Uncertainty Assessment using Monte Carlo Simulation in Net Thrust Measurement at AETF

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Abstract

In this paper, Monte Carlo Simulation (MCS) method was used as an uncertainty assessment tool for air flow, net thrust measurement. Uuncertainty sources of the net thrust measurement were analyzed, and the probability distribution characteristics of each source were discussed. Detailed MCS methodology was described including the effect of the number of simulation. Compared to the conventional sensitivity coefficient method, the MCS method has advantage in the uncertainty assessment. The MCS is comparatively simple, convenient and accurate, especially for complex or nonlinear measurement modeling equations. The uncertainty assessment result by MCS was compared with that of the conventional sensitivity coefficient method, and each method gave different result. The uncertainties in the net thrust measurement by the MCS and the conventional sensitivity coefficient method were 0.906% and 1.209%, respectively. It was concluded that the first order Taylor expansion in the conventional sensitivity coefficient method and the nonlinearity of model equation caused the difference. It was noted that the uncertainty assessment method should be selected carefully according to the mathematical characteristics of the model equation of the measurement.

Key Word : Uncertainty, Monte Carlo Simulation, Air flow, Net thrust, Traceability

Introduction

Measurement uncertainty should be estimated for the measured data because all the measurement values include systematic error and random error. Publication in late 1993 by the International Organization for Standardization (ISO), the Guide to the Expression of Uncertainty in Measurement (GUM), established a new international experimental uncertainty standard and some researcher have tried to apply the standard to the uncertainty assessment in their measurement.

In the uncertainty assessment, the method using sensitivity coefficient has been widely used because this method does not required computer's support and is easy to apply to simple mathematical modeling equation. The rule of the uncertainty propagation in the method is first order approximation equation of Taylor expansion, thus it is difficult to apply the method to the equations including sensitivity coefficient or requiring iterated calculation.

The Monte Carlo Simulation (MCS) is a uncertainty assessment method using probability distribution of data and it is known that this method can be applied to the cases of large number of inputs and outputs, non-linear cases requiring 2nd order partial derivatives in order to find sensitivity terms, cases with large measurement uncertainty where linear interpolation assumptions fails, or cases with untypical input and output distributions that cannot be treated by conventional ways.

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Main

1. Measured parameters and measurement

In this paper, the uncertainty assessment on the net thrust measurement system for small gas turbine engines in the altitude engine test facility (AETF) of Korea Aerospace Research Institute (KARI) has been studied. The net thrust measurement is one of the important part in developing gas turbine engine, and the measurement uncertainty for the test had been performed by the conventional sensitivity coefficient method. However, due to the nonlinearity, complicacy, and many inputs of the modeling equation for the net thrust, the conventional sensitivity method might not be appropriate for the uncertainty assessment on the net thrust measurement. In this paper, MCS was performed in order to estimate the uncertainty in the net thrust measurement.

In the altitude engine test, Mach number of engine is simulated by adjusting the total pressure and the temperature of inlet air, and the altitude environment is achieved by controlling the static pressure of the test cell.

Figure 1 shows the schematic of the direct-connected type altitude engine test cell. The measurement section of the inlet duct from inlet bell mouth to the engine intake was named as a sliding duct, 01, 02, 05, and 09 sections depending on the diameter of the section and the distance from the engine. In the altitude engine test facility (AETF), the gross thrust is calculated by momentum force of exhaust gas and the pressure difference between engine inlet and outlet as shown in Eq. (1).

$$F = M\Delta V + A\Delta P \tag{1}$$

If the sum of force applying to the engine is zero, the gross thrust (F_G) is calculated by Eq.(2)

$$F_G = F_{LC} + W_{A05} V_{05} + (P_{S01} - P_{S90})(A_{01} - A_{02}) + (P_{S02} - P_{S90}) A_{02} \tag{2}$$

The net thrust (F_N) is computed by subtracting the ram drag from the gross thrust as shown in Eq.(3).

$$F_N = F_G - W_{A05} V_\infty \tag{3}$$

It is important to make a modeling equation so that the measured value can be directly used in the uncertainty assessment. Areas of 01 and 02 sections are calculated from diameter of pipe (D) indirect.

$$A = \frac{\pi}{4} D^2 \tag{4}$$

In the AETF of KARI, P_{T05} and P_S at 01, 02, 05, and 90 sections are not measured directly, and gauge pressure $P_{T,a}$, $P_{S,a}$ and atmospheric pressure P_r are calculated by Eq. (5), respectively.

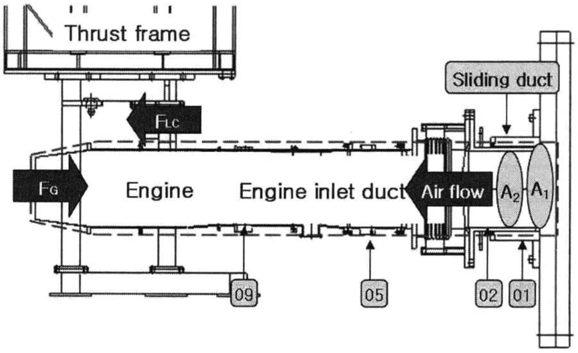


Fig. 1. Schematic diagram of engine thrust control surface

$$P_T = P_{T,a} + P_r\,,\ P_S = P_{S,a} + P_r \tag{5}$$

V_{05} and V_∞ in Eq.(2) are calculated by Eq.(6) and (7), respectively.

$$V_{05} = \sqrt{\frac{2\gamma R T_T}{\gamma - 1}\left(\frac{P_{S05}}{P_{T05}}\right)^{\frac{\gamma-1}{\gamma}}\left\{\left(\frac{P_{T05}}{P_{S05}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\}}\ . \tag{6}$$

$$V_\infty = \sqrt{\frac{2\gamma R T_T}{\gamma - 1}\left(\frac{P_{S90}}{P_{T05}}\right)^{\frac{\gamma-1}{\gamma}}\left\{\left(\frac{P_{T05}}{P_{S90}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\}} \tag{7}$$

Due to the short distance (500mm) between the 05 section and the engine intake, the pressure loss in the region is very small and it is reasonable to define the infinite velocity V_∞ as a velocity at the 05 section.

The measured parameters (inputs) used in the net thrust calculation and the measurement apparatus for the parameters are listed in Table 1.

2. The analysis of uncertainty sources

The uncertainty sources in the net thrust measurement in AETF of KARI are discussed in references (5), (6), and (7). In this paper, the application of MCS for the uncertainty assessment on the net thrust is focused and the detailed procedures are list below.

Uncertainty by traceability

The uncertainties of measuring apparatuses for F_{LC}, $P_{T,a}$, $P_{S,a}$, P_r, T_T and D at each section were estimated based on the calibration certification of each device and then, converted to the standard uncertainty. For P_r and T_T due to the signal converting procedure in A/D converters, the traceability of the sensors and A/D converters also were considered.

Uncertainty by temporal non-uniformity

During the measurement, there is fine fluctuation in $P_{T,a}$ and T_T and as a result, the net thrust and the uncertainty are affected by the fluctuation. Such temporally and spatiallyunpredicted variation is called as random effect. In order to compensate the random effect, the measurement was conducted for five seconds with a sampling rate of 10Hz and the mean value of measured data was used as a representative value. For this case, the uncertainty is the standarddeviation of the mean$(s/\sqrt{n})$ for the time variation of the measured data.

Table 1. Measured parameters for Net thrust calculation at AETF

Parameter	Apparatus	Description
F_{LC}	Interface 1110U	
$P_{S01,g}$	PSI 9816	
$P_{S02,g}$	PSI 9816	
$P_{S05,g}$	PSI 9816	
$P_{S90,g}$	PSI 9816	
$P_{T05,g}$	PSI 9816	
P_r	Setra 270 w/ HP VXI E1413C	
T_T	K-type thermocouple w/ HP VXI E1413C	
D	Vernier Calipus	
R	from theory	287.05
γ	from theory	1.4

Uncertainty by spatial non-uniformity

Due to the friction and heat losses in the pipe, the radial distributions of $P_{T,a}$ and T_T are not uniform and, results int the uncertainty of measured $P_{S,a}$. Uncertainties by such random error are expressed standard deviation of mean$(s/\sqrt{n})$ of measurement value at each measurement position.

Uncertainty by Quoting of previous measured value

In the net thrust measurement system, most of the measurements are conducted in real time, however, the diameter of pipe is quoted from the data of initial pipe manufacturing. In this case, the uncertainty is estimated as the half range of the significant figure of a measuring device.

Uncertainty by Quoting as constants

The physical values such as R, γ isted in Table 1 are treated as a constant. The uncertainty of R is estimated as the half range of the measuring limit of a significant figure.

3. Test and Monte Carlo Simulation Engine performance test

The engine used in the altitude tests was a single spool, turbojet engine. The performance test was conducted in standard day, sea level condition and inlet air Mach number was 0.7. The test results were mean values of 50 times repeatedly measured data.

Probability distribution function

Among the source of the uncertainties mentioned above, the probability distribution function of the traceability, the spatial non-uniformity, and the temporal non-uniformity are given as standard distributions. The calibration certificate was used in the calculation of the mean and the standard deviation of the standard distribution of the traceability of measurement apparatus.

For the spatial/temporal non-uniformity, the mean and the standard deviation of data measured in real engine performance test was used. For the parameters showing standard distribution, Box-Muller method(8) was used to generating random number having uniform distribution as shown in Eq. (8).

$$\begin{aligned} x_1 &= RA(0,1),\ x_2 = RA(0,1) \\ Z_1 &= s \times \sqrt{-2lnx_1} \times \cos(2\pi x_2) + m \\ Z_2 &= s \times \sqrt{-2lnx_2} \times \cos(2\pi x_1) + m \end{aligned} \tag{8}$$

In Eq. (8), $RA(0,1)$ is a random number generator function of ANSI C(10), which generates random number between 0 and 1 with rectangular distribution. In this case, Z1 and Z2 are uniformly distributed random numbers.

For the case of quoting existing value, probability distribution of input is assumed as a rectangular distribution ranged by maximum and minimum values. Thus, the data of diameter D can be expressed as Eq.(9).

$$D = L \times \{RA(0,1) - 0.5\} + m \tag{9}$$

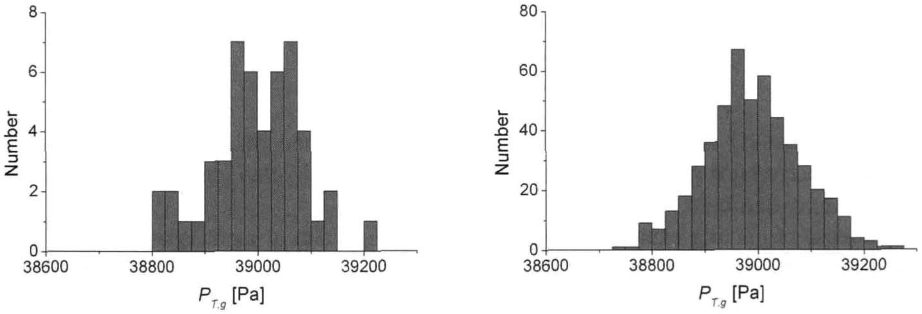
In Eq.(9), L is the range of maximum value and minimum value. Constant(R and γ) is assumed to be a rectangular distribution, and simulated by Eq.(9). The measurement uncertainty factors, the probability distribution, and the uncertainty value about inputs are listed in Table 2.

Determination of the number Monte Carlo Simulation

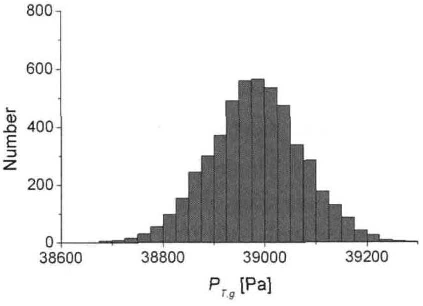
The standard deviation of the data by Monte Carlo simulation must simulate the actual standard deviation of inputs. However, when the number of Monte Carlo simulation is not enough, standard deviation of each input data is not consistent with real target value. Figure 2 shows various results of $P_{T05,a}$ according to the number of simulation. The actual standard deviation of $P_{T05,a}$ is 91.3 Pa(table 2). It is estimated that the number of MCS should be more than 5000 times in order to get less than 1% of the difference between the standard deviation by MCS and the actual standard deviation. Therefore, the number of MCS is determined as 5000 in this study.

Table 2. Uncertainty components for net thrust measurement ((N) : normal distribution, (R) : rectangular distribution, figures are standard deviations for normal distributions, full ranges for rectangular ones)

Uncertainty components / Inputs	Traceability	Spatial non-uniformity	Temporal non-uniformity	Quoting from other meas.	Quoting as constants	Combined uncertainty
	(N)	(N)	(N)	(R)	(R)	
F_{LC} [N]	0.3	0	9.32	0	0	9.33
$P_{s,01}$ [Pa]	51.71	39.29	44.65	0	0	78.81
$P_{s,02}$ [Pa]	51.71	24.27	43.06	0	0	71.53
$P_{s,05}$ [Pa]	51.71	49.31	32.25	0	0	78.39
$P_{s,90}$ [Pa]	51.71	0	14.32	0	0	53.66
$P_{T05,a}$ [Pa]	51.71	74.55	10.24	0	0	91.30
P_r [Pa]	15.0	0	0.6099	0	0	15.01
T_T [°C]	1.0	2.490	0.09170	0	0	2.685
D_o [m]	0(neglecting)	0	0	$0.05 \times 10^{-}$	0	$0.05 \times 10^{-}$
D_i [m]	0(neglecting)	0	0	$0.05 \times 10^{-}$	0	$0.05 \times 10^{-}$
R [J/kg · K]	0	0	0	0	0.005	0.005
γ	0	0	0	0	0.00277	0.00277



(a) 50 simulations(s =86.7 Pa) (b) 500 simulations(s =89.0 Pa)



(c) 5,000 simulations(s =90.7 Pa)

Fig. 2. MC results for $P_{T05,a}$ with various numbers of simulation

The measurement uncertainty assessment results

The uncertainty of the net thrust measurement was estimated using Monte Carlo Simulation. 5000 simulated results were averaged to evaluate the net thrust. The standard deviation was used as a standard uncertainty, and the coverage factor of the expanded uncertainty (k) of 2 was used.

The measurement uncertainties by sensitivity coefficient method and by Monte Carlo simulation are listed in Table 3.

Results show that the difference between the expanded uncertainties by these methods is higher than 0.303%. For linear and simple modeling equation, Basil and Jamieson showed that the estimated uncertainties by each method were very consistent. However, uncertainties of nonlinear modeling equation showed larger difference. It is concluded that the difference in the expanded uncertainty in Table 3 is caused by the approximation of sensitivity coefficient of non-linear modeling equation and first order Taylor expansion approximation in the conventional method.

Table 3. Statistical properties and uncertainties of measured and simulated Net thrust data

	Traditional method	MC
Average [N]	1109.2†	1106.1†
Standard uncertainty $u(F_N)$	5.028	6.686
Expanded uncertainty $U(F_N)$ [%]	0.906 %	1.209 %

† Measured data were multiplied by arbitrary number due to security reason

Conclusions

In this paper, uncertainty assessment using Monte Carlo Simulation for net thrust measurement is conducted and compared with results by traditional method. In the case of non-linear equation such as the net thrust measurement, traditional method and Monte Carlo simulation showed different result.

Results showed that the Monte Carlo simulation can be substituted for the conventional method if the probability distribution of input value and mathematical modeling equation are chosen appropriately. It is suggested that the uncertainty assessment method should be selected carefully according to the model of measurement system.

References

1. ISO, 1993, "Guide to the expression of uncertainty in measurements" Martin Basil & Andrew Jamieson, 1999, “Uncertainty of Complex Systems by Monte Carlo Simulation, Measurement & Control”, Vol. 32.

2. Minsoo Yoon, Sooseok Yang & Daesung Lee, 2001, “Uncertainty Analysis of Altitude Performance of a Gas Turbine Engine”, J. KSAS, Vol. 29, No. 1

3. Yongmin Jun, Yang, Samsik Nam, Chuntaek Kim, Sooseok Yang & Daesung Lee, 2002, "Small Turbojet Engine Test and Uncertainty Analysis," Proc. KSAS Annual Conference Daesung Lee, Inyoung Yang, Yongmin Jun, Chuntaek Kim & Sooseok Yang, 2002, "Improving the Measurement Uncertainty of Altitude Test Facility for Gas Turbine Engines", J. KSME, Vol. 26, No. 11

4. Daesung Lee, Inyoung Yang, Chuntaek Kim and Sooseok Yang, 2003, "Uncertainty assessment using Monte carlo simulation in gas flow measurement“, KSME, Vol.2 7, No. 12 pp. 1758~1765

5. John A. Blevins and Donald A. Wagner, 2000, "An uncertainty assessment of thrust determination in altitude ground test facilities", 38th aerospace sciences meeting & exhibit.

6. Press, William H., Teukolsky, Saul A., Vetterling, William T., Flannery, Brian P., 1992, "Numerical Recipes in C", Cambridge university press, pp. 288~290

7. L. M. Basil & P. Cox, 2000, "System Uncertainty Modelling using Monte Carlo Simulation", Measurement & Control, Vol. 33

8. Christos E. Papadopoulos & Hoi Yeung, 2001, "Uncertainty estimation and Monte Carlo simulation method". Flow Measurement and Instrumentation. Vol. 12