Minimum-Time Attitude Reorientations of Three-Axis Stabilized Spacecraft Using Only Magnetic Torquers

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Abstract

Minimum-time attitude maneuvers of three-axis stabilized spacecraft are presented to study the feasibility of using three magnetic torquers perform large angle maneuvers. Previous applications of magnetic torquers have been limited to spin-stabilized satellites or supplemental actuators of three-axis stabilized satellites because of the capability of magnetic torquers to produce torques about a specific axes. The minimum-time attitude maneuver problem is solved by applying a parameter optimization method for orbital cases to verify that the magnetic torque system can perform as required. Direct collocation and a nonlinear programming method with a constraining method by Simpson's rule are used to convert the minimum-time maneuver problems into parameter optimization problems. An appropriate number of nodes is presented to find a bang-bang type solution to the minimum-time problem. Some modifications in the boundary conditions of final attitude are made to solve the problem more robustly and efficiently. The numerical studies illustrate that the presented method can provide a capable and robust attitude reorientation by using only magnetic torquers. However, the required maneuver times are relatively longer than when thrusters or wheels are used. Performance of the system in the presence of errors in the magnetometer as well as the geomagnetic field model still good.

Key Word : Attitude Maneuver, Optimization, Magnetic torquer

Introduction

Optimal attitude reorientations of three-axis stabilized satellites have been studied as one of the most important subjects in mission design and operation [1]. The term 'optimal' in an attitude maneuver problem implies minimizing required maneuver time or control effort in fuel, power, and so on. Minimizing control efforts may be more critical than minimizing maneuver time, especially for some small satellite missions whose required accuracy is relatively loose, hence trading off between these two cost functions is a very important subject. The main objective of this study is to perform reorientation maneuvers using only magnetic torquers as quickly as possible with little or no fuel consumption. This study was motivated by the unexpected fuel consumption in the early operation of the Republic of Korea multipurpose satellite-1(KOMPSAT-1) that was launched in 1999 [2]. During its early operation, KOMPSAT-1 experienced a safehold mode several times due to an anomaly state, and thrusters had to be used for maneuvers to recover from safehold mode. This excessive fuel consumption caused difficulties in managing fuel usage [2]. It was noted that if magnetic torquers could play a role in reorientation maneuvers, such an unexpected consumption of fuel might be prevented.

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Magnetic torquers can be chosen as the main actuators for attitude reorientation maneuvers because a magnetic torquer has its own advantages of low cost, low weight, and low power consumption. Moreover, the simple structure of a magnetic torquer makes it highly reliable [3]. These characteristics make magnetic torquers more attractive as the main actuator of smaller satellites, because smaller satellites, such as micro/nano/pico satellites, are designed with the minimum hardware for proper mission operation in order to lower the mission cost [4].

The key challenges of using magnetic torquers for attitude control are (1)that the mechanical torque of a magnetic torquer can only be generated in a plane perpendicular to the local geomagnetic field vector, and (2)that the generated torque is relatively weak. Since the possibility of using the Earth's magnetic field for attitude actuator of spacecraft was first presented by Kristiansen [5], magnetic torquers have mostly been used as supplemental actuators for momentum dumping or momentum uploading due to the disadvantages [6,7,8]. As main actuators, magnetic torquers were considered as candidates for attitude torque sources in reorientation maneuvers in the literature of Junkins et. al [9]. However, their work was limited to maneuvering the spinning axis of a spin-stabilized satellite. Since then, magnetic torquers did not attract much interest as main actuators until recently. But, several authors have studied magnetic torquers as the main actuators for the attitude control system of small satellites. For a nadir-pointing satellite, Wisniewski [10] developed magnetic attitude control logic using the periodic property of the observation of the geomagnetic field. Lovera et. al [11] also used periodic property for the magnetic attitude control of small nadir-pointing satellite. Bushenkov et. al [12] presented the controller using only a magnetic torquer to stabilize the attitude of gravitationally stabilized satellites. These works showed that the magnetic torquer is a reliable actuator for a satellite whose required accuracy is about a few degrees. Because the previous researches were conducted assuming small angle and periodic property of the magnetic field, they can be applied to only polar orbits. Hence, it is still necessary to study the possibility of minimum-time maneuvering for large angle rotation fully using magnetic torquer as main actuators for arbitrary orbits. In this paper, the reliability of minimum-time reorientation maneuvers performed using only magnetic torquers is extensively investigated. This magnetic maneuver strategy is essential for a satellite having only magnetic torquers, and is also a very useful alternative attitude control system for maneuvering without fuel consumption or in cases of wheel or thruster failure. The appropriate implementation of the direct collocation method to minimum-time attitude maneuver is also investigated.

There are several numerical methods used to solve the dynamic optimal control problem numerically [13-17]. For this study, the resulting dynamic optimal control problem is solved by converting it into a parameter optimization problem that may be solved using NonLinear Programming (NLP), after setting the problem up using a collocation method. This method has not been commonly used for minimum-time attitude maneuver because of the difficulties of allocating nodes and the complexity of the resulting NLP in spite of its advantages such as it being less sensitive to initial guess and there being no need to drive necessary conditions. Since this combined method was first applied by Hargraves and Paris [18] to solve optimal orbit maneuver, some active investigations and applications have been implemented. Recently, Herman and Spencer [19] developed a high-order collocation method for low-thrust orbit maneuver. Herman and Conway [20] first used this method for attitude control problems with a minimum-control cost function. Scrivener [21] also used this combined method for an attitude reorientation method with cost function of maneuver duration. In Ref. 21, a uniformly divided phase interval was used to convert a problem to NLP, and a fixed-scale factor was used in a redefined penalty function. The method used in Ref. 21 made the problem more simpler, but often failed to yield the solutions for complex case like that treated in this paper. To overcome these difficulties, in this paper, the step size of each node is parameterized and the appropriate number of nodes is chosen from various numerical experiences.

In the next two sections, the dynamics of the attitude problem addressed here and its conversion algorithm that is used to obtain the NLP problem are described. Also, the ability of large angle reorientation maneuvers using magnetic torquers is investigated through various numerical examples in the following section. To solve the problem more robustly and effectively, implementation techniques are presented. The geomagnetic field modeling errors are also considered to observe how well the system will perform under pressure of the desired attitude.

Dynamics of the Problem

To solve the problem of interest, ordinary differential equations (ODEs) are derived from attitude dynamics and kinematics; thereafter, the derived ODEs are converted into a nonlinear programming by applying a collocation method. The attitude kinematics are expressed using a quaternion, which is of course derivable from the Euler axis $[\cos\alpha,\ \cos\beta,\ \cos\gamma]$ and the rotation angle(ϕ) in a body-fixed coordinate system with the origin at the center of mass [22]. The angle α, β, and γ are between the Euler axis and the axes of body-fixed frame. The quaternion system is given as;

$$\bar{\mathbf{q}} = [\mathbf{q}^T \quad q_0]^T = [\cos\alpha\sin(\phi/2),\ \cos\beta\sin(\phi/2), \cos\gamma\sin(\phi/2), \cos(\phi/2)]^T \tag{1}$$

The differential equation, which governs the evolution of the quaternion, is as follows;

$$\begin{aligned} \dot{\mathbf{q}} &= d\mathbf{q}/dt = \frac{1}{2}(q_0\boldsymbol{\omega} - \boldsymbol{\omega}\times\mathbf{q}) \\ \dot{q}_0 &= dq_0/dt = -\frac{1}{2}\boldsymbol{\omega}\cdot\mathbf{q} \end{aligned} \tag{2}$$

where, $\boldsymbol{\omega}$ is the angular velocity vector and the upper dot means derivative. Also, the attitude dynamic equations for a rigid body, often called Euler equations, may be written as [22]

$$\dot{\boldsymbol{\omega}} = d\boldsymbol{\omega}/dt = \mathbf{I}^{-1}(-\boldsymbol{\omega}\times\mathbf{I}\boldsymbol{\omega} + \boldsymbol{\Gamma}) \tag{3}$$

where, $\mathbf{I}$ is the moment of inertia matrix ($\mathrm{diag}(I_1, I_2, I_3)$),and $\boldsymbol{\Gamma}$ is the applied torque vector generated by onboard actuators. Three magnetic torquers are considered as active torquers; therefore the resulting torque can be expressed as follows [22] :

$$\boldsymbol{\Gamma} = [\mathbf{m}_1\times\mathbf{B}] + [\mathbf{m}_2\times\mathbf{B}] + [\mathbf{m}_3\times\mathbf{B}] \tag{4}$$

where, $\mathbf{B}(=[B_1, B_2, B_3]^T)$is the geomagnetic field vector in the body-fixed coordinate at given time and orbital position. The vectors $\mathbf{m}_{1,2,3}$ are magnetic dipole moment vectors that are aligned along each of principal axes respectively and have the following forms, as shown in Fig. 1 :

$$\mathbf{m}_1 = [mu_1, 0, 0]^T \quad \mathbf{m}_2 = [0, mu_2, 0]^T \quad \mathbf{m}_3 = [0, 0, mu_3]^T \tag{5}$$

In Eq. (5), m is a dipole moment and $u_{1,2,3}$ stand for the input currents of the magnetic torquers. In this paper, $u_{1,2,3}$ are bounded and their magnitude takes a normalized value in the range [- 1, 1].

Equations (2) and (3) are normalized for simplicity and generality using the method in Ref. 18, and normalization of the applied torque is given as;

$$\tau = \frac{m}{\Gamma_{\max}}\begin{bmatrix} 0 & B_3 & -B_2 \\ -B_3 & 0 & B_1 \\ B_2 & -B_1 & 0 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{6}$$

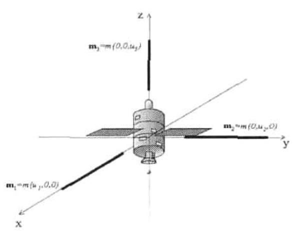

Fig. 1. Configuration of Magnetic Torquers

The maximum torque (Γ_{max})is set to $\sqrt{2}mB_{max}$,which is obtained when the geomagnetic vector has one of the following ideal values-$[-B_{max},B_{max},0]^T/\sqrt{2}$, $[B_{max},0,-B^T_{max}]/\sqrt{2}$ and $[0,-B_{max},B_{max}]^T/\sqrt{2}$, and the control inputs have their maximum value at that moment. According to this formulation, the magnitude of the normalized torque cannot exceed 1.

Optimization

Time-optimal attitude maneuver problem is known to be very difficult to solve analytically because of its high nonlinearity. In addition, magnetic torquer is able to generate torque only in the direction perpendicular to geomagnetic field vector [3]. To overcome these characteristics, a collocation method and NLP are used. The collocation method is one of the most useful methods to convert dynamic optimization problems into NLP problems. These parameter optimization methods are widely used due to their robustness, namely, their insensitivity to initial guess on parameter in spite of their complexity. Here only the brief algorithm described as follows, is used in this paper.

When $\mathbf{x}$ is defined as a state vector of spacecraft attitude, the two sets of ODEs about quaternion and angular velocities can be rewritten

$$\dot{\mathbf{x}}=\mathbf{f}(\mathbf{x},\mathbf{u}) \tag{7}$$

where, $\mathbf{u}(=[u_1,u_2,u_3]^T)$ is the control vector. Maneuver time is divided into N phases (nodes), and a defect vector is defined at each phase using Simpson's integration and Hermite interpolation. The defect vector ($\mathbf{d}$) of phase (= 0,...,N-1) is defined as follows [23];

$$\mathbf{d}^{(i)}=\mathbf{x}^{(i+1)}-\mathbf{x}^{(i)}-\frac{T^{(i)}}{6}[\mathbf{f}(\mathbf{x}^{(i+1)},\mathbf{u}^{(i+1)})+4\mathbf{f}(\bar{\mathbf{x}}^{(i)},\bar{\mathbf{u}}^{(i)})+\mathbf{f}(\mathbf{x}^{(i)},\mathbf{u}^{(i)})] \tag{8}$$

where $\bar{\mathbf{x}}^{(i)}$and $\bar{\mathbf{u}}^{(i)}$ are the state and control vectors at the center of nodes i and i+1, and are defined as follows ;

$$\bar{\mathbf{x}}=\frac{1}{2}(\mathbf{x}^{(i)}+\mathbf{x}^{(i+1)})+\frac{T^{(i)}}{8}[\mathbf{f}(\mathbf{x}^{(i)},\mathbf{u}^{(i)})-\mathbf{f}(\mathbf{x}^{(i+1)},\mathbf{u}^{(i+1)})] \tag{9}$$
$$\bar{\mathbf{u}}=\frac{1}{2}(\mathbf{u}^{(i)}+\mathbf{u}^{(i+1)})$$

and $T^{(i)}$ is the size of phase i.

Performance index (J) becomes the sum of each phase size, namely, the total maneuver time. The total number of nodes is fixed in the NLP and each phase size is parameterized as input variables to determine a more precise switching point. Consequently, the performance index is

$$J=\sum_{i=0}^{N-1}T^{(i)} \tag{10}$$

The input parameters of NLP problem consist of all time-steps between nodes, quaternion, angular velocity, and controls at each node. The lower and upper bounds of each parameter are given as

$$X=[T^{(0)},\cdots,T^{(N-1)},\bar{\mathbf{q}}^{(0)},\omega^{(0)},\mathbf{u}^{(0)},\bar{\mathbf{q}}^{(1)},\omega^{(1)},\mathbf{u}^{(1)},\cdots,\bar{\mathbf{q}}^{(N)},\omega^{(N)},\mathbf{u}^{(N)},\bar{\mathbf{q}}^{(N+1)},\omega^{(N+1)},\mathbf{u}^{(N+1)}] \tag{11}$$

bounded to

$$-1\le u^{(i)}_{1,2,3}\le 1,\quad i=0,\cdots,N+1$$
$$-1\le q^{(i)}_{0,1,2,3}\le 1,\quad i=0,\cdots,N$$

The initial (i=0)and final (i=N+1) states (quaternion and angular velocities) are fixed in the problem and the specific values of these parameters are assigned according to the maneuver cases.

Basically, the resulting NLP problem has the following simple form:

Minimize, $J(\mathbf{X})$
where
lower bound $\leq \mathbf{X} \leq$ upper bound
subject to
$\mathbf{d}^{(i)} = 0, \quad i = 0, \cdots, N-1$

In this way, a dynamic optimization problem is converted to NLP problem with constraints through collocation method. Note that the number of nodes should be greater than the required switching points. The above NLP problem is solved by NLP solver-NPSOL, which is based on Sequential Quadrature Programming (SQP) algorithm [24].

Numerical Implementation

The parameters achieved from the problem developed in the previous sections consist of quaternion components, angular velocities, and controls in each principle axes. Because the controls in this problem are electric currents leading to the magnetic torquers, the actual torque components to spacecraft are calculated from control history and magnetic field vector. This means that the applied torque is not a bang-bang history, even though the controls are in fact applied as a bang-bang form. To implement the method described in the previous section, certain parameters should be treated carefully, for they significantly affect the problem. The number of nodes is one of the key parameters of this test. Note that the number of nodes in this problem is different from the number of control switchings. The control history of a bang-bang type is sometimes solved by determining the switching function or the switching number instead of the control history directly [25]. The number of nodes in this problem is affected by the error arising from the constraining method of the Hermite-Simpson rule in phase interval, as well as by the switching numbers of control, because the response time of attitude motion is moves too quickly to be represented through the Hermite-Simpson rule with a few nodes. Too few nodes cannot help ascertain optimal control history, whereas too many nodes increase computational burden due to the increase of the input parameter number. An appropriate number of nodes can produce an optimal solution by allocating nodes properly. For the study in this paper, twenty-five nodes are chosen in most cases based on experimental experiences about the numbers of nodes. To generalize the solutions to this reorientation maneuver problem, the normalized unit is used in physical values of satellite as previously described. The orbital elements and the date performed should be specified in the tested examples because the observation of the geomagnetic field is a function of orbital position and time. The observed Earth magnetic field is simulated using IGRF2000 Model [26]. The accuracy of IGRF2000 Model is known ranges from 3~4 nT's to a few tens of nT's in intensity depending on the height. The Earth magnetic field data are tabulated for increasing computational efficiency. These data are used through cubic spline interpolation in all tests [27]. The open-loop controls obtained through the method formulated in this paper can be applied to Low Earth Orbit(LEO) satellites.

To verify the method developed in this study, numerical tests were conducted for diverse orbit cases: different altitude (400km, 600km), different orbital inclination (0°, 45°, and 98.3°), and different rotation axis (x, y, and z axis). For all the examples, circular orbit is assumed and the maneuver starts at the ascending node and finishes after 180° reorientation. The dipole moment of magnetic torquer is set to 32.6 Am^2 which is the typical value of a small satellite, the maximum magnetic field is set to $4.6472\times10^{-5}\mathrm{Wb/m}^2$, based on the data of one orbital period, and the resulting normalized time unit (NU) is 305.53sec. The maximum torque ($\Gamma_{\max} = \sqrt{2}mB_{\max}$) can be 2.143×10^{-3} N·m when all controls have a maximum value ($u_1 = u_2 = u_3 = 1$). In most tests, a rigid symmetrical body ($I_1 = I_2 = I_3 = I_0 = 200$ kg·m^2) is assumed, and orbit parameters are calculated using the KOMPSAT Simulator(SIM) [28] to obtain magnetic field observation data. In actual applications, the attitude orientation can be informed from three magnetometers, whereas the position and velocity of a satellite can be obtained from a GPS receiver or onboard propagator. To verify the attitude orientation, the initial attitude is integrated through the 4th order Runge-Kutta method with the control histories obtained from the solutions of NLP problem. The implementation of this method is first tested to refine the minimum-time reorientation maneuver problem of a rigid spacecraft with an independent three-axis control as is described in Ref. [17]. The minimum time (t_{min}=3.243) and control trajectory obtained by the implementation in this paper are identical with the results in Ref. [17].

Numerical Results

Permission of the error of desired attitude at final time

It is known to be very difficult to control a satellite's attitude accurately by only using magnetic torquers, because a magnetic torquer generates torque in the direction perpendicular to the geomagnetic field vector. Ref. [8] shows that the accuracy of attitude control using magnetic torquer is a few degrees. Considering these characteristics, a small error (about 0.5% of the reorientation angle) in attitude and angular velocity is permitted at the final time, T_f. By permitting this tolerance, the problem is solved more efficiently and robustly. Table 1 shows how the computing time and minimum maneuver time are affected by the error permitted at a final time of about 1°. When a 1° error permission is allowed at the final time, the computation time is considerably reduced. The amount of computing time reduction varies case by case, and it takes at least 150 seconds to solve a problem with a 1° error permission. All the numerical tests have been done on a 2.4GHz Intel Pentium4 Processor. The initial guessing for control and attitude is not a critical issue, because the robustness against the initial guessing is one of main advantage of the method presented in this study. By the way, the initial values for the control are set to zero, and the initial attitudes are set by linear interpolation between the initial and the final attitude.

Table 1. Comparison of the cases with and without attitude error permission at final time (25 nodes)

Inclination /Rotation Axis	Final time and computation time	No Error Permitted at Final Time	Error Permission(~1°) at Final Time
45°/y-axis	Tf	5.1587	5.0967
	computation time	279 sec.	158 sec.
98.3°/z-axis	Tf	4.2194	4.1915
	computation time	580 sec.	121 sec.
0°/y-axis	Tf	6.3175	6.2914
	computation time	1323 sec.	139 sec.

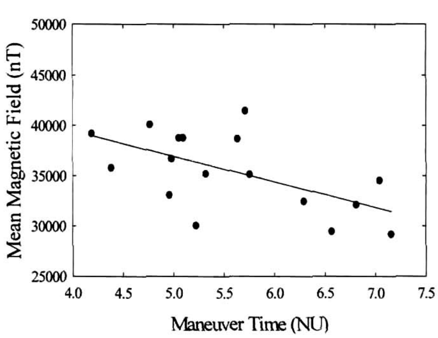

Fig. 2. The relation between maneuver time and mean magnetic field during maneuver. Linear regression line is depicted as a solid line

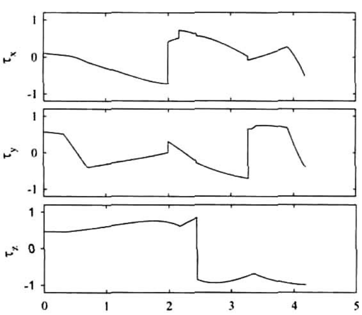

Fig. 3. Torque histories of 180° maneuver about z-axis of 98.3° inclination and 400 km altitude. The unit of tor torque is normalized by Γ_{max}

Table 2. The obtained maneuver time in various cases with 25 nodes and a final attitude error permission of about 1°.

Altitude(km)	Orbital Inclination(°)	Tf of x-axis rotation	Tf of y-axis rotation	Tf of z-axis rotation
400	i = 0	6.8082	6.2914	4.9574
	i = 45	5.6378	5.0528	5.0967
	i = 98.3	5.7110	4.7663	4.1915
600	i = 0	7.1595	6.5682	5.2221
	i = 45	7.0421	5.3154	5.7535
	i = 98.3	5.0949	4.9821	4.3798

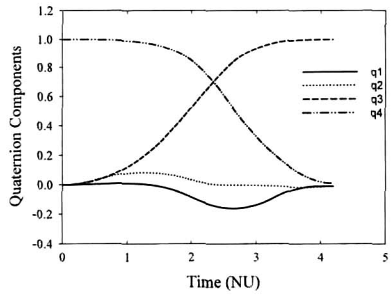

Fig. 4. Quaternion histories of 180° maneuver about z-axis of 98.3° inclination and 400 km altitude. Effects of Number of Nodes

The minimum time solutions of all the tested cases are achieved successfully and the results tabulated in Table 2. The required minimum-times vary in cases ranging from 20 minutes to 40 minutes. Fig. 2 shows the relation between the maneuver time and the mean strength of the geomagnetic field vector during maneuver. As expected, it can be concluded that the maneuver time is significantly dependent on the strength of the magnetic field. Among all the tested cases, the maneuver about z-axis in polar orbit requires the shortest duration to complete the reorientation. This occurs because the direction of rotation axis is perpendicular to the direction of the magnetic field vector. Consequently, the generated z-axis torque (Eq. 6) history appears similar to the case of independent torque in the rotation axis (see Fig. 3). These results also highly relate with the amount of nutational components in attitude motion. It is known that principle axis (eigenaxis) rotation is not the minimum-time solution for a reorientation of symmetric spacecraft with three independent controls along the principle axes. Minimum-time reorientation is achieved with nutational component of motion for such a spacecraft [17]. The solutions of minimum-time reorientation using only magnetic torquers also have a significant nutational component. However, the nutational component of motion in the maneuvers treated here resulted mainly from the limitations on generated torque directions. Namely, the nutational phenomenon in the reorientation discussed in this paper makes longer maneuver duration unlike that of three independent controls along the principle axes. Consequently, the required maneuver duration becomes shorter if the amount of nutational component of motion decreases. Quaternion histories of the shortest maneuver time case demonstrate that reorientation is performed with a smaller nutation as shown in Fig. 4.

Effects of Number of Nodes

Since the control history of minimum-time maneuver is generally bang-bang type, the problem treated in this paper should also have a bang-bang type control history. However, some tests yield solutions not having exact bang-bang control. These results are significantly related to the method used in this study. Because the NLP problem converted through direct collocation method optimizes only parameters in a finite set of nodes, the solutions are significantly affected by the number of nodes and the constraining method [29]. The same maneuver cases in Table 1 are tested for a different number of nodes to see how the solutions are affected with respect to the number of nodes. In Table 3, the corresponding changes in minimum maneuver time are tabulated for different number of nodes.

The control, the associated torque, and the quaternion histories including no attitude error permission case are also depicted in Figs. 5-7. The minimum-time slightly reduced with an increasing number of nodes, as seen in Table 3; moreover, the control histories are slightly closer to bang-bang type (see Fig. 5). While about twenty nodes might be thought sufficient to solve this problem, actually, more than that is needed to achieve the minimum-time solution for the treated cases. This is because of the constraining rule of the collocation method; approximation error of the Hermite-Simpson interpolation between adjacent nodes. This kind of numerical error is generally caused by a discretization of continuous system or linearization of a non-linear system. The results of Ref. [16] show the examples of near bang-bang control histories that were achieved through a linearization algorithm for a minimum-time attitude maneuver. However, for the cases having more than twenty-five nodes, the maneuvers treated in this paper have been little influenced by the number of nodes. The generated torque and quaternion histories are depicted in Fig. 6 and Fig. 7, and show almost the same trajectories for all cases. Actually, the freedom of the desired final attitude of about 1° has little effect on the maneuver trajectory. The more accurate solutions can be achieved by using more than thirty nodes, while the resulting NLP problem becomes more complex due to large number of parameters to be determined. Thus, the number of nodes should be chosen by considering these two conflicting characteristics. It can be said that the twenty-five nodes used in these numerical tests is a commonly acceptable number for various maneuvers.

Table 3. Comparison of maneuver times with a different number of nodes

Number of Nodes	Inclination: 0° Rot. Axis: y-axis	Inclination: 45° Rot. Axis: z-axis	Inclination: 98.3° Rot. Axis: z-axis
N = 15	Tf = 6.3517	Tf = 5.1223	-fail
N = 20	Tf = 6.2963	Tf = 5.0944	Tf = 4.1915
N = 25	Tf = 6.2914	Tf = 5.0967	Tf = 4.1915
N = 30	Tf = 6.2754	Tf = 5.0963	Tf = 4.1856

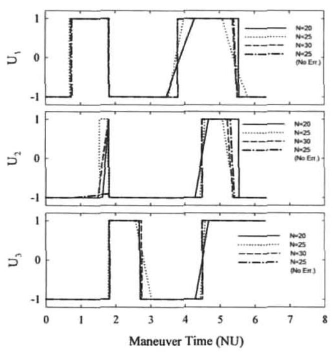

Fig. 5. Comparison of control histories using a different number of nodes for case of zero inclination, 400 km altitude, and 180° rotation maneuver about y-axis

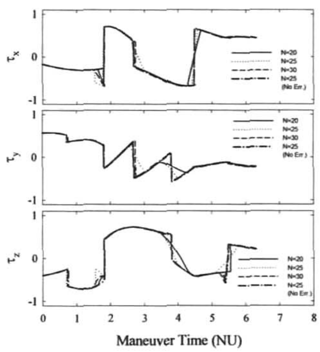

Fig. 6. Comparison of torque histories using a different number of nodes for the case of zero inclination, 400 km altitude, and 180° rotation maneuver about y-axis

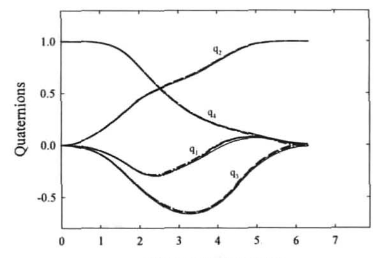

Fig. 7. Comparison of quaternion histories using a different number of nodes for the case of zero inclination, altitude of 400 km, and 180° rotation maneuver about y-axis

Table 4. Comparison of the Euler angles deviation due to the error of the geomagnetic field modeling The example is for the case of 98.3° inclination, altitude of 400 km, and 180° rotation maneuver about z-axis

Std. dev. (nT)	α (°)	β (°)	γ (°)
0	179.95	179.95	0.01
50	179.93	179.93	0.06
100	179.90	179.90	0.11
500	179.61	179.61	0.54
1000	179.24	179.24	1.07
2000	178.50	178.49	2.13
5000	176.30	176.25	5.27

Table 5. Parameters for the maneuver from sun-pointing to nadir-pointing of KOMPSAT-1

	Initial states	Required final states
Date	Value to be found	19 May 2004 17:36:50.0UTC
Quaternion	[-0.1966, 0.6803, 0.2893,0.6440]	[0.3435, -0.9364, -0.0655, -0.0257]
Orbital Elements (from TLE)	Epoch: 140.51213099 days 2004 Inclination: 98.0541° Argument Of Perigee: 327.4944° Eccentricity: 0.0007117 Right Ascension of Ascending node : 40.1234° Mean anomaly : 32.5821°	
Moment of Inertia(Kg×m2)	$\begin{bmatrix} 304.724 & 3.403 & -0.407 \\ 3.403 & 139.574 & 9.958 \\ -0.407 & 9.958 & 219.769 \end{bmatrix}$	

The Effects of the Geomagnetic Field Modeling Error

To apply the obtained open loop control solution to operating satellites, the effects of the observation error should be considered. This error can occur from a magnetometer's observation error and the geomagnetic field modeling error. To check the amount of attitude deviations due to these errors, the initial attitude with an error in observation data of magnetometer is propagated using the 4th order Runge-Kutta method with the control histories obtained by the method developed in this work. The errors are modeled as white-gaussian noises, and the deviations of the Euler angles at final time due to the noises are summarized in Table 4 for different noise levels. Table 4 shows that the errors of Euler angles α, β, and γ defined in the previous section increase by applying higher noise levels to the geomagnetic field. If the standard deviation of the observation error is less than 1000nT, which is about 3% error of the mean geomagnetic field of a given altitude, the maneuver can be achieved within one-degree deviation. Considering both the accuracy of IGRF model and magnetometer's measurement, the maneuver can be successfully performed through the method presented in this paper, even though the worst cases such as the occurrence of solar flare.

Maneuver to Earth pointing Attitude for KOMPSAT-1

A practical numerical test is conducted for the maneuver of KOMPSAT-1. Attitude control system of KOMPSAT-1 usually uses thrusters for attitude control, except for its normal operation. The required maneuver starts from a sun-pointing attitude and finishes near north-pole after rotating 90° about y-axis, and consequently, a nadir-pointing as displayed in Fig. 8. This maneuver is usually conducted to recover from a safehold mode or from maneuver mode attitude. Because the final time of maneuver must be terminated at 19 May 2004 17:36:50.0 when the satellite passes near north pole, the resulting problem becomes to determine the starting time of maneuver including the control histories. The orbit of maneuver duration is generated by using MSGP4 Propagator with Two Line Element. For a convenient implementation of the problem, the final and initial attitudes are exchanged, and the geomagnetic field data is generated through inverse orbit direction and tabulated.

Fig. 8. Maneuver to the nadir-pointing from sun-pointing

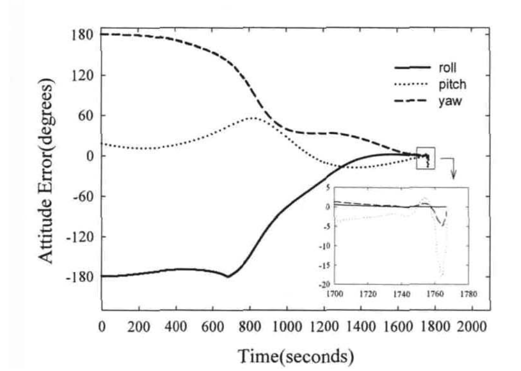

Fig. 9. Attitude errors about local vertical and local horizontal frame of reorientation maneuver from sun-pointing to nadir-pointing for KOMPSAT-1

Including final and initial attitudes, the physical properties are summarized in Table 5. Note that the moment of inertia of the spacecraft is approximated by ignoring the off-diagonal term. The achieved minimum maneuver time is successfully determined as 29 minutes and 18 seconds; about a third of the orbital period of KOMPSAT-1. The corresponding roll, pitch, and yaw angles with respect to local vertical and local horizontal show a convergence to zero, as shown in Fig. 9.

Conclusions

The capability of minimum-time attitude reorientations using only magnetic torquers was investigated and demonstrated for three-axis stabilized satellites. The 180 reorientation maneuvers are successfully completed within a half-orbital period. It is acceptable to state that the deviation of the Euler angles reaches about 1 after completing 180° reorientation with 1000 nT standard error in the geomagnetic field. Though it requires more maneuver time than maneuvers by using thrusters, the results show that a magnetic torquer can afford to maneuver of large angle reorientation in the context of their maneuver times. The methods developed here can also be used for an alternative maneuver strategy when main actuators like wheel and thruster fail. The strategy in this work was also numerically tested for the maneuver of currently operating satellite, KOMPSAT-1, by using actual orbital elements and physical parameters. The achieved minimum-time shows that the reorientation maneuver from sun-pointing to nadir-pointing takes about 30 minutes which is less than a third of the orbital period of KOMPSAT-1. It can be concluded that magnetic torquers can provide attractive and practical solutions for minimum-time rest-to-rest attitude maneuvers, especially, for small low-Earth satellites with limited control resources.

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