Thermal Buckling Characteristics of Composite Conical Shell Structures

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Abstract

Thermal buckling and free vibration analyses of multi-layered composite conical shells based on a layerwise displacement theory are performed. The Donnell's displacement-strain relationships of conical shell structure are applied. The natural frequencies are compared with the ones existing in the previous literature for laminated conical shells with several cone semi-vertex angles. Moreover, the thermal buckling behaviors of the laminated conical shell are investigated to consider the effect of the semi-vertex angle, subtended angle, and radius to thickness ratio on the structural stability.

Key Word : thermal buckling, composite, conical shell, layerwise displacement theory

Introduction

Conical shell panels are customarily used in various engineering applications, such as tanks, pressure vessels, missiles, spacecraft, and nuclear reactors. High-speed aerospace vehicles consisting of thin-shell elements are subjected to aerodynamic heating which induces a temperature distribution over the surface and thermal gradient through the thickness of the shell. The thermal stresses through temperature variation or boundary restrictions may cause buckling and dynamic instability. A thin-walled structure can become unstable at relatively lower temperatures and be buckled in the elastic region. Particularly, the structural behaviors of composite conical shell panels of launch vehicle are very complicated due to geometric complexity and high anisotropy unlike plates or columns.

The general subject area of composite laminated shells has been studied mainly using single layer theory, such as the classical theory and the first order shear deformation theory. However, in the case of curved composite shells, a refined theory is required owing to anisotropy, coupling, curvature and geometrical nonlinearity. Layerwise displacement theory[1-2] which represents the zigzag behavior of the in-plane displacements through the thickness has been highlighted. Analytical solutions for the free vibration analysis of composite cylindrical shells obtained by using layerwise theory have been presented in references[3-5].

The thermal buckling analysis of truncated isotropic conical shells was studied using the Galerkin method[6]. The relation of the critical temperature to the geometric parameters was evaluated. The radius to thickness ratio was the most significant factor on the critical temperature. Bendavid and Singer[7] carried out the study on the thermal buckling of isotropic conical shells heated along an axial strip employing the Rayleigh-Ritz method. The linear buckling analysis of laminated composite conical shells under thermal load using the finite element method was reported by Kari[8].

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The results indicated that the buckling behavior of laminated shell under thermal load was different from the one with respect to the angle of fiber orientation. Thermally induced dynamic instability of laminated composite conical shells is recently investigated employing perturbation approach to solve the linear three-dimensional equations of motion in terms of incremental stresses perturbed from the state of neutral equilibrium[9]. However, literature for the thermal buckling analysis of conical shells based on layerwise displacement theory is rarely found.

In this study, the thermal buckling characteristics for composite conical shell structures are numerically investigated. The structural modeling of composite conical shell panels employs layerwise displacement theory. The governing equations are obtained using the Hamilton's principle. The critical temperature values are obtained. The influences of semi-vertex angle, subtended angle, radius to thickness ratio, and length to thickness ratio on the natural frequency and the critical buckled temperature of the composite conical shells are presented.

Finite Element Modeling

Geometry and Coordinate Definition of Conical Panel

Figure 1 shows the geometric configuration of a truncated composite conical shell panel with slanted length L, thickness h, semi-vertex angle α, subtended angle ϕ, and radius of curvature at small end R_0. The displacement components u, v and w are taken along the x, ϕ and z coordinate directions.

For the conical shell, the radius at the mid-surface $R(x)$ is varying with respect to the x-direction. Moreover, in the case of a composite conical shell panel, the radius of curvature $g(z)$ in the ith layer at a distance z_i from the mid-surface is given as

$$\begin{aligned} R(x) &= R_0 + x sin\alpha \\ g(z) &= R(x) + z cos\alpha \end{aligned} \quad (1)$$

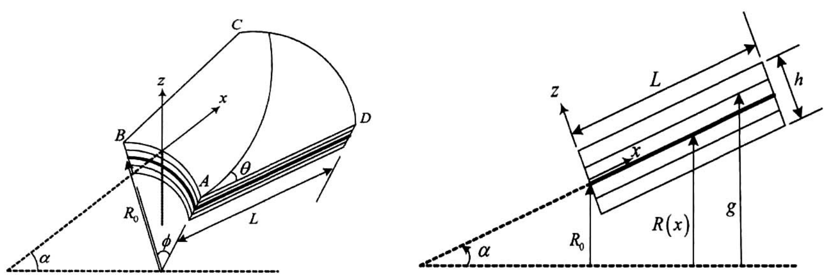


Fig. 1. Geometry and coordinate notation of a multi-layered conical shell structure

Layerwise Displacement Theory

Based on the layerwise laminated theory, the displacements (u, v and w) and temperature fields on the $x-\phi-z$ coordinate system can be expressed by layerwise continuous approximations as follows,

$$\begin{aligned} u_1 &= u(x,\phi,z,t) = \sum_{J=1}^{N_i} U^J(x,\phi,t)\Phi^J(z) \\ u_2 &= v(x,\phi,z,t) = \sum_{J=1}^{N_i} V^J(x,\phi,t)\Phi^J(z) \\ u_3 &= w(x,\phi,z,t) = W(x,\phi,t) \\ \Delta T(x,\phi,z,t) &= \sum_{J=1}^{N_i} \Delta T^J(x,\phi,t)\Phi^J(z) \end{aligned} \quad (2)$$

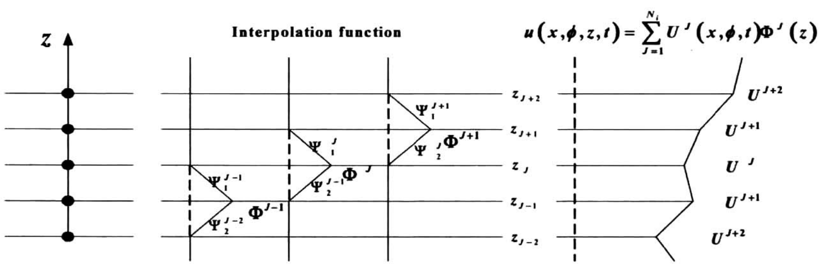


Fig. 2. In-plane displacement fields based on layerwise theory

where U^J, V^J and ΔT^J are axial and hoop displacements and temperature increment at the J-th interface, respectively. N_i is the number of degrees of freedom for the in-plane displacement along the thickness direction for element i. The displacement w is assumed to be constant through the thickness of the shell. The Lagrange interpolation function, $\Phi^J(z)$, is assumed to be of the following form,

$$\Phi^J(z) = \begin{cases} 0 & \text{for} \quad z < z_{J-1} \\ \Psi_2^{J-1}(z) = \dfrac{z - z_{J-1}}{z_J - z_{J-1}} & \text{for} \quad z_{J-1} < z < z_J \\ \Psi_1^J(z) = -\dfrac{z - z_{J+1}}{z_{J+1} - z_J} & \text{for} \quad z_J < z < z_{J+1} \\ 0 & \text{for} \quad z_{J+1} < z \end{cases} \tag{3}$$

where z_J denotes the global thickness coordinate of the J-th interface. Figure 2 shows the in-plane displacement through thickness direction based on the layerwise linear approximation. The present layerwise theory, which is by introducing zigzag in-plane displacements and accurate modeling of temperature fields, provides a more realistic description of the kinematics for composite conical shell than the equivalent single layer theories.

The displacement-strain relationships of conical shell structure based on the Donnell's theory[10] are applied. The relationships between strain and displacement can be written in Equation (4).

$$\begin{aligned} \varepsilon_{xx} &= \frac{\partial u}{\partial x} \\ \varepsilon_{\phi\phi} &= \frac{\partial v}{g\partial\phi} + \frac{w\cos\alpha}{g} + \frac{u\sin\alpha}{g} \\ \gamma_{xz} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \\ \gamma_{x\phi} &= \frac{\partial u}{g\partial\phi} + \frac{\partial v}{\partial x} - \frac{v\sin\alpha}{g} \\ \gamma_{\phi z} &= \frac{\partial w}{g\partial\phi} + \frac{\partial v}{\partial z} - \frac{v\cos\alpha}{g} \end{aligned} \tag{4}$$

where g is a distance from the central axis to an arbitrary point inside the conical panel as shown in Fig. 1.

Thermal Buckling Analysis

Constitutive equation of composite conical shell

Including the thermal effect, the corresponding constitutive equation for a composite laminated structure with the initial configuration $x-\phi-z$, can be obtained using the coordinate transformation with a fiber angle θ. The linear constitutive equation for a composite conical shell can be written in Equation(5).

$$\{\sigma\}_k = [\overline{Q}]_k(\{\varepsilon\}_k - \{\tilde{\alpha}\}_k \Delta T) \tag{5}$$

where a subscript k indicates the layer number. In addition, $[\overline{Q}]_k$ and $\{\tilde{\alpha}\}_k$ are the transformed reduced stiffness matrix and coefficients of thermal expansion vector, respectively.

Hamilton's principle and equation of motion

In order to derive the equation of motion for the composite conical panel, the Hamilton's principle is applied.

$$\begin{aligned}\delta\Pi &= \delta(U+V) - \delta K \\ &= \int_0^T \left\{ \int_V \left(\sigma_{ij}\delta\varepsilon_{ij} - f_i\delta u_i - \rho\dot{u}_i\delta\dot{u}_i\right)dV - \int_S \tau_i \delta u_i dS \right\} dt\end{aligned} \tag{6}$$

where U, V, K, ρ and T are the strain energy, the external work, the kinetic energy, the density and an arbitrary time, respectively. In addition, f_i is the volume force and τ_i is the surface traction.

Over each finite element, the displacements are expressed as a linear combination of shape function $\overline{\Psi}_k$ and nodal values W_k, U_k^J, V_k^J in the following form:

$$(W, U^J, V^J) = \sum_{k=1}^{NPE} (W, U_k^J, V_k^J)\overline{\Psi}_k \tag{7}$$

where NPE is the number of nodes per element.

By using the Hamilton's principle and finite element method, the linear finite element equation of motion for the composite conical panel can be obtained. Through the assembly procedure, global finite element equation of laminated shells subjected to a thermal load can be obtained,

$$M\ddot{u} + (KL - K^{\Delta T})u = F \tag{8}$$

where M, KL, $K^{\Delta T}$ and F are mass matrix, linear stiffness matrix, thermal geometric stiffness and external force vectors, respectively. The governing equation can be employed to find the critical buckling temperature.

$$(KL - \Delta T_{cr} K_0^{\Delta T})\{\Theta\} = 0 \tag{9}$$

where ΔT_{cr} is a critical buckling temperature and $\{\Theta\}$ is the buckling mode. $K_0^{\Delta T}$ is the equivalent geometric stiffness due initial thermal stress. In this study, the non-dimensional temperature parameter is defined.

$$\lambda_T = \alpha_T \times 10^3 \times \Delta T_{cr} \tag{10}$$

where α_T is a coefficient of thermal expansion in transverse direction.

Results and Discussion

Free vibration analysis

A truncated cross-ply [90/0/90] conical panel are compared with the previous results of Ramesh[11] and Srinivasan[12]. The geometric and material properties are given in Equation (11).

$$\begin{aligned}&E_1 = 177.4GPa,\ E_2 = E_3 = 70GPa, \\ &G_{12} = G_{23} = G_{13} = 7.027GPa,\ \nu = 0.3,\ \rho = 1600kg/m^3, \\ &L = 0.1m,\ R_0 = 0.1m,\ \alpha = 30^\circ,\ \phi = 360^\circ\end{aligned} \tag{11}$$

The boundary conditions are clamped at both ends. Non-dimensional frequency parameter, Ω is defined as

$$\Omega = \omega L^2 \sqrt{\frac{\rho h(1-\nu^2)}{E}} \tag{12}$$

where ω is natural frequency. The comparison of the non-dimensional frequency is given in Table 1. Ramesh's and present results are obtained by using the layerwise displacement theory while results of Srinivasan are obtained by using the first order shear deformation theory. The agreement of the results between the three cases is good.

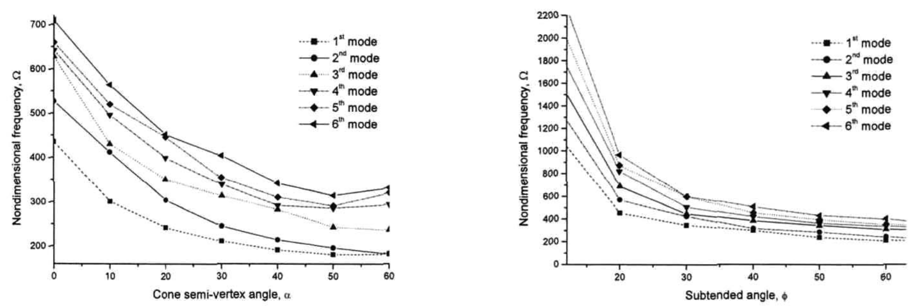
The effect of semi-vertex and subtended angles on the vibration characteristics of a conical shell panel is investigated. The geometric and material properties are given in Equation (13) and all edges of the structure are clamped.

$$\begin{aligned} &E = 70GPa,\ \nu = 0.3,\ \rho = 2700kg/m^3, \\ &L/h = 100,\ L/R_0 = 3,\ h = 2mm, \\ &\alpha = 30^\circ,\ \phi = 60^\circ \end{aligned} \tag{13}$$

The frequency variation rendered in terms of the non-dimensional frequency parameter versus cone semi-vertex angle α and subtended angle ϕ are presented in Fig. 3. As the larger end radius increases, the effective bending stiffness is decreased for all modes. The value of non-dimensional frequency is reduced by increasing α and ϕ.

Table 1. Comparisons of the non-dimensional frequency

Ω	Present	Ramesh[11]	Srinivasan[12]
Ω_1	0.909	0.909	0.908
Ω_2	1.147	1.149	1.144
Ω_3	1.721	1.726	1.726
Ω_4	2.190	2.188	2.195
Ω_5	2.4988	2.501	2.538



(a)Frequency vs. semi-vertex angle with $\phi = 60^\circ$ (b)Frequency vs. subtended angle with $\alpha = 30^\circ$

Fig. 3. Frequency variation for six modes of a conical shell structure

Thermal buckling analysis

The comparisons of the non-dimensional buckling temperature, λ_T, on a cross-ply $[0/90]_s$ laminated conical shell are shown in Table 2 and buckled mode shape at $\alpha = 45^\circ$ is illustrated in Fig. 4. The material properties and geometry are given in Equation (14), and all simply boundary conditions are subjected.

$$\begin{aligned} &E_1 = 172.25GPa,\ E_2 = E_3 = 6.89GPa, \\ &G_{12} = 3.445GPa,\ G_{13} = G_{23} = 1.379GPa \\ &\nu = 0.3,\ \phi = 360^\circ,\ L/R_0 = 0.2,\ R_0/h = 10, \\ &\alpha_1 = 6.3\times10^{-6}/^\circ C,\ \alpha_2 = \alpha_3 = 18.9\times10^{-6}/^\circ C \end{aligned} \tag{14}$$

Table 2. Comparisons of the critical buckling temperature

α	Wu and Chiu [13] 3D sol.	Wu and Chiu [13] Classical sol.	Patel[14]	Present
30°	4.4064	5.0550	4.3299	4.5015
45°	3.4750	3.9243	3.5409	3.6943
60°	2.5045	2.8196	2.5825	2.4721

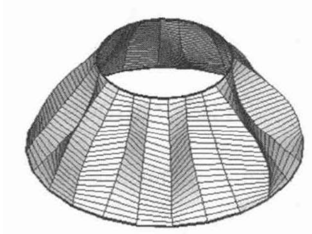
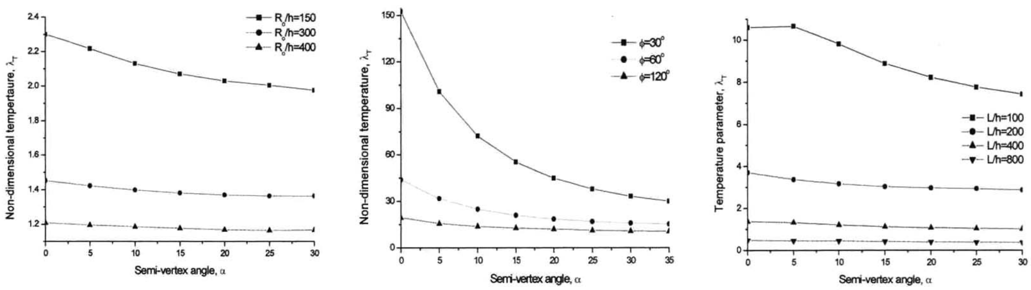


Fig. 4. Mode shape with $\alpha = 45°$



(a)Effects of radius to thickness with $\phi = 120°$ **(b)Effects of subtended angle thickness with $R_0/h = 25$** **(c)Effects of slanted length to ratio ratio with $\phi = 60°$**

Fig. 5. The variation of non-dimensional critical temperature

The detailed parametric study on cross-ply $[0/90]_s$ conical shell panel is carried out to examine the influences of the cone semi-vertex angle, subtended angle, slanted length to thickness and radius to thickness on the structural stability in Fig. 5. The material and geometric properties are given as follows,

$$\begin{aligned} &E_1 = 181\,GPa,\ E_2 = E_3 = 10.3\,GPa, \\ &G_{12} = G_{13} = 7.17\,GPa,\ G_{23} = 6.21\,GPa \\ &\nu = 0.28, \\ &\alpha_1 = 0.02 \times 10^{-6}/°C,\ \alpha_2 = \alpha_3 = 22.5 \times 10^{-6}/°C \end{aligned} \tag{15}$$

It is observed that non-dimensional critical temperature is reduced by increasing the cone semi-vertex angle. Moreover, the increase of radius to thickness ratio, subtended angle, and slanted length to thickness ratio induces the decrease of critical buckling temperature.

Conclusions

The vibration and thermal buckling characteristics of conical laminated panels are studied employing the layerwise theory. The displacement-strain relationships of conical shell structure based on the Donnell's theory are applied. The influences of cone semi-vertex angle, subtended angle, radius to thickness ratio and slanted length to thickness ratio on the thermal buckling characteristics of the composite conical shell structures are demonstrated. From the numerical results, the thermal buckling characteristics quite depend on the variation of geometric parameters. Therefore, the structural stability analysis of the conical shell structure should be performed at the preliminary design stage.

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