# Coordinated Simultaneous Attitude Pointing for Multiple Satellites Under Formation Flying

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## Abstract

In this paper, attitude control laws for simultaneous pointing of multiple spacecrafts are considered under a formation flying scenario. The basic approach lies in adaptive feedback gains using relative attitude information or maneuver time approximation for coordinated attitude control. Each control law is targeted to balancing mean motion or to correcting system response to the slowest satellite. The control gain adaptation is constructed by two approaches. The first one is using variable damping gain to manipulate speed of a fast system response, and the second one uses alternate natural frequency of the system under control. The validity and stability of the proposed approaches are examined analytically and tested through numerical simulations.

**Key Word** : coordinated attitude control, simultaneous pointing, formation control

## Introduction

In recent years, multiple spacecraft flying in formation has been studied extensively in the literature, and observed practical applications to many missions. Satellite formation flying offers improved feasibility, accuracy, robustness, flexibility, cost, energy efficiency, and probability of success for new innovative space missions. Spacecraft formation-flying applications using multiple micro spacecraft instead of a monolithic spacecraft can reduce mission cost improving system robustness and accuracy[1].

The formation flying can be divided into two main categories based on the ambient dynamical environment. In Deep Space(DS) mission, relative spacecraft dynamics reduce to a double integrator form. The second main category is Planetary Orbital Environments(POE), where spacecraft are subjected to significant orbital dynamics and environmental disturbances[2]. Especially, POE attempts to extend performance of earth observing satellites with agile actuators. For example, NASA's New Millennium program has an enhanced formation flying experiment in the Earth Observing Mission. ESA has developed an elaborate formation for its Cluster mission to study the Earth's magnetosphere[4]. Characteristically, control of satellites in formation comprises both orbit control and attitude control.

For precise pointing of earth observing satellites, coordinated attitude control for multi-agent has emerged as a key issue, and three primary methods have been investigated; leader-follower, behavior-based, and virtual structure with relative orbit control problems. Two primary objectives, formation-keeping and station-keeping have received attention in the

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coordinated attitude control problems. Station-keeping refers to driving the spacecraft to its absolute desired attitude. On the other hand, formation-keeping tries to align the spacecraft with other spacecraft in the formation[5]. Thanks to capability of formation flying, multi-mission has become available also in attitude control side with agile spacecraft systems. But most of formation control literatures are formulated under identical spacecraft model or in attitude formation. All of these can lead to satisfactory performance under attitude formation, but do not have proper operation for some special cases like coordinated attitude rotations under different conditions or other initial states.

This paper concerns simultaneous pointing of multiple satellites under formation flying configuration. That is, a group of multi-agent are commanded to the same or severally commanded direction from different initial conditions. Intuitively speaking, simultaneous pointing is analogous to the optimal control problem under a fixed final time. But because spacecraft attitude dynamics are fully nonlinear, it is hard to correct a fixed final time during attitude maneuvers. And solution of a single optimal problem has no direct connection to multi-agent control problems. Therefore, in this paper we introduce simultaneous attitude pointing with coordinated attitude control strategy with free maneuver time. Simultaneous pointing control may consist of two objectives. The first goal is to assess maneuver ability of present satellites compared with other satellites, and the second is to control the whole system to accomplish mission goal. Comparison of relative attitude and approximated maneuver time is made from the perspective of the first goal. Control gain adaptation technique in a behavior-based approach is proposed for simultaneous pointing maneuvers.

## Spacecraft Attitude Dynamics

First, a rigid body rotational equations of motion from Euler's equation can be written as

$$J\dot{\omega} = \Omega J \omega + u \tag{1}$$

where $J$ is the inertia matrix of total spacecraft system, $\omega = [\omega_1\ \omega_2\ \omega_3]^T$ denotes the angular rate vector of the spacecraft in body-fixed axes, and $u = [u_1\ u_2\ u_3]^T$ denotes the control torque vector. A skew-symmetric matrix $\Omega$, for simplification, is defined as

$$\Omega = -\begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{2}$$

For attitude representation, quaternion, sometimes called Euler parameters, is defined in terms of the principal rotation components such that

$$q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = e\sin(\phi/2), \quad q_4 = \cos(\phi/2) \tag{3}$$

where $e$ represents a unit principle axis vector and $\phi$ is the Euler angle of rotation. The quaternion also satisfies a unit-norm constraint and

$$q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1 \tag{4}$$

whereas the quaternion inverse is given by

$$q^{-1} = [-q_1\ -q_2\ -q_3\ q_4]^T$$

The kinematic differential equations for the quaternion are given by

$$\begin{aligned}\dot{q} &= \frac{1}{2}\Omega q + \frac{1}{2}q_4\omega \\ \dot{q}_4 &= -\frac{1}{2}\omega^T q\end{aligned} \tag{5}$$

## Quaternion Feedback Control Law

Spacecraft orientation is commonly described in terms of the quaternion. Some other options such as Modified Rodrigues Parameter(MRP) are readily available. However, the quaterion attitude parameter and associated feedback control laws design have taken a central position in spacecraft attitude control area. Probably, one of th most well-known attitude control law is in form of a PD control law which consistis of quaternion attitude error and body angular rate[7]

$$u = -Kq_e - D\omega \tag{6}$$

where $K$, $D$ are controller gain matrices, and $q_e = [q_{1e}\ q_{2e}\ q_{3e}]^T$ refers to attitude error between desired quaternion($q_c$) and present quaternion($q$), as follows;

$$\begin{aligned} q_e &= q^{-1} \otimes q_c \\ &= \begin{bmatrix} q_{4c} & q_{3c} & -q_{2c} & -q_{1c} \\ -q_{3c} & q_{4c} & q_{1c} & -q_{2c} \\ q_{2c} & -q_{1c} & q_{4c} & -q_{3c} \\ q_{1c} & q_{2c} & q_{3c} & q_{4c} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \end{aligned} \tag{7}$$

The closed-loop nonlinear systems of a rigid spacecraft with the quaternion feedback control logic is usually globally asymptotically stable in the Lyapunov sense. According to the convergency of the quaternion feedback control, the control logic applies to attitude control of not only a single spacecraft but also coordinated maneuver for multiple spacecraft in formation flying.

A coordinated attitude control law about $j$th spacecraft using the quaternion feedback under behavior-based approach was proposed by Mattew[5];

$$u_j = -\sum_{k=0}^{n} K_{jk} q_{jk} - \sum_{k=0}^{n} D_{jk}\omega_{jk} \tag{8}$$

where $q_{jk}$ and $\omega_{jk}$ correspond to attitude and angular rate errors between $i$th and $j$th spacecrafts. In addition, $q_{j0}$ and $\omega_{j0}$ denote errors between absolute reference and the $j$th spacecraft itself. Consequently, the control law in Eq.(8) is constructed by summing the control action for the station-keeping and formation-keeping objectives. Global asymptotic stability and convergence of the control law were established also in Ref. [5]. The principal idea of Eq.(8) is to take into account states of other satellites for the purpose of coordinated formation control.

In this paper, we propose a simultaneous pointing control strategy from different initial conditions based on the coordinated formation control concept. The proposed control law relies on an adaptive gain matrix instead of augmenting additional adaptive control input. Considering recent powerful capacities of onboard flight computers, the adaptive gain matrix algorithm can be implemented without much difficulty.

## Coordinated Attitude Pointing for Multiple Satellites

### Quaternion Feedback by damping adaptation

The proposed quaternion feedback controller of the $i$th satellite for simultaneous attitude pointing from different initial states is defined as

$$u_i = -\Omega_i J_i \omega_i - k_i J_i q_{e_i} - \xi_i(q) J_i \omega_i \tag{9}$$

where $k_i$ and $\xi_i$ are positive parameters for determining proportional and damping feedback gains, $J_i$ is moment of inertia matrix, and $q_{e_i}$ denotes the error quaternion of the $i$th satellite between initial and desired attitude. In addition, $\xi_i(q)$ is an adaptive damping gain as a function of average error quaternion. The adaptive damping gain takes the following form

$$\begin{aligned} \xi_i(q) &= \alpha_i + \beta_i \left( \frac{1}{n} \sum_{j=1}^{n} q_j^T q_j - q_i^T q_i \right) \\ &= \alpha_i + \beta_i \left( \frac{1}{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} q_j^T q_j - \frac{1-n}{n} q_i^T q_i \right) \\ & \textit{where } \alpha_i > 0,\ \beta_i > 0 \end{aligned} \tag{10}$$

where $\alpha_i$ and $\beta_i$ are tuning parameters to determine the damping gain of the $i$th satellite, $\frac{1}{n}\sum_{j=1}^{n} q_j^T q_j$ represemts quaternion error square average between reference and present attitude of each satellite, and $q_i^T q_i$ corresponds to attitude error square. The damping gains is adjusted based on the difference between quaternion error average and the $i$th quaternion error. Because quaternion satisfies $q^T q \leq 1$, the damping gain satisfies the following inequality constraint;

$$\alpha_i - \frac{n-1}{n}\beta_i \leq \xi_i(q) \leq \alpha_i + \frac{n-1}{n}\beta_i \tag{11}$$

If the tuning parameters are selected such that $\alpha_i > \frac{n-1}{n}\beta_i$, then $\xi_i(q)$ always remains as a positive scalar value. The principal idea of the proposed control law is to control multiple satellites to exhibit similar responses by variable damping of the satellites. If the quaternion error of the $i$th satellite is smaller than average, then the $i$th damping gain is increased to slow down its maneuver speed. Otherwise, the $i$th quaternion error is larger than others, the adaptation strategy tries to decrease the damping gain for improved response speed.

## Stability Analysis

The Lyapunov function candidate of the $i$th satellite for the proposed control law is defined as

$$V_i = \frac{1}{2}\omega_i^T (k_i J_i)^{-1} J_i \omega_i + q_i^T q_i + (1 - q_{i4})^2 + \frac{1}{\beta_i}\xi_i(q) \tag{12}$$

where $V_i$ remains positive definite under $\alpha_i > \frac{n-1}{n}\beta_i$ and radially unbounded. The time derivative of $V_i$ results in

$$\dot{V}_i = -\frac{1}{k_i}\xi_i(q)\omega_i^T \omega_i - \frac{1}{\beta_i}\dot{\xi}_i(q) \tag{13}$$

Note that $\dot{\xi}_i(q)$ can be solved by using Eq.(5), for which $\dot{\xi}_i(q)$ is

$$\begin{aligned} \dot{\xi}_i(q) &= \beta_i \left( \frac{2}{n} \sum_{j=1}^{n} q_j^T q_j - 2 q_i^T q_i \right) \\ &= \beta_i \left\{ \frac{2}{n} \sum_{j=1}^{n} q_j^T \left( \frac{1}{2}\Omega_j q_j + \frac{1}{2} q_{j4}\omega_j \right) - 2 q_i^T \left( \frac{1}{2}\Omega_i q_i + \frac{1}{2} q_{i4}\omega_i \right) \right\} \\ &= \beta_i \left\{ \frac{1}{n} \sum_{j=1}^{n} \left( q_{j4} q_j^T \omega_j \right) - \left( q_{i4} q_i^T \omega_i \right) \right\} \end{aligned} \tag{14}$$

For the stability of the entire system, the Lyapunov function candidate is defined as a linear combination of the candidate Lyapunov function of each system as follows;

$$V=\sum_{i=1}^{n} V_i \tag{15}$$

Obviously, $V$ is positive definite and radially unbounded since it consists of the Lyapunov function $V_i$ of each sub-system. The time derivative of $V$ is readily written as

$$\dot{V}=\sum_{i=1}^{n} \dot{V}_i \tag{16}$$

and $\dot{V}$ is solved by substituting Eq.(14) into Eq.(13).

$$\begin{aligned}\dot{V}&=-\sum_{i=1}^{n}\frac{1}{k_i}\xi_i(\boldsymbol{q})\boldsymbol{\omega_i^T}\boldsymbol{\omega_i}-\sum_{i=1}^{n}\frac{1}{\beta_i}\dot{\xi}_i(\boldsymbol{q})\\&=-\sum_{i=1}^{n}\frac{1}{k_i}\xi_i(\boldsymbol{q})\boldsymbol{\omega_i^T}\boldsymbol{\omega_i}-\sum_{i=1}^{n}\left\{\frac{1}{n}\sum_{j=1}^{n}\left(q_{j4}\boldsymbol{q_j^T}\boldsymbol{\omega_j}\right)-\left(q_{i4}\boldsymbol{q_i^T}\boldsymbol{\omega_i}\right)\right\}\\&=-\sum_{i=1}^{n}\frac{1}{k_i}\xi_i(\boldsymbol{q})\boldsymbol{\omega_i^T}\boldsymbol{\omega_i}-\left\{\sum_{j=1}^{n}\left(q_{j4}\boldsymbol{q_j^T}\boldsymbol{\omega_j}\right)-\sum_{i=1}^{n}\left(q_{i4}\boldsymbol{q_i^T}\boldsymbol{\omega_i}\right)\right\}\\&=-\sum_{i=1}^{n}\frac{1}{k_i}\xi_i(\boldsymbol{q})\boldsymbol{\omega_i^T}\boldsymbol{\omega_i}\end{aligned} \tag{17}$$

Because $\xi_i(\boldsymbol{q})/k_i$ is a positive definite function, $\dot{V}_i$ is guaranteed to be negative definite except for origin ($\boldsymbol{\omega_i}=[0\ 0\ 0]^T$). By Eq.(5), when angular rate is 0, attitude quaternion settles down to a constant value. From Eq.(1), for $\boldsymbol{\omega_i}=[0\ 0\ 0]^T$ to be an equilibrium point, the command input of the $i$th satellite should be equal to zero. Furthermore, because of the feedback control law, for zero command input($\boldsymbol{u_i}=[0\ 0\ 0]^T$), the error quaternion should be equal to zero as well. This implies that when maneuver time goes to infinity, each parameter satisfies

$$\begin{aligned}&\boldsymbol{\omega_i}=[0\ 0\ 0]^T\\&\boldsymbol{q_i}=[0\ 0\ 0\ 1]^T \text{ or } q_i=q_c\\&\xi_i(\boldsymbol{q})=\alpha_i\\&V(t)=\sum_{i=1}^{n}\alpha_i\end{aligned} \tag{18}$$

Therefore, the Lyapunov function candidate, being uniformly continuous, satisfies lower boundedness with negative semi-definite time derivative. From this property, the entire system is globally asymptotically stable.

## Quaternion Feedback using Maneuver Time Approximation

Because satellite dynamics are fully nonlinear it is difficult to estimate exact maneuver time under a feedback control. But assuming that the angular rate $\omega$ is small enough to allow the gyroscopic term to be negligible(or the satellite is under eigenaxis rotation), simplified governing equations for analytical derivations are feasible. When the control law is selected as

$$\boldsymbol{u}=-\boldsymbol{\Omega} \boldsymbol{J}\boldsymbol{\omega}-k_q\boldsymbol{J}\boldsymbol{q_e}-k_\omega \boldsymbol{J}\boldsymbol{\omega},$$

Eq.(1) can be approximated by[7]

$$\ddot{\phi}+k_\omega\dot{\phi}+k_q\frac{\phi}{2}=0 \tag{19}$$

where $\phi$ denotes the Euler axis rotation angle. By Laplace transform, Eq.(19) is rewritten as

$$\Phi(s) = \frac{s+k_\omega}{s^2+k_\omega s+k_q/2}\phi(0) + \frac{1}{s^2+k_\omega s+k_q/2}\dot{\phi}(0) \tag{20}$$

$$\phi(t) = \left\{\cos\alpha t + \frac{k_\omega}{2\alpha} sin\alpha t\right\}\phi(0)e^{-\frac{k_\omega}{2}t} + \left\{\frac{1}{\alpha} sin\alpha t\right\}\dot{\phi}(0)e^{-\frac{k_\omega}{2}t} \tag{21}$$

where $\alpha = \sqrt{\frac{k_q}{2} - \frac{k_\omega^2}{4}}$. Since sine and cosine terms directly affect oscillation of the system, one can assume that those terms are bounded 1, Eq.(21) can then be simplified intol

$$\phi(t) = \left\{\left(1+\frac{k_\omega}{2\alpha}\right)\phi(0) + \frac{1}{\alpha}\dot{\phi}(0)\right\}e^{-\frac{k_\omega}{2}t} \tag{22}$$

Therefore, the maneuver time under the quaternion feedback control is approximated by

$$t_m = \frac{2}{k_\omega}\left[\ln\left\{\frac{\left(1+\frac{k_\omega}{2\alpha}\right)\phi(0) + \frac{1}{\alpha}\dot{\phi}(0)}{\phi_f}\right\}\right] \tag{23}$$

when the satellite undergoes eigenaxis rotation, the angular rate $\boldsymbol{\omega}$ satisfies $\boldsymbol{\omega} = \dot{\phi}\mathbf{e}$. Consequently, the eigenaxis rotation angle and angular rate can be solved such that

$$\phi = \cos^{-1}\left\{\frac{1}{2}(C_{11}+C_{22}+C_{33}-1)\right\} \tag{24}$$

$$\dot{\phi} = \|\boldsymbol{\omega}\| \tag{25}$$

where $C$, attitude directional cosine matrix, is defined as[6]

$$C(\mathbf{q},q_4) = \begin{bmatrix} 1-2(q_2^2+q_3^2) & 2(q_1q_2+q_3q_4) & 2(q_1q_3-q_2q_4) \\ 2(q_2q_1-q_3q_4) & 1-2(q_1^2+q_3^2) & 2(q_2q_3+q_1q_4) \\ 2(q_3q_1+q_2q_4) & 2(q_3q_2-q_1q_4) & 1-2(q_1^2+q_2^2) \end{bmatrix} \tag{26}$$

The residual time during attitude maneuvers is solved through substitution of Eq.(24-25) into $\phi(t_0)$ and $\dot{\phi}(t_0)$ in Eq.(23). The $i$th control law for multiple satellites based upon the time approximation method is defined as

$$\mathbf{u_i} = -\boldsymbol{\Omega}_i J_i \boldsymbol{\omega}_i - (\alpha_i\varpi_i)^2 J_i \mathbf{q}_{\mathbf{e}_i} - 2\alpha_i\zeta_i\varpi_i J_i \boldsymbol{\omega}_i \tag{27}$$

where $\varpi_i$ is a natural frequency, $\zeta_i$ is a damping ratio of the $i$th system, and $\alpha_i$ is a ratio for the adaptive gains, such that

$$\alpha_i = \frac{t_{m_i}}{\max(T_m)} \tag{28}$$

where $T_m = \{t_{m_1}, t_{m_2}, \dots, t_{m_n}\}$. The main idea of the controller is to dictate all satellites possess similar maneuver times by adaptation of the natural frequencies of closed-loop system. By definition, $\alpha_i$ will lie between 0 and 1. If the $i$th satellite is very fast system compared with others or have small residual maneuver time, then $\alpha_i$ should be selected near 0. It means that control law tries to decrease the own natural frequency. As a result, the control law renders own satellite a slow system. Meanwhile, if the $i$th is subjected to the longest maneuver time, then the satellite is operated by predetermined control parameters because $\alpha_i$ is 1. When one of the satellite attitude happens to be smaller than the predetermined $\phi_f$, the proposed control law changes back into a original controller.

### Stability Analysis

Due to the highly nonlinear terms in Eq.(23), it is hard to prove stability of the entire system in the Lyapunov sense. In this section, we consider a boundedness theorem to show system convergence with some assumptions[9]. From the definition of Eq.(28), $\alpha_i$ always stays in the range $0 < \alpha_i \le 1$. Therefore, $\dot{\alpha}_i$ can be assumed by

$$\| \dot{\alpha}_i \| \le c \tag{29}$$

where $c$ is a constant positive value, $\dot{\alpha}_i$ is bounded by assumption. The Lyapunov function candidate for the $i$th satellite for showing boundeness of the solution is determined by

$$V_i = \frac{1}{2}\boldsymbol{\omega}_i^T\boldsymbol{\omega}_i + (\alpha_i\varpi_i)^2\left(\boldsymbol{q}_i^T\boldsymbol{q}_i + (1-q_{i4})^2\right) + \frac{1}{2}\alpha_i^2 \tag{30}$$

The function $V_i$ is positive definite and radially unbounded. Time derivative of $V_i$ of the system is given by

$$\begin{aligned} \dot{V}_i &= \boldsymbol{\omega}_i^T\dot{\boldsymbol{\omega}}_i + (\alpha_i\varpi_i)^2\boldsymbol{\omega}_i^T\boldsymbol{q}_i + 4\dot{\alpha}_i(\alpha_i\varpi_i)(1-q_{i4}) + \alpha_i\dot{\alpha}_i \\ &= -2\alpha_i\zeta_i\varpi_i\boldsymbol{\omega}_i^T\boldsymbol{\omega}_i + 4\dot{\alpha}_i(\alpha_i\varpi_i)(1-q_{i4}) + \alpha_i\dot{\alpha}_i \end{aligned} \tag{31}$$

the first term of $\dot{V}_i$ always satisfies negative definite since $(1-q_{4i}) \le 1$, and from Eq.(28) $\alpha_i$ is bounded by a constant $c$. Therefore, Eq.(31) can be rewritten in an inequality form

$$\begin{aligned} \dot{V}_i &\le -2\alpha_i\zeta_i\varpi_i\boldsymbol{\omega}_i^T\boldsymbol{\omega}_i + 4c(\alpha_i\varpi_i) + \alpha_i c \\ &\le 0, \quad \forall \, \|\boldsymbol{\omega}_i\|_2 \ge \sqrt{2\frac{c(\varpi_i + 1/4)}{\zeta_i\varpi_i}} \end{aligned} \tag{32}$$

and the solutions are bounded.

## Numerical Simulation

### Quaternion Feedback by Adaptive Damping Gain

The performance of the proposed adaptive damping quaternion feedback controller for simultaneous pointing of multiple satellites is investigated through numerical simulations. Inertia matrix of the rigid spacecraft model is given by

$$J_i = diag[1500,\ 1500,\ 1500](kg\ m^2) \quad \text{for } i = 1, 2, 3 \tag{33}$$

First simulation tests large angle maneuvers of 3 satellites with the same initial attitude but different control parameters. In this case, simulation is equivalent to a formation flying problem. The necessary parameters are listed in Table.1. For easy comparison of performance, $q_1$ histories of three spacecrafts by the classical several quaternion feedback law and proposed adaptive damping control law are presented in Figs. 1, and 2, respectively. Despite the same initial attitude and command, because of different control parameters, history of each $q_1$ appears different as shown in Fig. 1.

**Table 1. Parameter in same initial condition simulation**

| Symbol | value |
|---|---|
| $q_0$ | $[0.57\ 0.57\ 0.57\ 0.159]^T$ |
| $q_c$ | $[0\ 0\ 0\ 1]^T$ |
| 1st $[k_1,\ \alpha_1,\ \beta_1]$ | [0.0087, 0.062, 0.619] |
| 2nd $[k_2,\ \alpha_2,\ \beta_2]$ | [0.0061, 0.062, 0.619] |
| 3rd $[k_3,\ \alpha_3,\ \beta_3]$ | [0.0044, 0.062, 0.619] |

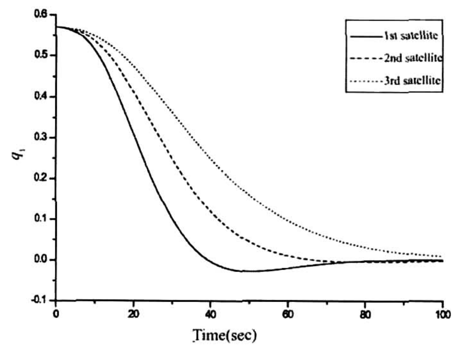


Fig. 1. Attitude histroy under quaternion feedback law

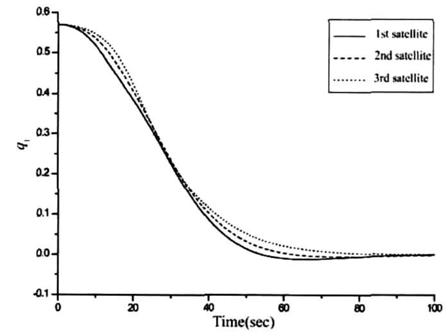


Fig. 2. Attitude histroy of adaptive damping gain application

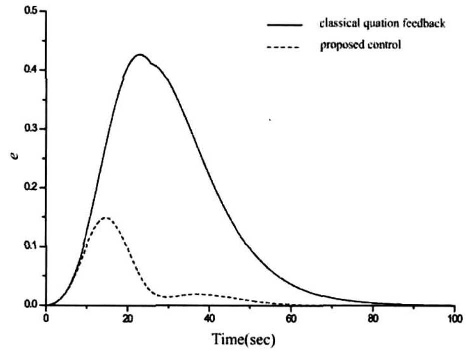


Fig. 3. Formation errors

On the other hand, it can be easily shown that results by the proposed control law attempts to make produce similar behaviors for other spacecraft. In the proposed algorithm, each system converges or the maneuver speed is determined by comparison of its own quaternion and average quaternion. In this simulation, the third satellite is taken as a slow system and, the control law decreases the damping gain for faster response. In this manner, the control law sequentially checks converge speed by magnitude error quaternion, and updates damping gain of each satellite to have proper response speed in every step. To highlight improvement of formation keeping error, the formation error is measured by

$$e = \sum_{i=1}^{n} \left| \frac{1}{n} \sum_{j=1}^{n} q_j^T q_j - q_i^T q_i \right| \tag{34}$$

and the error in this simulation is illustrated in Fig.3. Compared with a classical quaternion feedback control, the proposed control law attempts to decrease relative attitude error. After 60 seconds, attitude error of each satellite converges to 0. It means all satellites exhibit similar attitude properties and simultaneous pointing performance. Therefore, the new control law could be applied to formation flying control.

Second simulation is for different initial conditions. Each parameter of the satellites is in Table 2. The histories of $q_1$ are illustrated in Fig.4 and Fig.5. Figure 4 shows that in spite of largest initial attitude error, the first satellite attitude converges first to by the controller designed. But in Fig.5, because initial error of the first one is larger than others, the control law regards the first one as a baseline system and adjusts damping quickly. Therefore, the 1st one tries to minimize the attitude error between its own and mean attitude. Consequently, the new law contributes to achieving similar closed-loop attitude responses.

**Table 2. Parameter in different initial condition simulation**

| Symbol | value |
|---|---|
| 1st sat $q_0$ | $[0.57\ 0.57\ 0.57\ 0.159]^T$ |
| 2nd sat $q_0$ | $[0.5\ -0.5\ 0.5\ 0.5]^T$ |
| 3rd sat $q_0$ | $[0.2652\ 0.2652\ -0.6930\ 0.6157]^T$ |
| $q_c$ | $[0\ 0\ 0\ 1]^T$ |
| 1st $[k_1,\ \alpha_1,\ \beta_1]$ | $[0.0087,\ 0.062,\ 0.619]$ |
| 2nd $[k_2,\ \alpha_2,\ \beta_2]$ | $[0.0061,\ 0.062,\ 0.619]$ |
| 3rd $[k_3,\ \alpha_3,\ \beta_3]$ | $[0.0044,\ 0.062,\ 0.619]$ |

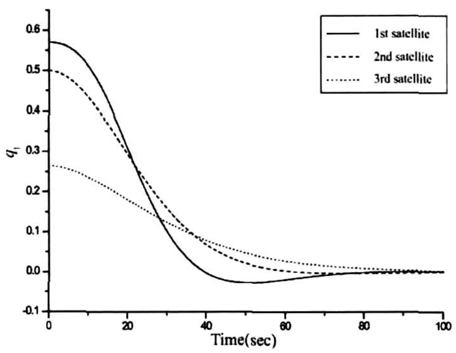


**Fig. 4. Attitude history under quaternion feedback law**

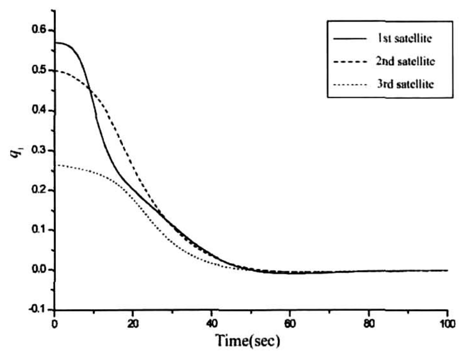


**Fig. 5. Attitude history of adaptive damping gain application**

## Quaternion Feedback using Time Approximation

Performance of the time approximation control law is illustrated through numerical simulation. For sure comparison, control parameters and initial angular rate are roughly selected, and given in Table 3. Initial attitude of each satellite is identical to the one in Table 2. Quaternion simulation result is illustrated in Fig. 6 and Fig. 7, respectively, under each application. By Eq.(23), the approximated maneuver times using the classical quaternion feedback control are estimated as $[t_{m_1}, t_{m_2}, t_{m_3}] = [77.7543, 14.6623, 37.7935]\text{sec}$. It can be easily seen that results from approximations in Eq.(23) are quite similar to those in Fig. 6. This implies feasibility of the approximation approach for the control law design purpose. Figure 7 shows the simulation results of the time approximation application. Because maneuver time of the 2nd and 3rd satellites are larger than the first one, the proposed control law reduces natural frequency. As a result, the 2nd and 3rd satellites rotate slowly trying to match with the first satellite in maneuver time. Trajectory of the reducing ratio, $\alpha_i$ is presented in Fig.8. Until 70sec, reducing or tuning ratio produce proper natural frequency. But, as the residual time approaches 0, $\alpha_i$ tends to loose accuracy of time comparison and changes dramatically.

When one of the satellites is in margin with the desired attitude, $\phi_f$, the control law sets $\alpha_i$ to 1(return to the original quaternion feedback control). The result from the time approximation control illustrates moderating mutual attitude maneuver time. Compared with the previous method, for which the adaptive damping gain approach tends to comply with mean motion, this algorithm tries to correct maneuver time to the slowest satellite, and does not care for mutual attitude trends during maneuvers.

**Table 3. Parameter in time approximation simulation**

| Symbol | value |
|---|---|
| 1st sat $\omega$ | $[0.1\ 0\ 0]^T$ |
| 2nd sat $\omega$ | $[-0.2\ 0\ 0]^T$ |
| 3rd sat $\omega$ | $[0.3\ 0\ 0]^T$ |
| $q_c$ | $[0\ 0\ 0\ 1]^T$ |
| 1st $[\varpi_1,\ \zeta_1]$ | $[0.1,\ 0.9]$ |
| 2nd $[\varpi_2,\ \zeta_2]$ | $[0.1,\ 0.9]$ |
| 3rd $[\varpi_3,\ \zeta_3]$ | $[0.1,\ 0.9]$ |

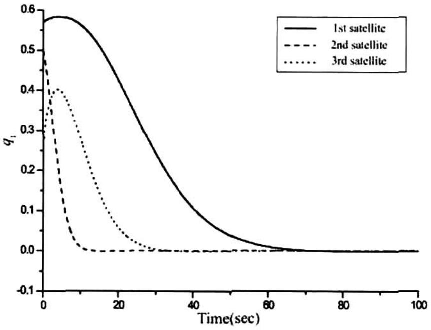


Fig. 6. Attitude history under quaternion feedback law

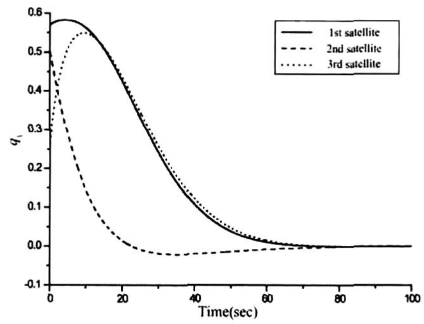


Fig. 7. Attitude history of time approximation method

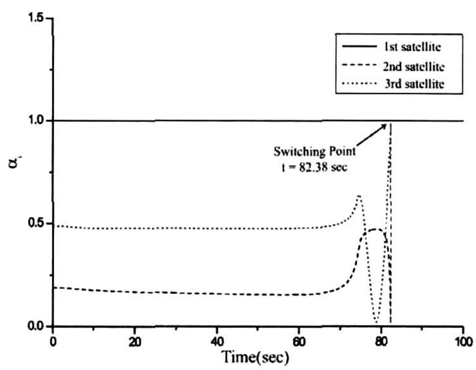


Fig. 8. Time ratio history

## Conclusions

In this paper, an coordinated control approach by changing of feedback control gain for simultaneous attitude maneuver of multiple satellites under formation flying scenario is demonstrated. The approach is built upon behavioral-based method which is one of formation flying methods. The main idea of this work is to continuously update control gains to control maneuver speed through comparison of attitude error and approximated maneuver time estimation. Conceptually, attitude error comparison seeks to make every satellite show average motion trend. On the other hand, the time approximation approach is targeted to correcting the largest maneuver time. The suggested control law guarantees global asymptotic stability in the Lyapunov sense.

The results of numerical simulations illustrate that control law is well suited to applications of coordinate attitude control especially, cooperative simultaneous pointing maneuver from different initial conditions.

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