Attitude Control of a Tethered Spacecraft

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Abstract

An attitude control problem for a tethered spacecraft is studied. The tethered spacecraft is viewed as a multi-body spacecraft consisting of a base body, a massless tether that connects the base body and an end mass, and tether actuator dynamics. Moments about the pitch and roll axes of the base spacecraft arise by control of the point of attachment of the tether to the base spacecraft. The control objective is to stabilize the attitude of the base spacecraft while keeping the perturbations of the tether small. Analysis shows that linear equations of motion for the tethered spacecraft are not completely controllable. We study two different control design approaches: (1) we decouple the attitude dynamics from the tether dynamics and we design a linear feedback to achieve stabilization of the attitude dynamics, and (2) we decouple the controllable modes from the uncontrollable mode using Kalman decomposition and we design a linear feedback to achieve stabilization of the controllable modes. Simulation results show that, although it is difficult to control the tether, the tether motion can be maintained within an acceptable range while stabilizing the attitude dynamics of the base spacecraft.

Key Word : tethered spacecraft, attitude control, stabilization, Kalman decomposition, multi-body dynamics

Introduction

Tethered spacecraft have been investigated by many researchers for their possible use in many space missions. Among proposed applications are generation of electricity or propulsion, collection of orbital debris, scientific experiments in a micro-gravity environment, and many others[1, 2, 3]. Over the last three decades, many dynamics and control problems have been proposed and studied for tethered spacecraft during the various phases of its deployment, station keeping and retrieval. It has been shown that, due to large tether forces, conventional attitude control schemes are inefficient for any mission that requires precision pointing[4]. Therefore, researchers have investigated more effective methods of implementing attitude control for tethered spacecraft, *e.g.* by using the tether tension force to generate control moments by moving the tether attachment point relative to the spacecraft center of mass.

Lemke et al.[4] first introduced the use of an attachment-point actuator for tethered spacecraft.

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Kline-Schoder and Powell[5] considered a planar model of a tethered spacecraft and developed an LQR controller for pitch control. They also demonstrated experimental results in a laboratory environment. Pradhan et al.[6] studied several applications of the attachment-point control scheme in controlling tethered spacecraft. Their model included spacecraft attitude dynamics, tether dynamics and tether vibrations, but their analysis was restricted to a fixed orbital plane. Ashenberg and Lorenzini[7] developed an LQR controller to achieve 3-axis attitude control of a tethered spacecraft in an elliptic orbit.

The tethered spacecraft models considered in the above references are incomplete. Either the motions of the spacecraft and the tether are restricted to an orbital plane, or the tether is considered solely to apply a tension force to the base spacecraft, that is the dynamics of the tether are neglected. In this study, we consider a three dimensional model for a tethered spacecraft including the dynamics of the base spacecraft, the tether and end mass dynamics, and the attachment point dynamics; we obtain equations of motion that can be used to design feedback controllers. The main contribution of this work is development of feedback control laws that stabilize the attitude of the tethered spacecraft while minimizing perturbations to the tether.

Model Formulation

In this section we derive a mathematical model for a tethered spacecraft in a circular orbit. Then we identify a relative equilibrium of interest, and we develop linearized equations of motion near that equilibrium.

Nonlinear Equations of Motion

We consider a three dimensional tethered spacecraft depicted in Figure 1. It consists of a base spacecraft and an end mass connected by a tether. The base spacecraft is considered to be a rigid body; its mass and inertia matrix are denoted by m_B and J_B, respectively. The tether of length l is assumed to be rigid and massless. The end mass is assumed to be a mass particle; its mass is denoted by m_2. The actuators for the spacecraft are two linear motors that change the positions of the tether attachment point relative to the center of mass of the base spacecraft, thereby generating control moments about the roll and pitch axes. We assume that the attachment point has a mass m_1 and control forces act on that mass.

Let R_B denote the rotation matrix from the local vertical local horizontal (LVLH) frame to the base spacecraft fixed frame, thus representing the attitude of the base spacecraft with respect to the LVLH frame (see [10]). Let ω denote the angular velocity vector of the base spacecraft in body coordinates and $q = (r_1, r_2, \phi_1, \phi_2)$ denote the generalized shape coordinates that describe the tether attachment position and the tether attitude angles with respect to the base body. Let r_1 and $r_2$denote the position of the tether attachment point along the roll and pitch axes as measured from the center of mass of the base spacecraft; the attitude of the tether relative to the base spacecraft is determined by the pitch rotation angle ϕ_2 and by the roll rotation angle ϕ_1.

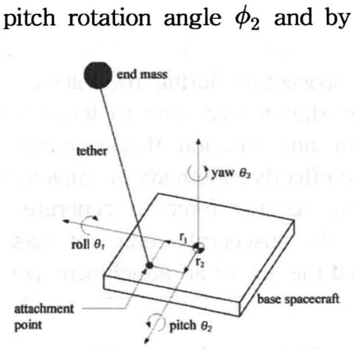

Fig. 1. Tethered Spacecraft

In this paper, we are interested in the attitude of the base spacecraft and the effect of tether dynamics and the attachment point dynamics. Therefore, we assume that there are no external control forces and moments acting on the tethered spacecraft, other than gravity, and the only control inputs are the internal forces τ_S acting on the attachment point. Also, we ignore the coupling between the translational dynamics of the spacecraft and the rotational and shape (tether and attachment point) dynamics. This can be done by assuming that the base spacecraft center of mass follows a circular orbit. In fact, the translational dynamics is coupled with the rotational dynamics and the shape dynamics; thus, the center of mass of the base spacecraft does not follow an exactly circular Keplerian orbit. However, if there is no external thrust acting on the spacecraft, these coupling effects are small and the deviation of the base spacecraft from a circular Keplerian orbit is insignificant[9]. The circular orbit is defined by a constant orbital radius r_e and a constant orbital rate $\dot{\nu}_e$. The nonlinear equations that describe the spacecraft motion can be derived as follows: (see [8] for the details.)

$$\dot{R}_B = R_B\left(\omega - \dot{\nu}_e R_B^{\mathrm{T}} e_2\right)^{\wedge} \tag{1}$$

$$\begin{aligned} J(q)\dot{\omega} + B_r(q)\ddot{q} = & -\dot{J}(q)\omega - \dot{B}_r(q)\dot{q} - \hat{\omega}J(q)\omega - \hat{\omega}B_r(q)\dot{q} \\ & + r_e \sum_{i=1}^{2} m_i \left(\dot{\nu}_e^2 - \frac{\mu}{d_i^3(R_B, q)} \right) \hat{\rho}_i(q) R_B^{\mathrm{T}} e_3 + \tau_g \end{aligned} \tag{2}$$

$$\begin{aligned} B_r^{\mathrm{T}}(q)\dot{\omega} + m(q)\ddot{q} = & \, r_e \dot{\nu}_e \left(B_t^{\mathrm{T}}(q)\hat{\omega} - \dot{B}_t^{\mathrm{T}}(q)\right) R_B^{\mathrm{T}} e_1 - \dot{B}_r^{\mathrm{T}}(q)\omega \\ & - \dot{m}(q)\dot{q} + \frac{\partial L(R_B, q, \omega, \dot{q})}{\partial q} + r_e \dot{\nu}_e^2 B_t^{\mathrm{T}}(q) R_B^{\mathrm{T}} e_3 + \tau_S \end{aligned} \tag{3}$$

where $e_2 = (0,1,0)$, $e_3 = (0,0,1)$; τ_g is the gravity gradient moment on the base spacecraft and τ_S is the generalized shape force vector; $J(q)$, $m(q)$, $B_r(q)$, $B_t(q)$ and $\rho(q)$ are defined by the geometry of the tethered spacecraft[8]. For column vector $a \in R^3$, $\hat{a}$ or $a^{\wedge}$ denotes the 3×3 skew-symmetric matrix formed from the column vector a.

The above equations of motion describe the attitude dynamics of the base spacecraft and the shape dynamics of the tether assuming that the base spacecraft follows exactly a circular orbit. Note that the only control force involved in these equations of motion is the force applied to the attachment point. If we denote its components along the roll axis and the pitch axis by τ_1 and τ_2, the generalized shape force vector τ_S is expressed as

$$\tau_S = (\tau_1 \;\; \tau_2 \;\; 0 \;\; 0)^{\mathrm{T}}$$

There are 7 degrees of freedom and 2 control inputs; therefore, equations (1)-(3) define an under-actuated multi-body control system.

Linearization of the Equations of Motion

We now consider small perturbations from the relative equilibria characterized by the following conditions:

- constant attitude of the base spacecraft with respect to the LVLH frame ($R_B = I$); this corresponds to the following attitude of the base spacecraft: the roll axis is tangent to the orbit, the yaw axis is aligned with the local vertical, and the pitch axis is normal to the orbital plane
- tether aligned with the local vertical and tether attachment point located at the center of mass of the base spacecraft ($q = \dot{q} = 0$)
- zero control forces ($\tau_S = 0$)

To the first order, we have

$$R_B = \begin{bmatrix} 1 & -\theta_3 & \theta_2 \\ \theta_3 & 1 & -\theta_1 \\ -\theta_2 & \theta_1 & 1 \end{bmatrix}, \qquad \omega = \begin{bmatrix} \dot{\theta}_1 + \dot{\nu}_e \theta_3 \\ \dot{\theta}_2 + \dot{\nu}_e \\ \dot{\theta}_3 - \dot{\nu}_e \theta_1 \end{bmatrix}$$

and the gravity gradient moment on the base spacecraft is [10]

$$\tau_g = (3\dot{\nu}_e^2 (J_3 - J_2)\theta_1,\ 3\dot{\nu}_e^2 (J_3 - J_1)\theta_2,\ 0)$$

Each scalar variable denotes a perturbation from the relative equilibrium. The angles θ_1, θ_2, θ_3 describe the roll, pitch, and yaw attitude of the base spacecraft with respect to the LVLH coordinates frame [10]. Substituting the above into equations (2)-(3) and ignoring higher order terms, we obtain the following linearized equations for the spacecraft roll and yaw dynamics:

$$(J_1 + m_2 l^2)\ddot{\theta}_1 - m_2 l \ddot{r}_2 + m_2 l^2 \ddot{\phi}_1 + (J_1 - J_2 + J_3)\dot{\nu}_e \dot{\theta}_3 + k_{11}\theta_1 + k_{15} r_2 + k_{16}\phi_1 = 0 \tag{4}$$

$$J_3 \ddot{\theta}_3 - (J_1 - J_2 + J_3)\dot{\nu}_e \dot{\theta}_1 + k_{33}\theta_3 = 0 \tag{5}$$

$$m_2 l \ddot{\theta}_1 + (m_1 + m_2)\ddot{r}_2 - m_2 l \ddot{\phi}_1 + k_{15}\theta_1 + k_{55} r_2 + k_{56}\phi_1 = \tau_2 \tag{6}$$

$$m_2 l^2 \ddot{\theta}_1 - m_2 l \ddot{r}_2 + m_2 l^2 \ddot{\phi}_1 + k_{16}\theta_1 + k_{56} r_2 + k_{66}\phi_1 = 0 \tag{7}$$

We also obtain the following linearized equations for the spacecraft pitch dynamics:

$$(J_2 + m_2 l^2)\ddot{\theta}_2 + m_2 l \ddot{r}_1 + m_2 l^2 \ddot{\phi}_2 + k_{22}\theta_2 + k_{24} r_1 + k_{27}\phi_2 = 0 \tag{8}$$

$$m_2 l \ddot{\theta}_2 + (m_1 + m_2)\ddot{r}_1 + m_2 l \ddot{\phi}_2 + k_{42}\theta_2 + k_{44} r_1 + k_{47}\phi_2 = \tau_1 \tag{9}$$

$$m_2 l^2 \ddot{\theta}_2 + m_2 l \ddot{r}_1 + m_2 l^2 \ddot{\phi}_2 + k_{27}\theta_2 + k_{47} r_1 + k_{77}\phi_2 = 0 \tag{10}$$

where the coefficients in the above equations depend on m_1, m_2, J_1, J_2, l, r_e, $\dot{\nu}_e$ and the gravity constant μ.

Note that the roll and yaw equations and the pitch equations each contain relevant tether effects, and these sets of equations are completely decoupled. It is easily shown that the equations for the pitch dynamics of the tethered spacecraft are completely controllable. The references [5, 6] in which the planar motion of a tethered spacecraft is studied treat this model. However, it can be shown that the equations for the roll and yaw dynamics of the tethered spacecraft are not completely controllable. This implies that we cannot control both the roll and yaw motion of the base spacecraft and the tether librational motion using linear feedback. (This does not necessarily exclude the existence of a nonlinear feedback controller that stabilizes the original nonlinear system.)

Feedback Control of the Tethered Spacecraft

In this section, we study control problems for the spacecraft roll and yaw dynamics given by equations (4)-(7), which is expressed in terms of configuration variables θ_1, θ_3, ϕ_1 and r_2. The equations (8)-(10), which describe the spacecraft pitch dynamics in terms of configuration variables θ_2, ϕ_2 and r_1, is completely controllable; therefore, linear control methods can be used to obtain a stabilizing feedback law. However, equations (4)-(7) are not completely controllable and the uncontrollable mode has imaginary eigenvalues. Thus it is not possible to find a linear asymptotically stabilizing feedback law. To resolve this issue, we propose two control ideas. The first idea is that we focus on controlling the base spacecraft attitude, which is considered most important, and we ignore the tether dynamics. The other idea is that we use a Kalman decomposition to control the controllable part of equations (4)-(7) and we ignore the part that is not controllable. These equations can be written in the standard state space form

$$\dot{\boldsymbol{x}}_1 = A_1 \boldsymbol{x}_1 + B_1 \tau_2 \tag{11}$$

Control of the Reduced Roll and Yaw Dynamics

Since equation (11) is not completely controllable, we cannot directly apply standard control design schemes. However, by focusing on the attitude control problem of the spacecraft base body and ignoring the tether dynamics, we can simplify the control problem to one that is completely controllable. If we subtract equation (7) from (4), along with equation (5) we obtain the following set of equations:

$$J_1\ddot{\theta}_1+(J_1-J_2+J_3)\dot{\nu}_e\dot{\theta}_3+(k_{11}-k_{16})\theta_1+(k_{15}-k_{56})r_2+(k_{16}-k_{66})\phi_1=0 \tag{12}$$

$$J_3\ddot{\theta}_3-(J_1-J_2+J_3)\dot{\nu}_e\dot{\theta}_1+k_{33}\theta_3=0 \tag{13}$$

Equations (12) and (13) only involve the roll and yaw dynamics of the base spacecraft and do not involve the dynamics of the tether or the attachment point. If we view the position r_2 of the attachment point as a control input, we can show that equations (12)-(13) are completely controllable. Therefore, we are able to design a feedback law that asymptotically stabilizes only the roll and yaw dynamics of the spacecraft. In fact, this reduction of the control system results in the roll-yaw system studied in [7].

The disadvantage of this approach is that we have neglected the dynamics of the tether. Control of the spacecraft roll and yaw dynamics might excite the motion of the tether. However, it is usual that the tether length is large compared to the displacements of the attachment point making it difficult to perturb the tether dynamics; in such cases, this feedback loop should be effective.

Now, we design a LQR control for the roll and yaw dynamics described by equations (12) and (13). It is convenient to add a double integrator to the input channel, so that

$$\ddot{r}_2=u \tag{14}$$

The acceleration u of the tether attachment point along the pitch axis is considered as the control input variable. We can find a simple relation between the acceleration u and the original force control input τ_2. It can be easily seen that the system defined by (12), (13) and (14) with u as a control input is completely controllable. We define a performance index for this LQR problem as follows:

$$J_r=\int_0^\infty\left(x_r^{\mathrm{T}}Q_r x_r+u^2\right)dt$$

where $x_r=(\theta_1,\theta_3,\dot{\theta}_1,\dot{\theta}_3)$ and Q_r is a positive semi-definite, symmetric matrix. The optimal feedback law is given in the form

$$u=-K_1\theta_1-K_2\theta_3-K_3r_2-K_4\dot{\theta}_1-K_5\dot{\theta}_3-K_6\dot{r}_2 \tag{15}$$

where the optimal gains are determined by solving a Riccati equation.

Feedback Law Using Kalman Decomposition

Another control approach can be developed for the not completely controllable system defined by equation (11). This approach utilizes a Kalman decomposition. The controllability matrix of the 8th order system defined by equation (11) has rank 6. This means that there are three modes that are controllable and one mode that is not controllable. The idea is to eliminate the uncontrollable mode from the linearized equations of motion. We then control the completely controllable part. For this tethered spacecraft model, the two poles of the uncontrollable mode are imaginary; that is, the uncontrollable mode is stable, although not asymptotically stable.

We now describe the control design process in detail. Given the system (11), which is not completely controllable, we can find eigenvalues and corresponding eigenvectors of the matrix A_1. Let Λ be the diagonal matrix composed of the eigenvalues, assumed distinct, and let the matrix V be the matrix whose columns are the linearly independent eigenvectors arranged so that they have the same order as the corresponding eigenvalues in the matrix Λ. Thus $A_1V=V\Lambda$. Now we define spectral coordinates η by $x_1=V\eta$.

The equations of motion can be written as:

$$\begin{aligned}\dot{\eta} &= V^{-1}A_1 V\eta + V^{-1}B_1\tau_2 \\ &= \Lambda\eta + V^{-1}B_1\tau_2\end{aligned} \tag{16}$$

Since the matrix Λ is diagonal, in the η coordinates the modes are decoupled. Therefore, by looking at the input matrix $V^{-1}B_1$, we can tell which modes are uncontrollable. If the i-th row of the matrix $V^{-1}B_1$ is null, then η_i is uncontrollable. Thus we can set up a new state vector by removing the two uncontrollable state variables, $\bar{\eta} = \bar{I}\eta$, where the 6×8 matrix $\bar{I}$ selects the controllable states. The reduced state equations, now expressed in terms of $\bar{\eta}$, are given by

$$\dot{\bar{\eta}} = \bar{\Lambda}\bar{\eta} + \bar{I}V^{-1}B_1\tau_2 \tag{17}$$

where $\bar{\Lambda}$ is a diagonal matrix which consists of the eigenvalues of the controllable modes. Equation (17) describes the controllable part of the original system; however, the equations may have complex coefficients due to the transformation defined by the complex matrix V. To obtain real equations, we define a new state vector p in the following way:

$$p = \bar{I}V\bar{I}^{\mathrm{T}}\bar{\eta} = \bar{V}\bar{\eta}$$

The equations expressed in the new state p are given by

$$\dot{p} = \bar{V}\bar{\Lambda}\bar{V}^{-1}p + \bar{V}\bar{I}V^{-1}B_1\tau_2 \tag{18}$$

The system (18) is real and represents the completely controllable part of the system given by (11). We can use any linear feedback control design method for this model; the controller is applied to the original system.

We design an LQR control using the state equation (18), with control input τ_2, which is the force acting on the attachment point along the pitch axis. We select a performance index for the LQR problem:

$$J_2 = \int_0^\infty \left(x_1^{\mathrm{T}} Q_2 x_1 + \tau_2^2\right)\mathrm{dt}$$

where Q_2 is a positive semi-definite, symmetric matrix. However, we need to express the performance index in terms of the transformed state p. The state p is obtained from the state x_1 by the transformation,

$$p = \bar{V}\bar{I}V^{-1}x_1$$

Since the size of the matrix $\bar{V}\bar{I}V^{-1}$ is 6 by 8, it does not have an inverse. However, if we assume that the uncontrollable modes are not disturbed, we have

$$x_1 = V\bar{I}^{\mathrm{T}}\bar{V}^{-1}p$$

Thus, the above performance index is equivalent to

$$J_2 = \int_0^\infty \left(p^{\mathrm{T}}\bar{Q}_2 p + \tau_2^2\right)dt$$

where

$$\bar{Q}_2 = \bar{V}^{-T}\bar{I}V^{\mathrm{T}}Q_2 V\bar{I}^{\mathrm{T}}\bar{V}^{-1}$$

The resulting optimal feedback law is given in the form

$$\tau_2 = -L_1\theta_1 - L_2\theta_3 - L_3 r_2 - L_4\phi_1 - L_5\dot{\theta}_1 - L_6\dot{\theta}_3 - L_7\dot{r}_2 - L_8\dot{\phi}_1 \tag{19}$$

where the optimal gains are determined by solving a Riccati equation.

Simulations

In this section, we illustrate the effectiveness of the two feedback controllers to achieve stabilization of a tethered spacecraft. We present simulations and we compare the results. The physical parameters of the spacecraft used in the simulations are m_B = 300 (kg), m_1 = 1.0 (kg), m_2 = 50 (kg), l = 100 (m), J_B = diag(180, 150, 200) (kg · m^2). The radius of the circular orbit is r_e = 6,678 (km), and from this value the orbital rate ν_e = 0.001157 (rad/sec). For this base spacecraft, it is easy to show that the roll and yaw dynamics, without control by the tether attachment point, is unstable. Thus the use of the tether is essential to stabilize the desired attitude of the base spacecraft.

We consider an initial configuration of the spacecraft given by

$$(\theta_1, \theta_3, \phi_1, r_2) = = (0.05 \text{ (rad)}, 0.05 \text{ (rad)}, -0.05 \text{ (rad)}, 0.0 \text{ (m)})$$

with the spacecraft initially at rest. Note that the initial condition for $(\phi_1+\theta_1)$ is zero so that the tether is initially unperturbed from the local vertical. Since the tether is not initially perturbed, we can see how the controller affects the resulting tether dynamics.

We first show simulation results for the closed loop system using the control method expressed by equation (15). By choosing the performance index as

$$J_1 = \int_0^\infty \left(10\theta_1^2 + \theta_3^2 + u^2\right) dt$$

the control gains are as follows:

[1.0 -0.752 0.034 -26.8 -15.22 0.259]

Figure 2 shows the time histories of the roll and yaw response of the base spacecraft. We can see that the trajectories are asymptotically stable. Figure 3 shows the time histories of the attachment point r_2 and the tether angle $(\phi_1+\theta_1)$ measured from the local vertical. The attachment point moves initially to control the attitude but it eventually tends to zero as the base spacecraft is stabilized. The tether is perturbed due to the control action and it has a small oscillatory motion thereafter. Since we are not controlling the tether motion, it continues oscillating. However, the perturbation in the tether angle $(\phi_1+\theta_1)$ in this case is less than 2×10^{-3} (rad) and is negligible.

Now, consider the feedback controller using the Kalman decomposition approach. The performance index is chosen as

$$J_2 = \int_0^\infty \left(100\theta_1^2 + \theta_3^2 + 100\phi_1^2 + r_2^2 + \dot{r}_2^2\right) dt$$

This gives the control gains

[-16.6 5.85 1.60 -9.66 1565 13030 0.383 976.6]

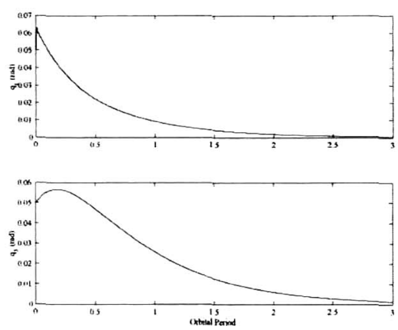

Fig. 2. Time Response for the Roll Angle Θ_1 and the Yaw Angle Θ_3 (Controller I)

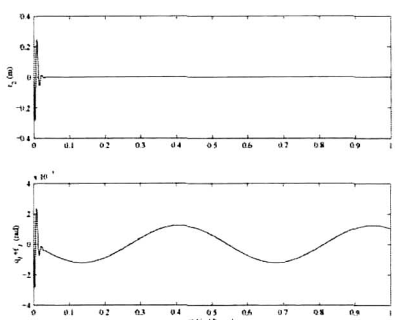

Fig. 3. Time Response for the Attachment Point r_2 and the Tether Libration Angle $\Phi_1+\Theta_1$ (Controller I)

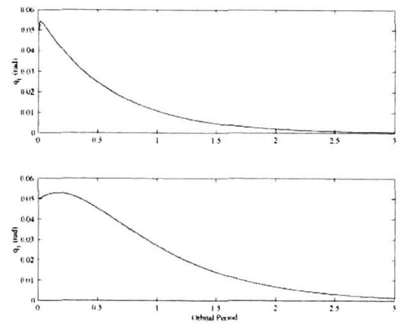

Fig. 4. Time Response of the Roll Angle Θ_1 and the Yaw Angle Θ_3 (Controller II)

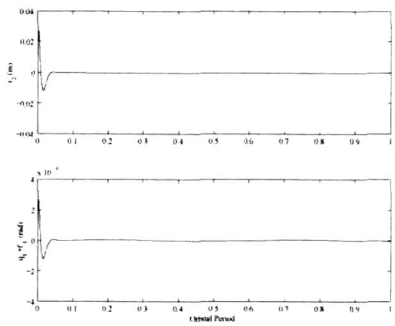

Fig. 5. Time Response for the Attachment Point r_2 and the Tether Libration Angle $\Phi_1+\Theta_1$ (Controller II)

Figure 4 shows the time histories of the roll and yaw response of the base spacecraft. We can see that the trajectories are asymptotically stable and are very similar to the responses in Figure 2. Figure 5 shows the time histories of the attachment point r_2 and the tether angle $(\phi_1+\theta_1)$ measured from the local vertical. The attachment point moves initially to control the attitude but it eventually tends to zero as the base spacecraft is stabilized. The tether is perturbed due to the control action and it oscillates thereafter. However, compared to the previous closed loop, the attachment point movement and the amplitude of the tether libration are much smaller. This is because this feedback design method incorporates those responses in the performance index.

Conclusion

We have developed a mathematical model for attitude control of a tethered spacecraft. By including the tether dynamics and the attachment point dynamics, we have obtained more realistic equations of motion than have previously been studied. These equations show that the pitch control problem can be decoupled from the roll and yaw control problem. Since the pitch control problem is controllable and easy, we have studied the roll-yaw control problem, that is not completely controllable. Using the formulated equations, we obtained two different feedback laws for roll and yaw control of the spacecraft. We conclude that the feedback using the Kalman decomposition approach is more effective in the sense that it requires less actuator movement and it has less effect on the tether dynamics.

We note that we have considered the case when the initial perturbation to the uncontrollable mode is small. The uncontrollable mode not only involves the tether motion but also the attitude motion of the base spacecraft. If there is a perturbation to the tether dynamics, it also affects the rotational motion of the base spacecraft; however, since the Kalman decomposition based feedback controller does not affect the uncontrollable mode, the feedback is unable to achieve exact attitude stabilization. The first feedback controller obtained for the reduced roll and yaw dynamics equations, though it might excite the tether motion, decouples the attitude of the base spacecraft from the tether dynamics, thus always keeping the base spacecraft stabilized.

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