Optimal Path Planning for UAVs to Reduce Radar Cross Section

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Abstract

Parameter optimization technique is applied to planning UAVs(Unmanned Aerial Vehicles) path under artificial enemy radar threats. The ground enemy radar threats are characterized in terms of RCS(Radar Cross Section) parameter which is a measure of exposure to the radar threats. Mathematical model of the RCS parameter is constructed by a simple mathematical function in the three-dimensional space. The RCS model is directly linked to the UAVs attitude angles in generating a desired trajectory by reducing the RCS parameter. The RCS parameter is explicitly included in a performance index for optimization. The resultant UAVs trajectory satisfies geometrical boundary conditions while minimizing a weighted combination of the flight time and the measure of ground radar threat expressed in RCS.

Key Words : UAV path planning; RCS(Radar Cross Section); RCS reduction; Parameter optimization; Optimal path

Introduction

During the last decades, Unmanned Aerial Vehicles(UAVs) technologies have been evolving rapidly with the development of unmanned systems. Since UAVs are suited to the tasks that are too dangerous for human pilots, their usefulness has been received significant attention. In the military purpose, we can conceive different levels of missions such as reconnaissance, surveillance, enemy radar jamming, decoying, suppression of enemy air defense(SEAD), fixed and moving target attack, and air-to-air combat. In order to fulfill such challenging mission requirements, efficient path planning of UAVs under adversary conditions is needed[1]-[4]. Coordinated and cooperative path planning for multiple UAVs has been addressed in previous studies[2]-[4]. Real-time motion planning under dynamic environment for agile autonomous vehicles was discussed in [5].

Since a large portion of the military UAV missions are to be operated in the areas under ground radar threats, it is necessary to plan the path avoiding the threats to complete mission objectives[6]-[8]. And the path should be the shortest one, if possible, from a start location to a target destination. At the same time, it should be made safe from operational viewpoint. Planning an appropriate path which meets such principal demands is becoming an essential element for the UAV operation. In this paper, a path planning approach, which considers RCS(Radar Cross Section) of the UAVs, based upon a parameter optimization

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scheme is addressed.

RCS is a parameter of primary importance assessing the level of exposure to enemy radars[9]. There have been some previous work concerning on the RCS parameter in path planning for UAVs[6]-[8]. Combination of route planning and optimum attitude planning to minimize RCS risk was addressed in [7]. The RCS parameter was incorporated into a potential-like function for path planning of UAVs[8]. In [10] and [11], the radar threat constraint was implemented in terms of geometric inequality constraints. Since the strength of radar depends upon range between radars and a target, the range constraint was posed in the optimization.

In general, the RCS level of an aircraft is strongly dependent upon the geometric configuration and material properties of the aircraft[9]. It is expressed as a function of azimuth as well as elevation angles in the aircraft body frame[9]. It is not a simple matter, in general, to accurately model the RCS characteristics. In this study, we attempt to roughly model the RCS using a combination of ellipsoids in 3-dimensional space. The ellipsoid shape is presumed to be determined by the measured values of RCS prior to the mission operation of UAVs. The approximate mathematical model of the RCS is established as a function of the attitude of the UAVs. Thus, in the process of trajectory design, the UAVs is commanded to reduce the RCS level by adjusting their attitude angles such as bank and heading angles.

As a typical mission scenario of UAVs in a risky area, let us consider a mission area being enclosed with threats such as enemy radars and anti-air fires. Basically, threats are assumed to be known *a priori*, but there is a possibility that unknown threats also may appear. Moreover, it is important that the trajectory generation be made autonomous to the extent possible. With such requirements, this paper deals with an automated path planning strategy for the UAVs. In contrast with the previous studies[10,11], for which the RCS was alternatively handled by range constraints, we analyze the RCS in terms of attitude angles of the UAVs. However, the problem is limited to a plane motion at a constant altitude. The bank angle command is engaged to generate lateral acceleration as a typical bank-to-turn strategy.

The path planning is performed by a parameter optimization technique. The desired path and associated control input are discretized at finite points. Those discretized states and control input are employed as parameters required to satisfy geometric boundary conditions while minimizing a performance index. In the optimization process, the RCS parameter, as a critical measure of the threat, is directly taken into account. This is made possible by incorporating the mathematical model of the RCS into the performance index to reflect the influence of radar threats.

This paper is organized as follows. In section 2, a brief introduction to trajectory optimization is presented. Section 3 discusses dynamic equations of motion of a UAV as a model system for this study. Mathematical modeling of the RCS is described in section 4, while the trajectory optimization based upon the RCS model is addressed in section 4. Simulation study and concluding remarks are presented in sections 5 and 6, respectively.

Trajectory optimization

In this section, a brief introduction to trajectory optimization technique is made. The trajectory optimization is largely defined as constructing control history and resultant trajectories that minimize a given performance index[12,13]. A number of theories and algorithms are available already, and many of them have been implemented for a wide range of applications. Numerical solution techniques have been a viable approach to solving complex trajectory optimization problems[12]. Commercial software tools and software modules developed by laboratories are exploited to investigate aerospace problems such as launch vehicles, guidance weapons, aircraft, reentry vehicles, and UAVs.

Recently, the parameter optimization techniques have been utilized as a standard tool for the trajectory optimization. Original system dynamics, which are continuous in nature, are first parameterized into discrete static problems. Conversion process from the dynamic problems to static ones is required before application of the parameter optimization algorithms. With the advance of computing power, the parameter optimization techniques have evolved toward solving highly complex problems.

The equations of motion for trajectory optimization are described by system states and input variables with limited bounds. Each state can have initial and terminal conditions. General trajectory optimization problems can be summarized as minimizing a scalar performance index[13]

$$J = \phi(x_f, b) \tag{1}$$

subject to a dynamic differential constraint

$$\dot{x} = g(t, x, u, b) \tag{2}$$

together with equality and inequality constraints

$$c(x) = 0 \tag{3}$$

$$d(x) \geq 0 \tag{4}$$

where t, x, u, and b represent time, state variable, control input, and a design parameter, respectively. Equations of motion of the system are described by Eq. (2). Equation (3), and Eq. (4) represent equality and inequality constraints, which tend to cause difficulty in the process of numerically obtaining solutions.

The conversion of such optimal control problems to a parameter optimization formulation starts from defining N-fixed times or nodes.

$$t_0 = t_1 < t_2 < \cdots < t_k < \cdots < t_{N-1} < t_N = t_f \tag{5}$$

Like other parameter optimization strategies, the control input $u(t)$ as functions of time are replaced by their values at the nodes. In general, the unknowns include the design parameter b, and some combinations of the control parameters u_k and state parameters x_k. Let X, which consists of x, $u(t)$, and b, denote a vector of the unknown parameters. The corresponding parameter optimization problem, then, is redefined as finding X that minimizes the performance index[13];

$$J = F(X) \tag{6}$$

with the constraints

$$C(X) = 0, \; D(X) \geq 0 \tag{7}$$

Conversion of the optimal control problem to a parameter optimization problem can be classified into four general categories. First, the unknowns are the control parameters; second, the control parameters and the state parameters at some nodes; third, the control parameters and the state parameters, and, last, the state parameters only [12, 13]. After the conversion of the optimization problem to a parametric form, standard parameter optimization tools can be applied to solve the optimization problem.

UAV equations of motion

The principal objective of this section is to establish governing equations of motion for UAVs heading toward a target destination avoiding enemy radar threats. For this purpose, dynamics equations should be constructed first. The vehicle is assumed to be a point mass for trajectory optimization in a two-dimensional plane. Since our goal for the RCS-based

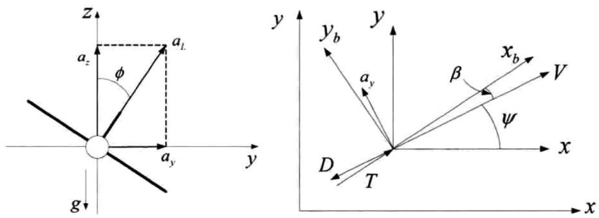

Fig. 1. Motion of UAV about the vertical plane

path planning is focused on analyzing attitude trend to reduce the RCS, the dynamics is restricted to plane motion assuming a constant altitude. Turning motion of the vehicle is controlled by acceleration a_L generated from the lift as shown in Fig. 1. By assuming that the vehicle flies at a constant altitude, the vertical acceleration a_z has a magnitude of gravity acceleration, g. Thus, the required horizontal acceleration for turning motion without sideslip satisfies $a_y = a_z \tan\phi = g\tan\phi$, where ϕ represents bank angle. If the magnitude of lift acceleration is restricted, then the bank angle should be limited also.

By assuming the bank angle by the command as a first-order time lag system with a time constant, τ, the equations of motion of the vehicle in the horizontal plane are represented as follows:

$$\dot{x} = V\cos\psi \tag{8}$$

$$\dot{y} = V\sin\psi \tag{9}$$

$$\dot{\psi} = g\tan\phi/V \tag{10}$$

$$\dot{\phi} = (\phi_c - \phi)/\tau \tag{11}$$

where ϕ_c represents the bank angle command, ψ is heading angle, V denotes speed of the UAV, and (x, y) is a position of the UAV.

The motion of the UAV in the horizontal plane is also illustrated in Fig. 1. The horizontal acceleration a_y is acting in the direction perpendicular to the vehicle's velocity vector. The sideslip angle β, defined as the angle between the velocity vector and x-axis of the body frame, can be approximated as $\beta \approx 0$. Therefore, the thrust and drag are in the same direction so that the velocity of the vehicle can be fixed as a constant.

Radar Cross Section(RCS) modeling

Introduction to RCS

In general, there could be many threats existing in the operational area for the UAVs. The ground threats targeted to the UAVs can be replaced by radars in usual cases[6]. Radar cross section(RCS) is a measure of a target's ability to reflect radar signal in the direction to radar receivers. In other words, the RCS of a target amounts to a comparison of the signal reflected from a target to the signal from a perfectly smooth sphere of unit cross sectional area[9]. The unit of RCS(σ) commonly adopted is decibels relative to a square meter (dBsm) as defined in Eq (12), and repeated here for tutorial purpose [9].

$$\sigma, \mathrm{dBsm} = 10 \cdot \log(\sigma, \mathrm{m}^2) \tag{12}$$

Typical values of RCS range from 40dBsm (10,000 m^2) for ships and large bombers to $-$30dBsm (0.001 m^2) for insects [9]. The signal reflected form a target depends on the following three parameters; i) geometric cross section, ii) reflectivity, and iii) directivity. Mathematically, they are expressed as [9]

$$P_{\text{intercepted}} = (A)P_{\text{incident}} \tag{13}$$

$$\text{Reflectivity} = \frac{P_{\text{scatter}}}{P_{\text{intercepted}}} = \frac{P_{\text{scatter}}}{(A)P_{\text{incident}}} \tag{14}$$

$$\text{Directivity} = \frac{P_{\text{backscatter}}}{P_{\text{isotropic}}} = \frac{P_{\text{backscatter}}}{(1/4\pi)\,P_{\text{scatter}}} \tag{15}$$

for which $P_{incident}$ and $P_{\text{intercepted}}$ represent the transmitted and intercepted powers. Also, P_{scatter} refers to the power scattered by the target, and $P_{\text{backscatter}}$ denotes the power reflected to the radar. Finally, $P_{\text{isotropic}}$ is defined as the power that is scattered in a perfect sphere over a unit solid angle of that sphere. Three parameters defined above are combined to determine the RCS of a target [9].

$$\begin{aligned} \text{RCS} = \sigma &= \text{Geometric Cross Section} \times \text{Reflectivity} \times \text{Directivity} \\ &= A \times \frac{P_{\text{scatter}}}{(A)P_{\text{incident}}} \times \frac{P_{\text{backscatter}}}{(1/4\pi)\,P_{\text{scatter}}} = 4\pi \frac{P_{\text{backscatter}}}{P_{\text{incident}}} \end{aligned} \tag{16}$$

It is difficult to accurately evaluate the RCS level of an aircraft because the shape of the aircraft is generally too complicated to analyze by simple mathematical functions. For this reason, traditionally, the RCS is assessed from measurement through experiments and sophisticated mathematical modeling. An example RCS pattern of an aircraft, in two-dimensional horizontal and vertical planes, is shown in Fig. 2. In this figure, 0° and 90° represent the nose direction and left wing of the aircraft, respectively. The peaks are observed in the direction of a couple of wings for the large projected area. Some aircraft designed by stealth technology is known to have the peaks at 60°, 120°, 240° and 300°.

Not only the distance from a radar, but also the RCS of an aircraft is a crucial parameter for detection probability [7,8]. Therefore, the UAV should fly far from the threats at its maximum capability, and maintain attitude to reduce the RCS and associated detection probability. However the flight path is coupled with the UAV's attitude also, so it has to be designed based upon a compromise between the desired flight path and RCS reduction.

In [10,11], an optimal path planning was studied by satisfying constraints on radar threats. The threat constraints are formulated in the form of geometric range constraints between a UAV and the radar threats. The UAV path should maintain a certain range of distance to the radars to preclude the possibility of detection. Attitude angles are not considered in those approaches. However, the RCS of aircraft should be evaluated relative

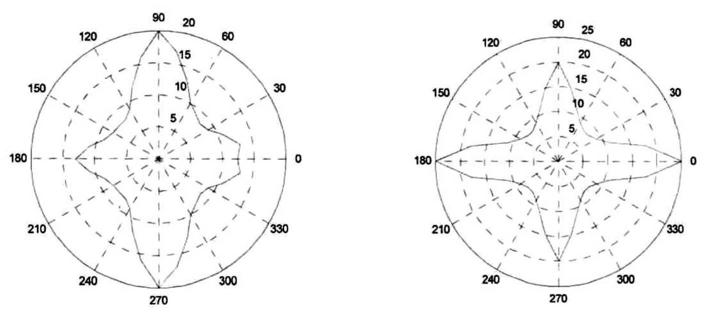

Fig. 2. Horizontal(left) and vertical(right) RCS patterns for general aircraft

to the radar line of sight as one can see the RCS pattern in Fig. 2, which is a function of azimuth and elevation angles of the aircraft body. The relative attitude angle between the UAVs and radars can be mapped to the azimuth and elevation angles for the RCS evaluation.

Our approach, in this study, is to consider the UAV attitude angles to reduce the RCS parameter in the course of flying over the areas of enemy radars. For this goal, the variation of RCS parameter is formulated in terms of UAVs attitude angles. This is motivated by the observation that the RCS pattern(Fig. 2) is characterized by azimuth and elevations angles of the aircraft body frame relative to the radars.

Mathematical modeling of RCS

For the path planning with enhancement of UAV's survivability, appropriate evaluation of the RCS should be performed first. Albeit the planned path is defined in the separate two-dimensional planes, the vector from the UAVs to a radar is generally a three-dimensional quantity. Therefore, the RCS of UAVs is modeled in the three-dimensional space. To mathematically approximate the RCS in Fig. 2, the RCS pattern is specified as a combination of three ellipsoids laid in each axis. The equation of an ellipsoid is simply given by

$$\frac{x_b^2}{a^2}+\frac{y_b^2}{b^2}+\frac{z_b^2}{c^2}=1 \tag{17}$$

where (x_b, y_b, z_b) is a point on the RCS distribution about the body frame of the aircraft. For instance, the constants *a*, *b*, and *c* for the ellipsoid about each axis are taken as follows:

For *x*-axis ellipsoid: $a_1 = 10$, $b_1 = 5$, $c_1 = 5$

For *y*-axis ellipsoid: $a_2 = 5$, $b_2 = 15$, $c_2 = 5$

For *z*-axis ellipsoid: $a_3 = 5$, $b_3 = 5$, $c_3 = 20$

The reason for the ellipsoid model can be explained by the 2-dimensional pattern of the RCS, both in azimuth and elevation planes in Fig. 2. The final complete RCS pattern is approximated through an ellipsoid configuration. Let us note that the ellipsoid in $x_b - y_b$ plane corresponds to the RCS pattern in the azimuth(horizontal) plane. Whereas, the ellipsoid in $x_b - z_b$ plane reconstructs that of the elevation(vertical) plane. Thus, a complete RCS pattern of a UAV over each body axis is generated by a simple mathematical function. The constants *a*, *b* and *c* are arbitrary numbers that characterize the shape of the RCS for general aircraft. Using those constants for the mathematical model, the example RCS configuration is illustrated in Fig. 3. Obviously, the mathematical model does not reflect every detailed characteristic of a real RCS pattern. Instead, it can be used as an approximation for the purpose of path planning for the UAVs.

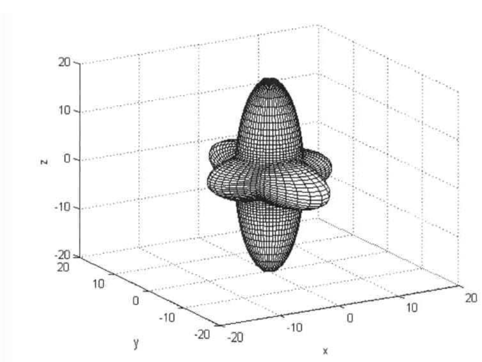

Fig. 3. RCS pattern model in three-dimensional space for general aircraft

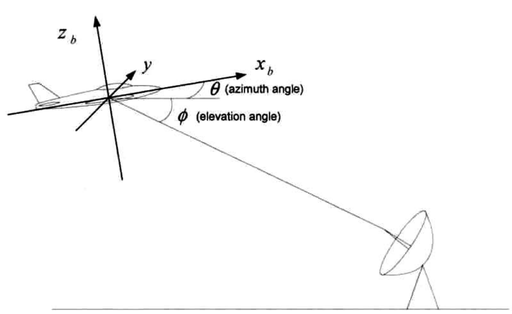

Fig. 4. Geometric relationship between an aircraft and a radar station

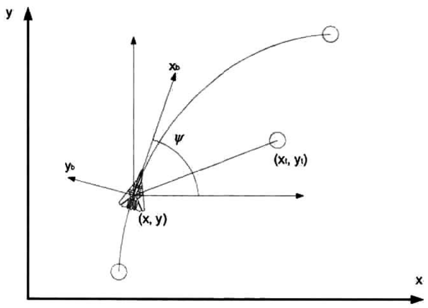

Fig. 5. Top view of geometrical relationship between of aircraft and radar

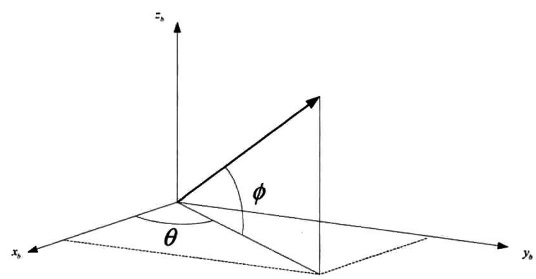

Fig. 6. Polar coordinates to locate the RCS distribution

Consequently, by considering a vector pointing from an UAV to a ground-based radar(see Fig. 4), one can model the RCS distribution in terms of the azimuth and elevation angles defined in the body frame of the aircraft. The azimuth and elevation angles can be derived as follows. First, by assuming x, y and z-axis pointing the East, North and Up directions, respectively, the geometrical configuration of the aircraft and radar is described in Fig. 5. The current heading and bank angles are denoted as ψ and ϕ, respectively. The nose direction is set to be the x-axis of the body frame, so that the heading angle is defined as the rotation of the x-axis about the positive z-axis.

With the origin of the inertial frame located at the center of the UAV, a relative coordinate of the radar is introduced as (x_t, y_t, z_t). Then, one can easily transform this coordinate to the body frame, (x_{tb}, y_{tb}, z_{tb}). Coordinate transformation is performed as follows. First, the aircraft rotates with respect to the z-axis, and then rotates about the x-axis. So the resultant transformation can be written such that

$$\begin{pmatrix} x_{tb} \\ y_{tb} \\ z_{tb} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x_t \\ y_t \\ z_t \end{pmatrix} \tag{18}$$

Furthermore, the distance from the center of the body frame of the UAV to the radar is

$$R = \sqrt{x_{tb}^2 + y_{tb}^2 + z_{tb}^2} \tag{19}$$

Finally, we can calculate the azimuth and elevation angles such as

$$\text{azimuth} = \tan^{-1}\left(\frac{x_{tb}}{y_{tb}}\right), \quad \text{elevation} = \sin^{-1}\left(\frac{z_{tb}}{R}\right) \tag{20}$$

The vector from the UAV to the radar intersects the ellipsoid previously defined. There can be multiple points of intersect. Among those contact points, the point of the longest distance from the origin of the body frame is adopted as a RCS value from a practical reason.

The equation for a vector from the UAV to the radar site in three-dimensional space, from Fig. 6, satisfies

$$\frac{x_b}{\cos\phi\cos\theta} = \frac{y_b}{\cos\phi\sin\theta} = \frac{z_b}{\sin\phi} \tag{21}$$

The z_b-intercept contacting line and a resultant ellipsoid are calculated as follows:

First, x_b and y_b in terms of z_b are given by

$$x_b = \frac{z_b}{\sin\phi}\cos\phi\cos\theta\ ,\ \ y_b = \frac{z_b}{\sin\phi}\cos\phi\sin\theta \tag{22}$$

Substituting these relationships into the ellipsoid equation in Eq. (17) yields

$$\frac{z_b^2\cos^2\phi\cos^2\theta}{a^2\sin^2\phi} + \frac{z_b^2\cos^2\phi\sin^2\theta}{b^2\sin^2\phi} + \frac{z_b^2}{c^2} = 1 \tag{23}$$

and

$$z_b^2 = \frac{a^2b^2c^2\sin^2\phi}{b^2c^2\cos^2\phi\cos^2\theta + a^2c^2\cos^2\phi\sin^2\theta + a^2b^2\sin^2\phi} \tag{24}$$

After determining the z_b −coordinate, then x_b, y_b are subsequently determined from

$$x_b = \frac{z_b}{\sin\phi}\cos\phi\cos\theta = \frac{z_b}{\tan\phi}\cos\theta\ ,\ \ y_b = \frac{z_b}{\sin\phi}\cos\phi\sin\theta = \frac{z_b}{\tan\phi}\sin\theta \tag{25}$$

where (x_b, y_b, z_b) represents the point of contact between the ellipsoid and the vector. Among three points on the three ellipsoids, the point of the largest value is taken as the estimated RCS value. As a final result, the RCS is represented as the distance from the origin to the point of contact;

$$RCS = \sqrt{x_b^2 + y_b^2 + z_b^2} \tag{26}$$

Substituting x_b, y_b, and z_b, evaluated previously, into Eq. (26) yields

$$\begin{aligned} RCS &= \sqrt{\frac{a^2b^2c^2}{b^2c^2\cos^2\phi\cos^2\theta + a^2c^2\cos^2\phi\sin^2\theta + a^2b^2\sin^2\phi}} \\ &= \frac{abc}{\sqrt{b^2c^2\cos^2\phi\cos^2\theta + a^2c^2\cos^2\phi\sin^2\theta + a^2b^2\sin^2\phi}} \end{aligned} \tag{27}$$

Therefore, given the azimuth and elevation angles for the vector from the UAV to the radar, the RCS value is evaluated through the simple algebraic equation.

Path planning by trajectory optimization with RCS

Path planning for the UAV by a trajectory optimization scheme is conducted hereafter by taking the RCS effect into consideration. As discussed in the preceding section, one can mathematically approximate the RCS via an ellipsoid function. So the RCS parameter can be directly incorporated into a performance index.

First, the proposed performance index, for the optimization without RCS, is represented by

$$\begin{aligned} J &= C_t t_f + (1 - C_t)\int_{t_0}^{t_f}\left(\sum_{i=1}^{N}\frac{\alpha_i}{R_i^4}\right)dt \\ &= C_t t_f + (1 - C_t)\int_{t_0}^{t_f}\left(\sum_{i=1}^{N}\frac{\alpha_i}{\left\{(x - x_i)^2 + (y - y_i)^2\right\}^2}\right)dt \end{aligned} \tag{28}$$

where $C_t > 0$ is a weighting parameter, and (x, y) represents a position for the UAV, whereas (x_i, y_i) for the i-th radar threat in a plane at constant altitude. The cost function is a weighted combination of the flight time (t_f) and the radar threat exposure, which is inversely proportional to the fourth power of range between the aircraft and radars[9]. Especially, the threat exposure cost is as follows:

$$\int_{t_0}^{t_f}\left(\sum_{i=1}^{N}\frac{\alpha_i}{\left\{(x-x_i)^2+(y-y_i)^2\right\}^2}\right)dt$$

This is obtained from the assumption that all the parameters, in the radar power equation, except for the distance(d) are constant[9].

$$P_r = \frac{P_t G A_e \sigma}{(4\pi)^2 d^4} \tag{29}$$

where σ represents the radar RCS parameter. With constant parameters, which affect the radar power signal such as σ, d, and other parameters, then the threat exposure cost can be rewritten as

$$\int_{t_0}^{t_f}\left(\sum_{i=1}^{N}\frac{\alpha_i\sigma}{\left\{(x-x_i)^2+(y-y_i)^2\right\}^2}\right)dt$$

And the performance index in conjunction with the RCS measure(σ) is written in the form

$$J = C_t t_f + (1-C_t)\int_{t_0}^{t_f}\left(\sum_{i=1}^{N}\frac{\alpha_i\sigma_i}{\left\{(x-x_i)^2+(y-y_i)^2\right\}^2}\right)dt \tag{30}$$

Next the path planning task is performed by seeking a set of control and state parameters which minimize the performance index defined in Eq. (30). By adopting the cost function, it is expected that we can reduce the cumulative radar power along the whole flight path. The time integral in the performance index implies that duration of exposure to radars is evaluated for the consideration of detection probability. The weighted combination of the two terms in Eq. (30) allows for a trade-off between the flight time, which typically leads to a straight line for minimum-time problem, and avoidance trajectory of a non-straight pattern depending upon the threat environment such as the number of enemy radars and their locations.

Depending upon situations, it might be sometimes necessary to reduce the peak value of the RCS[7]. In such a case, the performance index can be modified as

$$J = C_t t_f + (1-C_t)\int_{t_0}^{t_f}\left(\sum_{i=1}^{N}\frac{\alpha_i}{\left\{(x-x_i)^2+(y-y_i)^2\right\}^2}\right)dt + C_r\sigma_{\max} \tag{31}$$

or

$$J = C_t t_f + (1-C_t)\int_{t_0}^{t_f}\left(\sum_{i=1}^{N}\frac{\alpha_i\sigma_i}{\left\{(x-x_i)^2+(y-y_i)^2\right\}^2}\right)dt + C_r\sigma_{\max} \tag{32}$$

where C_r is a positive weighting factor for adjusting the scale of the maximum RCS, $\sigma_{\max}$. It is a designer's choice; which performance index, Eq. (30) and (32), should be taken. It would be primarily judged by the probability of detection due to either the duration of exposure to threats or the maximum value of RCS.

Simulations

Simulation study has been performed to validate the proposed approach in the preceding section. In particular, the effect of explicitly including the RCS parameter in the optimization is carefully analyzed. For simulation, a virtual operation area of a square type with 250km side length is assumed. For numerical solution, the parameter optimization technique, based on the Sequential Quadratic Programming, (CFSQP[14]) is used. In addition, the Runge–Kutta 4^{th} order numerical integration algorithm is applied to the evaluation of the dynamic equations of motion for the UAV. Since the two terms in Eq. (28) are linked with different scales, they are adjusted to a similar scale of dimension. The velocity of the UAV is fixed constant at 150m/s, and α_i s representing the strength of radars are also held constant. The time constant, $\tau = 1s$, is assumed for the bank angle output from a command.

First, there are two threats assumed in the operation area. Under multiple–threat cases, the weight parameter is fixed at $C_t = 0.5$ for optimization. With the two threats, the desired path is established by including the RCS term. For the multiple–threat case, Eq. (30) comprises the RCS of all the threats. But, in practice, the nearest threat adds most significant effect on the survivability of the UAVs. Even though Eq. (30) is a function of the distance from the UAV to every radar, it is highly complicated to handle all the RCSs from every threat. So, in this study, we evaluate only the significant RCS source from the nearest radar at the present location of the UAV. The performance index is then slightly modified as follows:

$$J = C_t t_f + (1 - C_t) \int_{t_0}^{t_f} \left(\sum_{i=1}^{N} \frac{\alpha_i \sigma_{\text{min\_dist}}}{\left\{ (x - x_i)^2 + (y - y_i)^2 \right\}^2} \right) dt \tag{33}$$

where $\sigma_{\text{min\_dist}}$ denotes the RCS from the nearest radar.

For the optimal path planning based upon Eq. (33) as the performance index, the simulation results are presented in Figs. 7~9. The RCS value tends to decrease near the first radar, which appears as the first peak of the RCS response. It can be easily validated from the comparison of the RCS response in Fig. 9. Total summation under the RCS curve is obviously smaller for the case where the RCS is quantified in the process of optimization. From this result, reduction of RCS by the RCS–based optimization technique is demonstrated again. A similar tendency can be also observed for the case with three radars. The simulation results are presented in Figs. 10 to 12. The threats affect the resultant UAV's trajectory as the result of the performance index to be minimized.

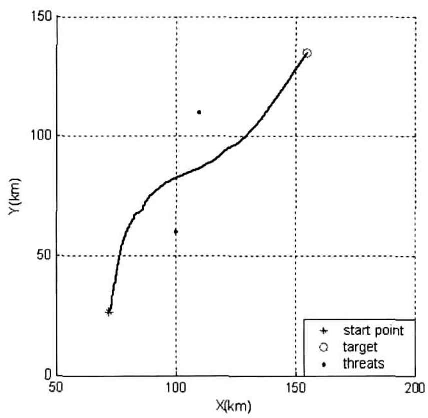

Fig. 7. Trajectory designed with RCS considered against two radars

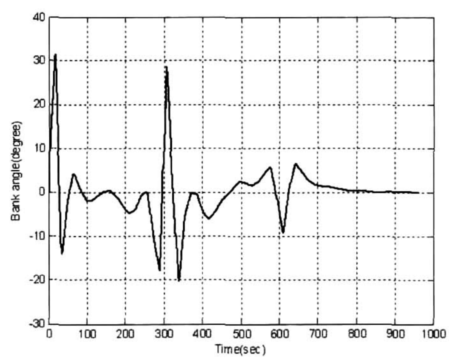

Fig. 8. Input bank angle with RCS index included in the performance index

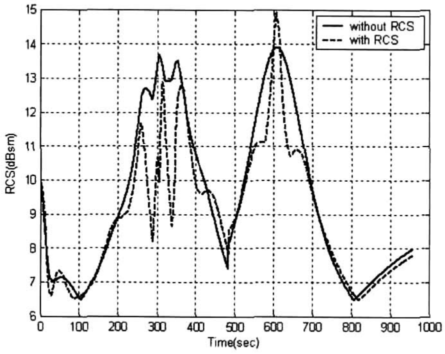

Fig. 9. Comparison of RCS for the two -radar case

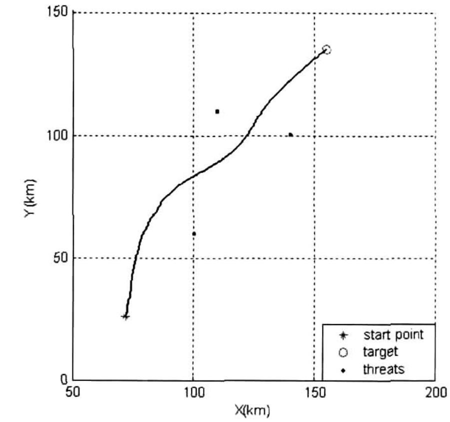

Fig. 10. Trajectory with RCS considered against three radars

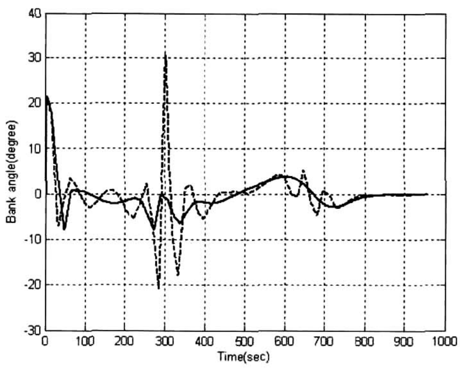

Fig. 11. Comparison of commanded bank angle input (solid line-without RCS, dashed line-with RCS)

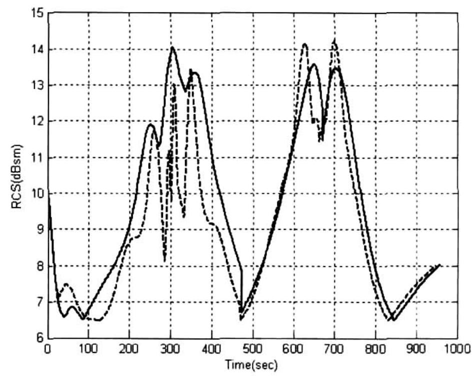

Fig. 12. Comparison of RCS for the three -radar case (solid line-without RCS, dashed line-with RCS)

Comparison of the RCS evaluated for the two cases - with and without RCS in the performance index, is presented in Fig. 12. Again, the solid line denotes the case without RCS considered and dashed line with RCS. The difference is quite distinct compared to the preceding double-threat example. The relatively large variation in the commanded bank angle input and resultant RCS level is probably due to the number of threats at different locations, which contribute to the total RCS and associated flight path.

Under many threats, for instance as many as twenty, then it was difficult to arrive at convergent solutions because of highly nonlinear nature of the performance index. If we select some threats near the UAV, then it led to a solution. This is attractive in the sense that a convergent solution is guaranteed even for multiple-threat cases to quickly construct a feasible flight path.

Conclusions

Optimal path planning approach, for UAVs under ground radar threats, has been successfully demonstrated. The new technique takes into account the radar threat sources by incorporating the RCS measure into the performance index. By employing a trajectory optimization technique in conjunction with UAV dynamics, we were able to construct feasible path of the UAVs. And the resultant path turns out to be globally optimal. However, a relatively large amount of computational time was required before arriving at convergent solutions. Further study is needed to ensure real-time or near-real-time applications of the proposed approach. Since all the threats are presumed as radars, the radar RCS parameter is an important parameter for UAV survivability. It is approximated as a function of the relative attitude between the UAVs and radars by an algebraic function of ellipsoids in three-dimensional space. The proposed ellipsoid-type RCS model is believed to be a convenient approximation of generic RCS patterns. The mathematical RCS model is explicitly analyzed in the process of the path planning. For efficient reduction of RCS, the UAV makes attempt to change its bank angle attitude, and corresponding trajectory is altered from the case without consideration of the RCS. The proposed RCS modeling and associated numerical optimization for optimal path planning provides a possibility of further applications in the future.

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