

Dynamic Control Allocation for Shaping Spacecraft Attitude Control Command

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Abstract

For spacecraft attitude control, reaction wheel(RW) steering laws with more than three wheels for three-axis attitude control can be derived by using a control allocation(CA) approach.¹⁻² The CA technique deals with a problem of distributing a given control demand to available sets of actuators.³⁻⁴ There are many references for CA with applications to aerospace systems. For spacecraft, the control torque command for three body-fixed reference frames can be constructed by a combination of multiple wheels, usually four-wheel pyramid sets. Multi-wheel configurations can be exploited to satisfy a body-axis control torque requirement while satisfying objectives such as minimum control energy.¹⁻² In general, the reaction wheel steering laws determine required torque command for each wheel in the form of matrix pseudo-inverse.

In general, the attitude control command is generated in the form of a feedback control. The spacecraft body angular rate measured by gyros is used to estimate angular displacement also.⁵ Combination of the body angular rate and attitude parameters such as quaternion and MRPs(Modified Rodrigues Parameters) is typically used in synthesizing the control command which should be produced by RWs.¹ The attitude sensor signals are usually corrupted by noise; gyros tend to contain errors such as drift and random noise. The attitude determination system can estimate such errors, and provide best true signals for feedback control.⁶ Even if the attitude determination system, for instance, sophisticated algorithm such as the EKF(Extended Kalman Filter) algorithm⁶, can eliminate the errors efficiently, it is quite probable that the control command still contains noise sources. The noise and/or other high frequency components in the control command would cause the wheel speed to change in an undesirable manner. The closed-loop system, governed by the feedback control law, is also directly affected by the noise due to imperfect sensor characteristics. The noise components in the sensor signal should be mitigated so that the control command is isolated from the noise effect. This can be done by adding a filter to the sensor output or preventing rapid change in the control command.

Dynamic control allocation(DCA), recently studied by Härkegård, is to distribute the control command in the sense of dynamics⁴; the allocation is made over a certain time interval, not a fixed time instant. The dynamic behavior of the control command is taken into account in the course of distributing the control command. Not only the control command requirement, but also variation of the control command over a sampling interval is included in the performance criterion to be optimized. The result is a control command in the form of a finite difference equation over the given time interval.⁴ It results in a filter dynamics by taking the previous control command into account for the synthesis of current control command. Stability of the proposed dynamic control allocation(CA) approach was proved to ensure the control command is bounded at the steady-state.

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In this study, we extended the results presented in Ref. 4 by adding a two-step dynamic CA term in deriving the control allocation law. Also, the strict equality constraint, between the virtual and actual control inputs, is relaxed in order to construct control command with a smooth profile. The proposed DCA technique is applied to a spacecraft attitude control problem. The sensor noise and/or irregular signals, which are existent in most of spacecraft attitude sensors, can be handled effectively by the proposed approach.

Problem Formulation

Governing Equation of Motion

Let us consider a rigid spacecraft model with a set of pyramid-type RWs, for which the wheel angular momentum about the spacecraft body frames^{1,2}

$$\mathbf{h}_{RW} = C(\beta)I_{RW}\boldsymbol{\omega}_{RW} \quad (1)$$

where \mathbf{h}_{RW} represents wheel angular momentum about the spacecraft body frame, I_{RW} , $\boldsymbol{\omega}_{RW}$ are moment of inertia and angular velocity vector of the wheels, and $C(\beta)$ is a 3×4 rotational matrix that maps the wheel spin axis to the body frame, and β is a skew angle of the wheel spin axis. The rotational governing equations of motion for a generic rigid spacecraft model equipped with multiple reaction wheel actuators are described as^{1,2}

$$J_s\dot{\boldsymbol{\omega}} + \dot{\mathbf{h}}_{RW} + \boldsymbol{\omega} \times (J_s\boldsymbol{\omega} + \mathbf{h}_{RW}) = \mathbf{u}_d \quad (2)$$

where \mathbf{u}_d denotes external disturbance, and the internal wheel dynamics satisfy the torque (\mathbf{v}) equilibrium equation;

$$-\dot{\mathbf{h}}_{RW} = -C(\beta)I_{RW}\dot{\boldsymbol{\omega}}_{RW} = \mathbf{v} \quad (3)$$

where I_{RW} , $\boldsymbol{\omega}_{RW}$ are moment of inertia and angular velocity vector of the spacecraft body, and $\mathbf{h}_{RW} = I_{RW}\boldsymbol{\omega}_{RW}$ is the angular momentum of the wheels. Note that the angular rate of the spacecraft body is neglected in deriving Eq. (3).

Feedback Control Law

As a most popular feedback attitude control law, the quaternion-based attitude command is introduced as^{1,2}

$$\mathbf{v} = -\mathbf{K}_e\mathbf{q}_e - \mathbf{K}_\omega\boldsymbol{\omega} \quad (4)$$

where the vector component of error quaternion, \mathbf{q}_e , defined as $\delta\mathbf{q} = [\mathbf{q}_e, \delta q] = \mathbf{q}^* \otimes \mathbf{q}_t$, is a vector component of the quaternion error ($\delta\mathbf{q}$) with respect to the current (\mathbf{q}) and target (\mathbf{q}_t) quaternion. A vector \mathbf{q}^* is conjugate of the current quaternion (\mathbf{q}). Note that the quaternion attitude parameter is usually obtained from a kinematics supplied by gyro sensor measurement⁶

$$\dot{\mathbf{q}} = \frac{1}{2}\boldsymbol{\Omega}(\boldsymbol{\omega})\mathbf{q} \quad (5)$$

where $\boldsymbol{\Omega}(\boldsymbol{\omega})$ is given by

$$\boldsymbol{\Omega}(\boldsymbol{\omega}) = \begin{bmatrix} -[\boldsymbol{\omega} \times] & \boldsymbol{\omega} \\ \boldsymbol{\omega}^T & 0 \end{bmatrix}$$

and $[\boldsymbol{\omega}\times]$ is a skew-symmetric matrix for vector multiplication. As can be shown, the feedback control law in Eq. (5) requires gyro measurement and quaternion information, for which the gyro measurement is modeled as⁶

$$\boldsymbol{\omega}_{gm} = \boldsymbol{\omega} - \mathbf{b}_g - \boldsymbol{\eta}_a \quad (6)$$

and the bias (\mathbf{b}_g) is usually modeled as random-walk such that

$$d\mathbf{b}_g / dt = \boldsymbol{\eta}_r \quad (7)$$

where $\boldsymbol{\eta}_a$ and $\boldsymbol{\eta}_r$ are random noise vectors satisfying $\Xi\{\boldsymbol{\eta}_a \boldsymbol{\eta}_a^T\} = \Sigma_a$ and $\Xi\{\boldsymbol{\eta}_r \boldsymbol{\eta}_r^T\} = \Sigma_r$. The feedback signal in Eq. (5) is, therefore, controlled by the noise input, $\boldsymbol{\eta}_a$, and $\boldsymbol{\eta}_r$. Hence, direct application of the control command to the attitude dynamics results in

$$\mathbf{v} = -\mathbf{K}_e \mathbf{q}_e - \mathbf{K}_\omega (\boldsymbol{\omega} - \mathbf{b}_g - \boldsymbol{\eta}_a) \quad (8)$$

thus

$$J_s \dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (J_s \boldsymbol{\omega} + \mathbf{h}_{RW}) = -\mathbf{K}_e \mathbf{q}_e - \mathbf{K}_\omega (\boldsymbol{\omega} - \mathbf{b}_g - \boldsymbol{\eta}_a) + \mathbf{u}_d \quad (9)$$

The closed-loop system in Eq. (9) together with the kinematics in Eq. (5) constitutes a stabilized dynamic system excited by noise ($\boldsymbol{\eta}_a, \boldsymbol{\eta}_r$) and other disturbance sources, \mathbf{u}_d . After removal of the bias term, the pointing accuracy at steady-state is determined by the level of the noise and disturbance sources.

Typical attitude determination system is designed to estimate the bias component, \mathbf{b}_g , and eliminate it in the feedback control command. However, the wheel torque command is still subject to noise even after application of attitude determination algorithms. Furthermore, in a large number of cases, there are unmodeled disturbance sources, internal or external, which can cause wild variation in the control command. For instance, static and dynamic unbalancing of reaction wheels cause jitter in the spacecraft body, and the jitter is likely to be picked up by gyros. In order to mitigate rapid wheel control command changes due to noise and other high frequency components, we propose the dynamic control allocation (DCA) approach motivated by the recent work in Ref. 4.

Dynamic Control Allocation

Fundamentals of control allocation

The feedback control command constructed is allocated to the individual reaction wheels. Control allocation is defined as distributing a given control demand among available sets of actuators. The relationship between the desired or commanded control input vector (\mathbf{v}) and actuator's control input vector (\mathbf{u}) is formulated as

$$\mathbf{v}(t) = A(t)\mathbf{u}(t) \quad (10)$$

where \mathbf{v} is the desired input, called 'virtual control', from attitude controllers such as quaternion feedback control command. The purpose of the control allocation is to find a feasible solution, \mathbf{u} , satisfying the control torque requirement. In general, $A(t) \in \mathcal{R}^{m \times n}$ where $m \leq n$ and $\text{rank}(A) = m$, and there is no unique solution. A common practice to make the vector \mathbf{u} unique is to adopt a minimum-norm solution by solving¹

$$\begin{aligned} & \min_{\mathbf{u}} \|\mathbf{u}\| \\ & \text{subject to } A\mathbf{u} = \mathbf{v} \end{aligned} \quad (11)$$

This problem is well known, and has the unique solution

$$\mathbf{u} = A^\dagger \mathbf{v} \quad (12)$$

where $A^\dagger = A^T(AA^T)^{-1}$ is the pseudo-inverse of A . The resultant steering law determines the torque commands (\mathbf{u}) for individual actuators.

Dynamic control allocation(DCA)

The DCA algorithm recently suggested by Härkegård is a solution to a constrained optimization problem given by⁴

$$\begin{aligned} \min_{\mathbf{u}(t)} J = & \frac{1}{2} \mathbf{u}^T W_1 \mathbf{u} + \frac{1}{2} (\mathbf{u} - \mathbf{u}(t - \Delta T))^T W_2 (\mathbf{u} - \mathbf{u}(t - \Delta T)) \\ \text{subject to: } & \mathbf{v} = A \mathbf{u} \end{aligned} \quad (13)$$

where ΔT is a control sampling time, and $\mathbf{u}(t - \Delta T)$ is a previous input vector. The first term of Eq. (13) seeks to minimize the norm of \mathbf{u} , while the second term is for minimizing the difference between the input vector at the current and previous steps. Furthermore, W_1 and W_2 are positive definite weighting matrices determining the relative importance of the two terms. The solution to Eq. (13) using the Lagrange multiplier can be formulated such that⁴

$$\mathbf{u} = W^{-1} (I - A^T H^{-1} A W^{-1}) W_2 \mathbf{u}(t - \Delta T) + A^T H^{-1} \mathbf{v} \quad (14)$$

where $W = W_1 + W_2$, and $H = A W^{-1} A^T$. Stability of the dynamic control allocation(DCA), which is in the form a recursive equation over the sampling interval, was verified in Ref. 4. The DCA algorithm adds an extra flexibility by dynamically distributing the control command over the sampling period. The recursive form of allocation law is analogous to a digital filter, for which the input signal is \mathbf{v} and output is \mathbf{u} . The filter contributes to minimizing rapid dynamical change in the resultant control command as can be shown in the performance index.

To further extend the DCA algorithm tailored to the spacecraft attitude control program, we propose a two-step DCA algorithm.

$$\begin{aligned} \min_{\mathbf{u}(t)} J = & \frac{1}{2} \mathbf{u}^T W_1 \mathbf{u} + \frac{1}{2} \sum_{i=1}^2 (\mathbf{u} - \mathbf{u}(t - i \Delta T))^T W_{i+1} (\mathbf{u} - \mathbf{u}(t - i \Delta T)) \\ \text{subject to: } & \mathbf{v} = A \mathbf{u} \end{aligned} \quad (15)$$

The solution to the above two-step DCA algorithm, as an extension of the original Härkegård algorithm, can be also derived by the Lagrange multiplier approach.

$$\mathbf{u} = \sum_{i=1}^2 \{W^{-1} (I - A^T H^{-1} A W^{-1}) W_{i+1} \mathbf{u}(t - i \Delta T)\} + A^T H^{-1} \mathbf{v} \quad (16)$$

where $W = W_1 + \sum_{i=1}^2 W_{i+1}$, and $H = A W^{-1} A^T$.

Note that the resultant solution, Eq. (16), is a two-step version of the original algorithm in Ref. 4. The command input (\mathbf{v}) goes through a two-step filter dynamics. Stability proof for the new allocation law can be made also motivated by Härkegård's approach in Ref. 4. Using the same technique, singular value decomposition, it can be shown that

$$\begin{aligned} 0 \leq \lambda(F_1) \leq 1, \quad F_1 &= W^{-1} (I - A^T H^{-1} A W^{-1}) W_2 \\ 0 \leq \lambda(F_2) \leq 1, \quad F_2 &= W^{-1} (I - A^T H^{-1} A W^{-1}) W_3 \end{aligned} \quad (17)$$

where F_1 and F_2 denote the coefficient matrices of $\mathbf{u}(t - \Delta T)$ and $\mathbf{u}(t - 2\Delta T)$, respectively, and λ refers to the maximum eigenvalue of the coefficient matrices. Hence,

the recursive dynamics, as a linear combination of two stable dynamics, is guaranteed to be stable with the maximum eigenvalue less than unity in a discrete form of state equation.

The proposed algorithm is focused on dealing with the sensor noise or high frequency disturbance effect that is directly fed to the control command. The direct pseudo-inverse form CA technique, for instance, Eq. (16), may not be suitable for handling the control command subject to noise sources. This is because there is no dynamic characteristics in the strict algebraic allocation law. The purely algebraic allocation law, $\mathbf{v} = \mathbf{A}\mathbf{u}$, passes through not only the desired control command but also unwanted signals such as high frequency noise and disturbances.

Dynamic allocation to reduce virtual control input noise

To reduce the wild variation for the virtual control command, the inherent advantage of the original dynamic allocation technique is exploited. The new control allocation strategy is also designed in the form of a discrete system. A conceptual diagram of the proposed idea is illustrated in Fig. 1. The suggested allocation module consists of two parts; the first part is targeted to smoothing the control input vector \mathbf{v} that may contain noise and/or disturbance sources, and the second part seeks a unique minimum-norm solution for \mathbf{u} using the output of the DCA module.

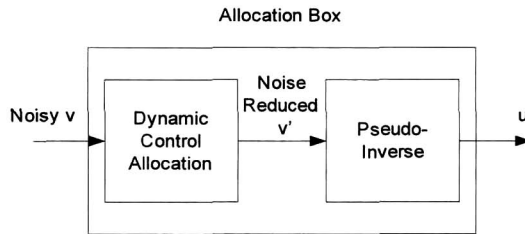


Fig. 1. Dynamic control allocation for command input noise reduction

First, for the design of the DCA module for the virtual control input \mathbf{v} , the following cost function is constructed.

$$\begin{aligned} \min_{\mathbf{v}'} J = & \frac{1}{2} (\mathbf{v}(t) - \mathbf{v}'(t))^T \mathbf{W}_1 (\mathbf{v}(t) - \mathbf{v}'(t)) \\ & + \frac{1}{2} \sum_{i=1}^2 (\mathbf{v}'(t) - \mathbf{v}'(t - i\Delta T))^T \mathbf{W}_{i+1} (\mathbf{v}'(t) - \mathbf{v}'(t - i\Delta T)) \end{aligned} \quad (18)$$

where $\mathbf{v}(t)$ is the original virtual control input, and $\mathbf{v}'(t)$ denotes the output of the DCA strategy. Furthermore, $\mathbf{v}'(t - \Delta T)$, $\mathbf{v}'(t - 2\Delta T)$ correspond to control inputs at previous steps. The above cost function attempts to minimize the control command variation, namely chattering.

Based upon the new cost function, a solution can be obtained from $\frac{\partial J}{\partial \mathbf{v}'} = 0$ with the final result as

$$\mathbf{v}'(t) = \mathbf{W}^{-1} \left(\mathbf{W}_1 \mathbf{v}(t) + \sum_{i=1}^2 \mathbf{W}_{i+1} \mathbf{v}'(t - i\Delta T) \right) \quad (19)$$

where $\mathbf{W} = \mathbf{W}_1 + \sum_{i=1}^2 \mathbf{W}_{i+1}$. The original virtual control command is allocated to $\mathbf{v}'(t)$ first by

DCA. The tentative control command \mathbf{v}' is now filtered virtual control command. From Eq. (12), the final allocated control torque input of RWs are solved by

$$\mathbf{u}(t) = A^\dagger \mathbf{v}'(t) \quad (20)$$

Thus, the proposed DCA algorithm involves two-step; one is dynamic smoother, and the other is allocation. The transfer function of Eq. (19) about the input and output is given by

$$\frac{v_i'}{v_i}(z) = \frac{w_i^{-1} w_1^i}{1 - w_i^{-1} w_2^i z^{-1} - w_i^{-1} w_3^i z^{-2}} \quad (21)$$

where the subscript and superscript i denotes i th component of the corresponding vectors, whereas w refers to the diagonal component of all W s. As one can see, the transfer function response is a typical second-order low-pass filter in z -domain. The proposed control allocation passes components of low frequency range, whereas high frequency components are filtered. Obviously, it is at the sacrifice of the speed of response of the control command, i.e., delayed response due to the lag component.

Mixed-form dynamic control allocation

Furthermore, a combination of the DCA and the pseudo-inverse approach can be made by introducing the following cost function;

$$\begin{aligned} \min_{\mathbf{u}(t)} J = & \frac{1}{2} (\mathbf{v}(t) - A(t)\mathbf{u}(t))^T W_1 (\mathbf{v}(t) - A(t)\mathbf{u}(t)) \\ & + \frac{1}{2} \sum_{i=1}^2 (\mathbf{u}(t) - \mathbf{u}(t - i\Delta T))^T W_{i+1} (\mathbf{u}(t) - \mathbf{u}(t - i\Delta T)) \end{aligned} \quad (22)$$

where $\mathbf{u}(t)$ represents control command allocated to each reaction wheel, ΔT is a control sampling time, and $\mathbf{u}(t - \Delta T)$, $\mathbf{u}(t - 2\Delta T)$ are defined similar to $\mathbf{v}(t)$, respectively. The principal ideal behind the cost function in Eq. (22) is to relax the strict equality constraints in Eqs. (13) and (15). The control torque command requirement is approximately satisfied by a judicious selection of the weighting parameters. This is intuitively achieved by choosing a large value for W_1 . The new optimization problem with the modified cost function can be simply obtained from $\frac{\partial J}{\partial \mathbf{u}} = 0$, which yields

$$\mathbf{u}(t) = W^{-1} \left(A W_1 \mathbf{v}(t) + \sum_{i=1}^2 W_{i+1} \mathbf{u}(t - i\Delta T) \right) \quad (23)$$

where $W = A^T W_1 A + \sum_{i=1}^2 W_{i+1}$. The final control command consists of the virtual control command and resultant control commands at the previous step. The derived control allocation law is essentially analogous to that of Eq. (17) from the perspective of the filter structure. One can also generalize the allocation strategy over a time window of a finite length. Namely,

$$\mathbf{u}(t) = W^{-1} \left(A W_1 \mathbf{v}(t) + \sum_{i=1}^m W_{i+1} \mathbf{u}(t - i\Delta T) \right) \quad (24)$$

where $W = A^T W_1 A + \sum_{i=1}^m W_{i+1}$, and m is a window length. As we can see, the weighting parameter W_{i+1} plays the role of a forgetting factor. As the weighing parameter, W_1 , becomes large, the solution approaches that of the exact pseudo-inverse. The proposed algorithm considering previous commands is composed of weighted sum of previous ones.

The computational burden of Eq. (24) could be slightly bigger than that of the pseudo-inverse method due to the increased mathematical operations in the second term of Eq. (24).

Numerical Simulation

The proposed approach is applied to an example spacecraft attitude control problem under a quaternion feedback control. The overall block diagram for the control allocation applied to the spacecraft attitude control is illustrated in Fig. 2. A Kalman filter-based attitude determination (AD) module using a standard AD technique⁶ is inserted in the total system. The information, such as gyro signal and quaternion parameter from the AD module, is used to construct the control command.

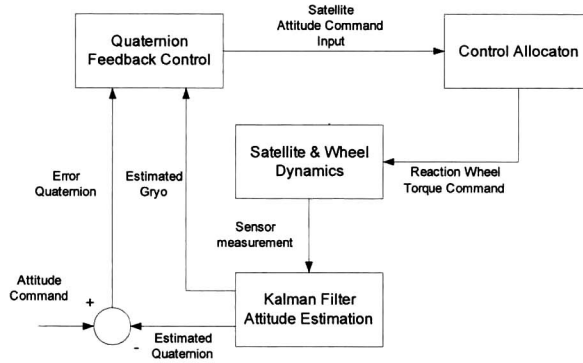


Fig. 2. Overall block diagram for the attitude determination and control allocation

The basic control input is derived by the quaternion feedback control.⁷ A set of reaction wheel command to develop such required control torque satisfies

$$\mathbf{v} = -C(\beta)I_{RW}\dot{\boldsymbol{\omega}}_{RW} = A\mathbf{u} \quad (25)$$

where, for notational simplicity, $A = -C(\beta)$ and $\mathbf{u} = I_{RW}\dot{\boldsymbol{\omega}}_{RW}$ are used.

Simulation under gyro signal with noise

The quaternion feedback control is law designed such that

$$\mathbf{v} = -\mathbf{K}_e\hat{\mathbf{q}}_e - \mathbf{K}_\omega\boldsymbol{\omega} \quad (26)$$

where $\hat{\mathbf{q}}_e$, estimated quaternion error, is estimated from the AD module by the Kalman filter algorithm⁶ while $\boldsymbol{\omega}$ is measured by gyros with bias corrected. The estimated parameters in Eq. (26) are optimal in the sense of EKF (Extended Kalman Filter) criterion. Nevertheless, they are not perfect with some random noise from original gyro signals. As a result, the final filtered output tends to deliver rough command to the RWs actuator. For simulation, the spacecraft and RWs material properties are assumed as $J_s = \text{diag}(6292, 5477, 2687) \text{ kg} \cdot \text{m}^2$, $I_{RW} = \text{diag}(1, 1, 1)$, $\beta = 45^\circ$, $\mathbf{K}_e = \text{diag}(126, 110, 54)$, $\mathbf{K}_\omega = \text{diag}(889, 774, 380)$, $\mathbf{q}_c = [0 \ 0 \ 0 \ 1]^T$, and $\mathbf{q}_0 = [0.5 \ 0.5 \ 0.5 \ -0.5]^T$.

At first, the simulation presented in Fig. 3 is performed based upon the conventional pseudo-inverse allocation, for which the control parameters are estimated quaternion and body angular rate. It is easily seen that the estimated body angular rate contains the gyro random walk noise. Since general control allocation methods are designed such that they

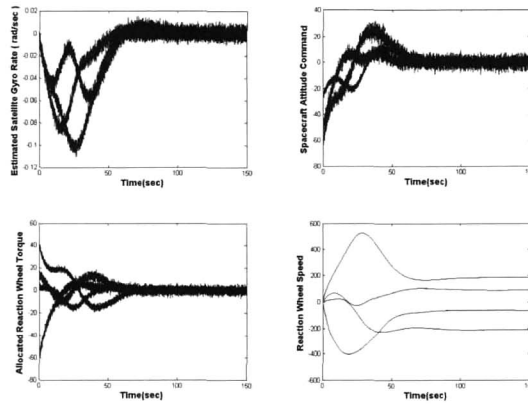


Fig. 3. Control allocation results for the case noise gyro signal with pseudo-inverse control allocation strategy applied

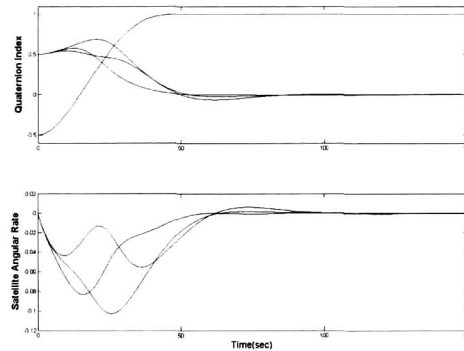


Fig. 4. Attitude quaternion and angular rate for the case of noisy sensor and pseudo-inverse allocation

transmit noise components of the virtual control to actuators, each actuator is issued chattered command in every step. As a consequence, spacecraft attitude control input and allocated reaction wheel torque, formulated as an equality constraint problem, are affected. Figure 4 shows attitude and angular rate generated by the control parameters in Fig. 4. It turns out that small attitude errors still exist caused by the wild control command.

The next simulation responses illustrated in Figs. 5 and 6 are used to validate the two proposed approaches. First, Fig. 5 corresponds to the simulation by the DCA strategy, for which the estimated angular rate is perturbed to the extent similar to the general allocation method.

Dramatic noise reduction is observed for the allocated RWs torque command, compared to the pseudo-inverse counterpart. Due to the structure of the cost function, the initial torque command of each wheel is zero, and resultant performance is quite smooth as in Fig. 5. The second graph is constructed as the result of attitude control by a combination of four wheel torques. Since the new control allocation is formulated in terms of the weighting parameters, tuning of those parameters is very important. Larger weighting parameter implies smooth control commands, whereas increasing the equality constraint weighting parameters cause the command to follow the original noisy control command. If the weighting parameters of dynamic allocation are much larger than that of the equality constraint, then the allocated wheel torque can not follow the desired control command. So a

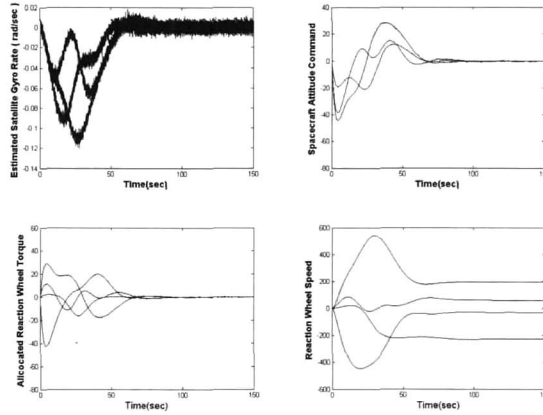


Fig. 5. Body angular rate and control allocation results with the new allocation law

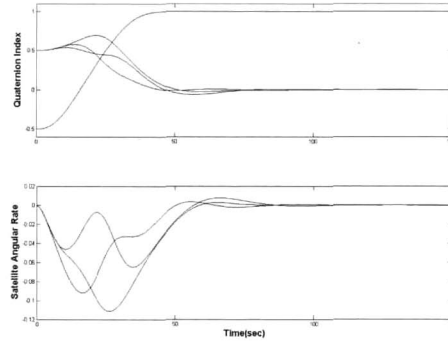


Fig. 6. Attitude quaternion and angular rate with the new allocation law

proper tuning of the weighting parameters is crucial in ensuring satisfactory performance of the closed-loop system under the proposed allocation law.

Attitude control responses with the allocation law applied are presented in Fig. 6. The original noise component, appearing in the gyro signal even after attitude determination, is handled efficiently in the process of dynamically allocating the required control command.

Simulation under disturbance source

Another application of the DCA is handling sinusoidal disturbance signals acting on the spacecraft body. For instance, dynamic unbalancing of RWs usually produces multiple harmonic disturbance sources. The disturbance torque by spinning RWs can be modeled as⁸

$$T(t) = \sum_{i=1}^N C_i^D \Omega_{RWA}^2 \cos(h_i \Omega_{RWA} t + \phi_i^D) \quad (27)$$

where C_i^D represents the amplitude due to dynamic imbalance for the i -th harmonic, Ω_{RWA} is angular rate of RWA, ϕ_i^D is random phase angle, and h_i denotes harmonics of the RWs spinning dynamics. The disturbance torque in Eq. (27), directly affecting attitude dynamics of the spacecraft, corresponds to \mathbf{u}_d in Eq. (2). The RWs disturbance torque is usually picked up by gyros in the form of high frequency components body angular rate. As such,

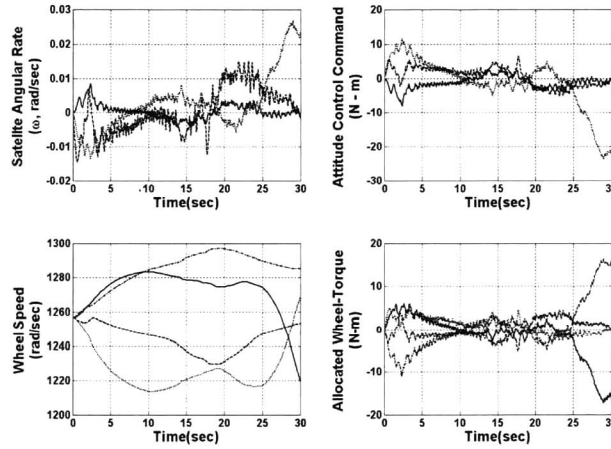


Fig. 7. Control allocation results for the case jitter with pseudo-inverse control allocation strategy applied

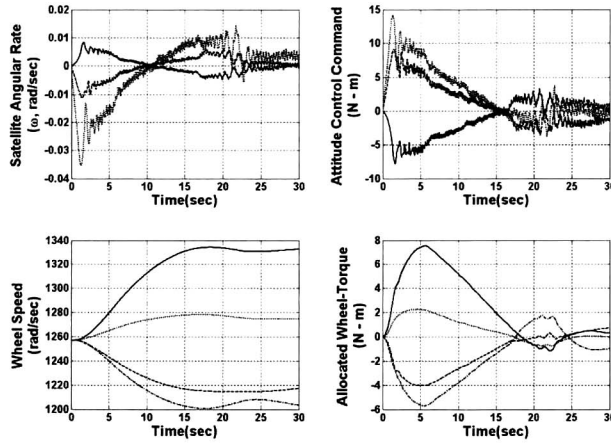


Fig. 8. Control allocation results for the case jitter with pseudo-inverse control allocation strategy applied

high frequency body angular rates, when fed to the control command, are liable to cause wild variation in the resultant control command. Thus, DCA strategy is applied to cope with such an undesirable situation. For simulation, the disturbance torque parameters are $C_i^D = [41.56 \ 8.32 \ 5.43 \ 6.21 \ 10.97 \ 5.42 \ 6.90] \times 10^{-6}$, $\Omega = 12,000 \text{ rpm}$, $h_i = [1.0 \ 2.0 \ 3.0 \ 4.0 \ 4.42 \ 5.37 \ 5.57]$. Also, a four-wheel in pyramid configuration is used in the simulation. The simulation results are displayed in Figs. 7 and 8.

As one can see, the proposed DCA algorithm results in a smoothed control torque profile under high frequency RW disturbance sources due to dynamic unbalancing. The simulation result does not reflect resultant attitude response of the spacecraft. Instead, it just demonstrates how the DCA approach can handle the disturbing sources to lead to a smooth control command, which is a factor of primary importance in real spacecraft operations.

The body angular rate is quite oscillatory, and so is the feedback control command due to the disturbance torque. The control torque command is smoothed handling those high

frequency components measured by the gyros. This again illustrates the useful feature of the proposed spacecraft RWs allocation algorithm, which was motivated by Ref. 4.

Conclusion

Dynamic control allocation for reaction wheel actuators has been validated by showing its capability of reducing original sensor noise effect. Unlike general equality constraint problems, the proposed control allocation is solved by penalizing the torque requirement error. The proposed control allocation technique, with the dynamic control allocation method, provides torque command to each reaction wheel simultaneously filtering the noise source from imperfect sensors. Typical spacecraft attitude control systems, represented by gyro-based attitude determination and pyramid-type multi-wheel configuration, could be target systems of the new approach of this paper. It will ensure stable wheel command torque at the sacrifice of torque command requirement, which can be adjusted by selecting appropriate weighting parameters in the cost function. It can handle undesirable noise or high frequency disturbance sources. Simulation results demonstrate the effect of the newly introduced method on satellite attitude control performance. As a further study, system way of selecting a best set of design parameters, especially, weighting parameters, should be investigated.

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References

1. Hanspeter Schaub and John L.Junkins, "Analytical Mechanics of Space System", AIAA Education Series, pp. 151-160, and 353-373, 2003.
2. Bong Wie, *Space Vehicle Dynamics and Control*, , AIAA Education Series, pp. 435-445, 1998.
3. Bodson, M., "Evaluation of Optimization Methods for Control Allocation", AIAA Guidance, Navigation, and Control Conference and Exhibit, Montreal, Canada, Aug. 2001.
4. Ola Härkegård, "Dynamic Control Allocation using Constrained Quadratic Programming", *Journal of Guidance, Control, and Dynamics*, vol.27 no.6 ,pp. 1028-1034, 2004.
5. James R.Wertz, *Spacecraft Attitude Determination and Control*, D.Redel Publishing Company, 1986.
6. E. J. Lefferts, F. L. Markley, and M. D. Shuster, "Kalman Filtering for Spacecraft Attitude Estimation", *Journal of Guidance, Control, and Dynamics*, Vol 5, No. 5, pp. 471-429, 1982.
7. Wie, B., and Barba, P. M., "Quaternion feedback for Spacecraft Large Angle Maneuvers", *Journal of Guidance, Control, and Dynamics*, vol.12, no.3, pp.375-380, 1989.
8. A. Y. Lee, et al, "Space Interferometry Mission Spacecraft Pointing Error Budgets", *IEEE Trans. on Aerospace Electronic Systems*, Vol.38, no.2, April 2002.