Moon. The objective of station keeping is to maintain the orbital elements within the allowable limits for a given period. The classical strategies for east-west station keeping, for example, were drift rate compensation targeting method and perigee Sun tracking method[8, 9]. These strategies make use of the orbital elements to obtain target orbit and keep spacecraft's position indirectly within tolerance box limit. However, it is necessary to control position and velocity of spacecraft directly for precise station keeping.

The objective of this paper is to propose a new method of modeling the orbital motion and to show that it can be applied in conjunction with an optimization technique for station keeping. For this, an innovative method of orbit propagation is introduced. A simple way of predicting the future position and velocity of the orbital motion using closed form algebraic function is applied to the problems of station keeping. Combining with the optimization algorithm, a variety of station keeping strategies can be devised. A proper combination of objective and constraint functions may lead to several and flexible methods for station keeping that will mitigate the rigidity of the conventional station keeping procedure.

Hybrid Orbit Representation for Geostationary Spacecraft

Relative Motion Modeling between Neighboring Orbits

The method for orbit modeling using relative motion usually defines reference orbit and introduces residual vector that is the difference of real orbit and reference orbit. Then, the residual vector is linearized with respect to the reference orbit and expressed with the analytical function. This method provides the easy way to find the analytic solution because it describes relative motion of the real orbit with respect to well-known reference orbit. For example, Hill Clohessy-Wiltshire(CW) equation has been widely used, but its application is limited because it does not include orbital perturbation.

In this paper, the orbit compression method is proposed to model the relative motion of the geostationary spacecraft. The orbit compression method represents the position vector of a spacecraft in terms of simple power and trigonometric functions[10]. For this, the real orbit is obtained by integrating the nonlinear equations of motion numerically. Perturbations due to the various sources can be included as needed. Then, a reference orbit that should remain close to the real orbit is introduced. Finally, position residual vector is approximated to the closed-form solution with the various basis functions considering orbital motion. In case of geostationary orbit, ideal geostationary orbit that means equatorial and circular orbit is a candidate for the applicable reference orbit. In this case, semi-major axis simply can be written as

$$a=\left(\frac{\mu_e}{n^2}\right)^{1/3}=\left(\frac{\mu_e}{\omega_e^2}\right)^{1/3} \tag{1}$$

where μ_e and n, respectively, denote the Earth's gravitational parameter and mean motion of spacecraft. And, ω_e is the rate of the Earth's rotation.

Figure 1 represents several orbits to be used in this paper. The real orbit is defined as that obtained by performing a highly precise numerical integration of the equations of motion that include various perturbation forces. Then, the reference orbit is introduced as an ideal geostationary orbit which has constant value of semi-major axis over time.

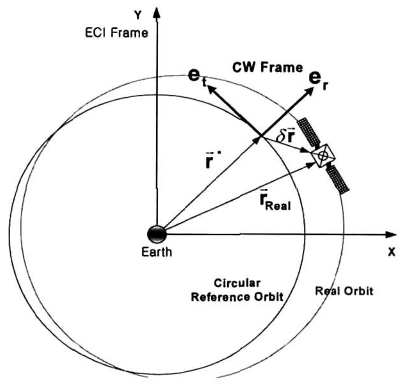

Fig. 1. Real Orbit and Reference Orbit

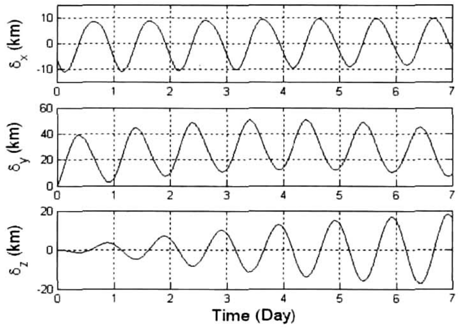

Fig. 2. Time Histories of Residual

Referring to figure 1, the reference position and velocity in CW Frame are expressed as:

$$\vec{r}^{\,*} = a\vec{e}_r \tag{2}$$

$$\vec{v}^{\,*} = an\vec{e}_t \tag{3}$$

The residual vector is defined as the difference between the real and the reference position vector, and figure 2 shows the time histories of the residual vector along radial, tangential and normal directions, respectively.

$$\vec{\delta r} = \vec{r}_{Real} - \vec{r}^{\,*} \tag{4}$$

If this residual vector can be expressed using characteristic frequency that reflects orbital motion, it is possible to represent real orbit accurately in combination with the well-known reference orbit. The selection of the basis function of residual vector is derived from analytical solution of geostationary orbit[11]. We have adopted residual reconstruction function for radial direction using polynomial and Fourier series of the following form by considering trade-off between accuracy and simplicity from iterative numerical simulation.

$$\begin{aligned}\delta x &= A_1 + A_2 t + A_3 t^2 \\ &+ A_4 \sin\omega_e t + A_5 \sin 2\omega_e t + A_6 \sin 3\omega_e t + A_7 \cos\omega_e t + A_8 \cos 2\omega_e t + A_9 \cos 3\omega_e t \\ &+ A_{10} \sin\Omega_s t + A_{11} \sin 2\Omega_s t + A_{12} \sin 3\Omega_s t + A_{13} \cos\Omega_s t + A_{14} \cos 2\Omega_s t + A_{15} \cos 3\Omega_s t \\ &+ A_{16} \sin\Omega_m t + A_{17} \sin 2\Omega_m t + A_{18} \sin 3\Omega_m t + A_{19} \cos\Omega_m t + A_{20} \cos 2\Omega_m t + A_{21} \cos 3\Omega_m t \\ &+ A_{22} t \sin\omega_e t + A_{23} t \cos\omega_e t + A_{24} \sin\omega_m t + A_{25} \cos\omega_m t + A_{26} \sin 2\omega_m t + A_{27} \cos 2\omega_m t\end{aligned} \tag{5}$$

Where Ω_s and Ω_m , respectively, are the relative angular velocity of the Sun and the Moon with respect to the Earth. And the residual reconstruction functions for the tangential and normal direction are expressed by using the same basis function as above radial direction. The normal equations that can be used to find the least square solution is applied to find the coefficients in Eq (5). The normal equations give the value of coefficients that minimize the sum of squares of error which means the difference between true orbit and reconstructed orbit[12].

The procedure for hybrid orbit propagation method using relative orbit modeling is described as follows. At ground station, residual vector is generated numerically from high precision orbit propagation data and reference orbit information. And then, orbit compression procedure is implemented to obtain the related coefficients. These coefficients are transmitted to the spacecraft, and the spacecraft receives them and determines the residual vector using Eq. (5). Finally, the spacecraft can calculates its reconstructed orbit as follows:

$$\vec{\vec{r}} = \vec{r}^{\,*} + \vec{\delta r} = [\,a\ 0\ 0\,]^T + [\,\delta x\ \delta y\ \delta z\,]^T \tag{6}$$

If the reconstructed orbit using Eq. (6) is nearly close to the real orbit, it is possible to predict the orbital information by simple operation instead of complex numerical integration process.

Performance Evaluation of Hybrid Orbit Propagation

The performance of the proposed relative orbit propagation method is validated through numerical simulation. For this purpose, the perturbing forces used for the real orbit are the Earth's potential of 4th order, the solar and lunar attractive forces, and solar radiation pressure. To support the modeling accuracy, time histories for the position and velocity error, which is the difference real orbit and reconstructed orbit, are presented and root mean square values are provided. And, pointing error, which is important parameter to determine nadir direction, is considered. Figure 3 depicts the time histories of the position, velocity, and pointing error.

The RMS(Root Mean Square) of the final error with respect to the operation period is summarized in Table 1. Because the basis function of the residual reconstruction function in Eq. (5) is based on 7-day operation, the best performance is archived when the same operation period is applied to the orbit propagation. Therefore, residual reconstruction functions should be redefined considering long term orbital motion for long term operation.

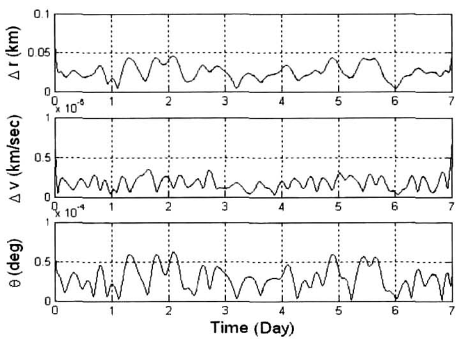

Fig. 3. Difference between Real and Reconstructed Orbit

Table 1. RMS Error of Relative Orbit Propagation

Propagation Period	Position (km)	Velocity (m/sec)	Pointing (Deg.)
7-days	0.027131	0.001922	0.000032
14-days	0.188487	0.012879	0.000243
28-days	0.605172	0.038914	0.000805

Station Keeping Control using Relative Orbit Modeling and Optimization Technique

Station Keeping Strategy

The main purpose of station keeping is to maintain the spacecraft within the predefined longitude and/or latitude limit. Therefore, station keeping maneuvers should be performed regularly during the mission lifetime to compensate for the natural perturbations that tend to change the orbit. This can be achieved by using thrusters of the spacecraft. For this, it is necessary to determine the precise position and to calculate firing time and velocity increment considering mission schedule and remaining fuel mass.

For the station keeping planning, it is easier to analyze the orbital motion in local frame than inertial frame. So, relative motion with respect to the nominal longitude and/or latitude should be employed and analyzed. As described in the previous section, relative orbit propagation method may be the candidate for the applicable orbit modeling. The relative motion of spacecraft is the summation of free drift term and orbit correction term. It is assumed that both quantities can be linearly added to denote the total change, and written as follows:

$$\delta\vec{r}(t) = [\delta x(t)\delta y(t)\delta z(t)]^T = \delta\vec{r}^{\,*}(t) + \sum^{N} \Psi(t-T_i)\Delta\vec{V}_i\, u(t-T_i) \tag{7}$$

where $\vec{\delta r}^*$ and N are, respectively, the relative position vector due to free drift and total maneuver number. From the Hill's Equation, the state transition matrix $\Psi(\tau)$ can be written[13].

$$\Psi(\tau) = \begin{bmatrix} \frac{\sin\omega_e\tau}{\omega_e} & \frac{2}{\omega_e}(1-\cos\omega_e\tau) & 0 \\ -\frac{2}{\omega_e}(1-\cos\omega_e\tau) & \frac{4\sin\omega_e\tau - 3\omega_e\tau}{\omega_e} & 0 \\ 0 & 0 & \frac{\sin\omega_e\tau}{\omega_e} \end{bmatrix} \tag{8}$$

The east-west and north-south direction is only considered for the velocity increment vector due to impulsive maneuver ΔV_i. And, $u(\tau)$ is the unit step function, which is 1 if τ is greater than zero, and 0 otherwise.

In classical station keeping method, orbital configuration is controlled using drift rate, eccentricity vector, and inclination vector. This means that the station keeping control for the spacecraft's position is performed indirectly. However, it is possible to control spacecraft's position directly because the closed form function of relative motion with respect to the nominal position is readily available. For this, the velocity increments and firing time could be determined using Eq. (7) such that relative motion should be maintained within specific boundary conditions.

Optimization Technique for Objective and Constraint Function

The direction and size of the station keeping maneuver may be calculated by means of an optimization technique. Generally, fuel consumption is particularly important to geostationary orbit operation because it is directly related to the given mission lifetime. The objective function for fuel minimum is commonly used as follows:

$$J = \sum_{i=1}^{N} \Delta V_i^2 \tag{9}$$

Otherwise, if one may want to minimize the maximal size of relative motion with respect to the nominal longitude and/or latitude, it is possible to design the objective function employing mini-max problem.

$$\textit{Minimize the Maximum of } |\vec{\delta r}(t)|, \quad t_o < t < t_f \tag{10}$$

The selection of objective functions depends on the requirement of the mission. Using these functions, the various station keeping planning strategies can be implemented.

In optimization procedure, constraint function should be defined to maintain spacecraft within the allowed box limit. In case of using relative orbit modeling, predefined tolerance box size can be simply converted to the distance range as follows:

$$\begin{aligned} |\delta x(t)| &< \delta x_{\max}, & t_o < t < t_f \\ |\delta y(t)| &< \delta y_{\max}, & t_o < t < t_f \\ |\delta z(t)| &< \delta z_{\max}, & t_o < t < t_f \end{aligned} \tag{11}$$

The tolerance size and boundary conditions are easily determined according to the operational requirements. The main feature of these constraint conditions is to define station keeping box in connection with relative position vector. Other constraints such as maneuver number and time can be added to the optimization problem.

Nonlinear Simulation and Discussion

Nonlinear simulation has been performed using the relative orbit modeling and optimization technique to support the proposed method. It is assumed that the 2-burn east-west station keeping maneuver is performed once a week and north-south station keeping maneuver is implemented biweekly. For optimization, Matlab function 'fminmax' and 'fminmax' have been used to find the solution for a constrained minimization and mini-max optimization respectively.

Firstly, fuel minimum strategy is considered for the objective function. In case of rectangular tolerance box, constraint function is applied to maintain the spacecraft within box limit from current planning time to the next planning time. It can be written as

$$J_{EW} = |\Delta V_{EW}^{1}| + |\Delta V_{EW}^{2}| \quad \text{and} \quad J_{NS} = |\Delta V_{NS}| \tag{12}$$

$$|\delta y(t)| = \left| \delta y^{*}(t) + \sum_{i=1}^{N} \Psi_{y}(t - T_{EW}^{i}) \Delta V_{EW}^{i} \; u(t - T_{EW}^{i}) \right| < \delta y_{\max} \quad (\text{Where, } t_{EW}^{P} < t < T_{EW}^{P+1}) \tag{13}$$

The constraint function for the north-south maneuver planning can be considered in similar way. And, the nominal longitude is set to 116E° and the tolerance range is ±0.05°. Therefore, the maximum range of relative motion is approximately ±30km. Figure 4 shows the time histories of controlled relative motion and spacecraft's trajectory with station keeping box in CW frame for 100 days. The relative motion is well controlled within the given boundary condition. The control for radial direction of spacecraft is not applied, but it remains within specific range due to the coupling effect with tangential direction.

Secondly, the minimum separation distance strategy for the objective function is considered. It is to control the spacecraft more accurately by minimizing drift distance from the nominal position. It can be written as

$$\textit{Minimize the Max. of } J_{EW} = |\delta y(t)| = \left| \delta y^{*}(t) + \sum_{i=1}^{N} \Psi_{y}(t - T_{EW}^{i}) \, \Delta V_{EW}^{i} \, u(t - T_{EW}^{i}) \right| \tag{14}$$

The objective function for the north-south maneuver planning, also, can be considered in similar way, and other constraint condition is the same as the previous case. The simulation results are shown in figure 5. As shown in the figure, the tangential component of relative motion is more tightly kept within boundary condition, and the performance of station keeping is

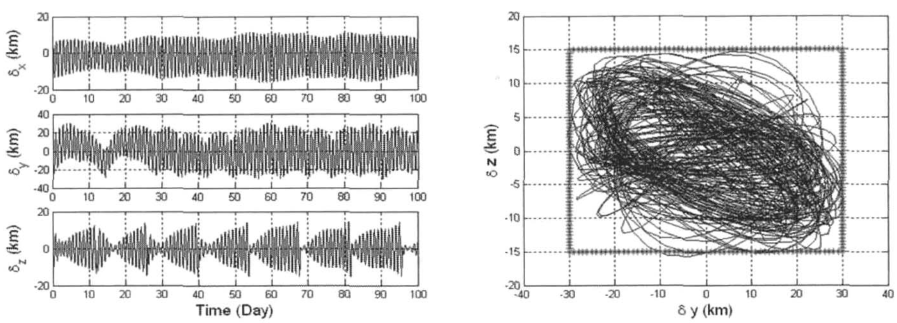

Fig. 4. Controlled Relative Motion and Spacecraft's Trajectory for Fuel Minimum Case

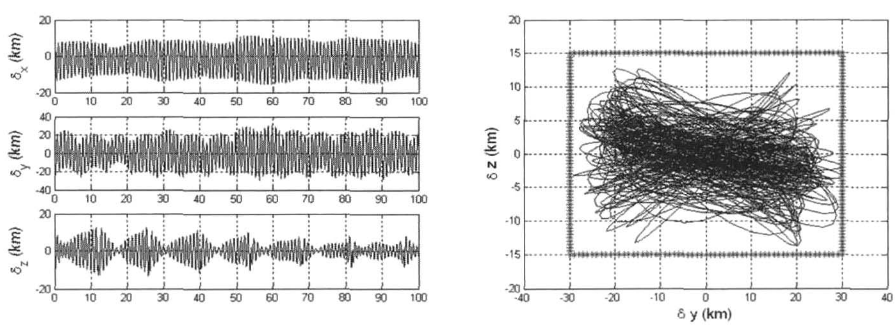

Fig. 5. Controlled Relative Motion and Spacecraft's Trajectory for Distance Minimum Case

quite excellent in the sense that the spacecraft remains accurately within the tolerance box. However, the total amount of velocity increment for station keeping is slightly greater than that of previous case because there is no consideration with respect to fuel consumption.

Conclusions

In this paper, a new method for station keeping of geostationary satellite using relative orbit modeling and optimization technique has been developed. The relative orbit modeling provides the simplicity for orbit prediction, and results in very small final error between the true and approximated orbit during the period of orbit propagation. The various station keeping planning strategies have been applied and demonstrated by combination with objective function and constraint condition. Using this method, spacecraft's position can be directly controlled based on the relative motion with respect to the nominal position. Nonlinear simulation results have demonstrated that the spacecraft can be tightly controlled within station keeping box.

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