

# Numerical Analysis of SMA Hybrid Composite Plate Subjected to Low-Velocity Impact

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## Abstract

The fiber reinforced laminated composite structures are very susceptible to be damaged when they are impacted by foreign objects. To increase the impact resistance of the laminated composite structures, shape memory alloy(SMA) thin film is embedded in the structure. For the numerical impact analysis of SMA hybrid composite structures, SMA modeling tool is developed to consider pseudoelastic effect of SMAs. Moreover, the damage analysis is considered using failure criteria and a simple damage model for reasonable impact analysis. The numerical results are verified with the experimental ones. Impact analyses for composite plate with pre-strained SMAs are numerically performed and the damage areas are investigated.

**Key Word** : shape memory alloy, low velocity impact, composite, pseudoelasticity

## Introduction

The fiber reinforced laminated composite structures are widely used for their good specific strength and specific stiffness. However, owing to their weak impact resistance, laminated composite structures are very susceptible to be damaged when they are impacted by a foreign object. Especially, the damages caused by low velocity impact are hard to be detected and decrease the mechanical properties of the composite structures. Embedding shape memory alloys(SMAs) in the laminated composite structures is one possible way to increase impact resistance of the composite structure. SMAs have a remarkably high strain-to-failure ratio about 20% with large recoverable strain up to 10%. This characteristic of SMAs is primarily due to the phase transformation into stress-induced martensite phase. This enables SMAs to absorb and dissipate large amount of energy. Therefore, SMAs embedded in the composite structures can absorb large amount of impact energy and increase the impact resistance of composite structures.

Several experimental studies show that the damage areas of the composite structures can be reduced and especially the perforation resistance of the composite structures can be remarkably increased by embedding SMA wires in the composite structures. Paine and Roger [1] performed impact experiment with graphite/epoxy and glass/epoxy composite plates embedding different kinds of fibers such as nitinol(one kind of SMAs), kevlar and aluminum, and found that the nitinol wires can increase impact resistance of the composite plate effectively comparing with other wires. Tsoi et al.[2] performed impact test of glass/epoxy composite plates embedding nitinol wires with different pre-strain(0%, 1.5%, 3%), positions in thickness direction(1/4, 1/2, 3/4) and volume fractions(50%,100%). Also, the results are compared with those of using stainless steel wires. They found that the delamination areas were decreased with increasing pre-strain and embedding SMA wires at 3/4 position was better than other positions to reduce the damage area.

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In the most numerical studies for increasing impact resistance of composite structures with SMAs, contact laws are used to calculate the contact force and only recovery stresses of SMA wires are simply considered for the effect of SMAs [3]. Recently Meo [4] performed the impact analysis of SMA hybrid composite plate using DYNA3D and he considered the pseudoelastic effects using experimental data.

In this study, low velocity impact analyses of the SMA hybrid composite plate are numerically performed. To simulate more precise behavior of SMAs, the mathematical model which is based on the Lagoudas' 3D model[5] is modified into plane stress conditions, for the 2D problems [6]. Moreover, the damage analysis is considered for reasonable impact analysis. And impact analyses for composite plate with pre-strained SMAs are numerically performed and the damage areas are investigated.

## Constitutive Modeling of SMAs

### Constitutive Equation of SMAs

SMAs show thermomechanically complicated behaviors, such as shape memory effect and pseudoelasticity. To simulate this complicated behaviors of SMAs, the modeling of constitutive equation is necessary. The constitutive equation of SMAs based on the Lagoudas' model[5] can be expressed as Equation (1). As can be seen, the strain( $\epsilon$ ) is related with stress( $\sigma$ ), temperature( $T$ ) and martensite fraction( $\xi$ ) which is internal variable indicating the amount of martensite phase.

$$\dot{\epsilon}_{ij} = S_{ijkl} \dot{\sigma}_{kl} + \alpha_{ij} \dot{T} + Q_{ij} \dot{\xi} \quad (1)$$

where, the compliance tensor  $S_{ijkl}$  and thermal expansion coefficient  $\alpha_{ij}$  are expressed in Equation (2). These are function of martensite fraction. The relationship between strain and martensite fraction  $Q_{ij}$  is expressed in Equation (3).

$$S_{ijkl} = S_{ijkl}^A + \xi \Delta S_{ijkl} \quad (2)$$

$$\alpha_{ij} = \alpha_{ij}^A + \xi \Delta \alpha_{ij}$$

$$Q_{ij} = \Delta S_{ijkl} \sigma_{kl} + \Delta \alpha_{ij} (T - T_o) + A_{ij} \quad (3)$$

where, the superscript A means austenite phase and  $\Delta$  means the difference of the values between martensite and austenite phase. And  $A_{ij}$  is the transformation tensor which decides the direction of phase transformation.

The thermodynamic force which is derived from Gibbs free energy assuming linear thermoelastic behavior and nonlinear transformation hardening behavior can be expressed in Equation (4).

$$\begin{aligned} \pi = & \sigma_{ij} A_{ij} + \frac{1}{2} \sigma_{ij} \Delta S_{ijkl} \sigma_{kl} + \Delta \alpha_{ij} \sigma_{ij} (T - T_o) \\ & + \rho \Delta c [(T - T_o) - T \ln(T/T_o)] + \rho \Delta s_o T - \partial f / \partial \xi - \rho \Delta u_o \end{aligned} \quad (4)$$

where,  $\rho, c, s_o, u_o$  means density, specific heat, specific entropy and specific internal energy, respectively. And  $f$  is transformation hardening function which represents the behavior of phase transformation.

Transformation function can be defined as Equation (5) using thermodynamic force and  $Y^*$  which means the maximum energy that the material can have in elastic domain.

$$\Phi = \begin{cases} \pi - Y^* & , \dot{\xi} > 0 \\ -\pi - Y^* & , \dot{\xi} < 0 \end{cases} \quad (5)$$

Kuhn-Tucker inequality is expressed in Equation (6) with the transformation function. It decide whether the phase transformation occurs or not.

$$\begin{aligned} \dot{\xi} &\geq 0, \Phi(\sigma, T, \xi) \leq 0, \Phi \dot{\xi} = 0 \\ \dot{\xi} &\leq 0, \Phi(\sigma, T, \xi) \leq 0, \Phi \dot{\xi} = 0 \end{aligned} \quad (6)$$

For  $\Phi < 0$ , elastic response is obtained without phase transformation by the constraint  $\Phi \dot{\xi} = 0$ . On the other hand, the forward phase transformation ( $\Phi = 0, \dot{\xi} > 0$ ) or reverse transformation ( $\Phi = 0, \dot{\xi} < 0$ ) occurs.

For the numerical analysis, return-mapping algorithm[7] is used. It is composed of elastic predictor and transformation corrector. At elastic predictor, new state is calculated elastically with given increments of strain(or stress) and temperature. If the newly calculated state is out of the elastic domain, it is corrected with calculating phase transformation in transformation corrector stage. The detailed information can be seen in the reference [5].

For the plane stress assumption, the stress, strain, thermal expansion coefficient vectors and compliance tensor are expressed as follows,

$$\sigma = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix}, \epsilon = \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{Bmatrix}, \alpha = \begin{Bmatrix} \alpha_{xx} \\ \alpha_{yy} \\ 0 \end{Bmatrix}, S = \begin{bmatrix} 1/E & -\nu/E & 0 \\ -\nu/E & 1/E & 0 \\ 0 & 0 & 2(1+\nu)/E \end{bmatrix} \quad (7)$$

### Verification of the SMA modeling

The ABAQUS finite element program is used for the impact analysis and the user material subroutine(VUMAT) for the SMA modeling is developed. The developed numerical model of SMAs is verified with experimental results[9]. The specimen used in the experiment is nitinol(Ti-48.0%) thin film which is  $1mm \times 5mm$  and the thickness is  $0.006mm$ . The strain-stress curves at 305K and 325K are numerically calculated using  $3 \times 15$  membrane elements (M3D4R). As can be seen in Figure 1, There are good agreements between the numerical and experimental results.

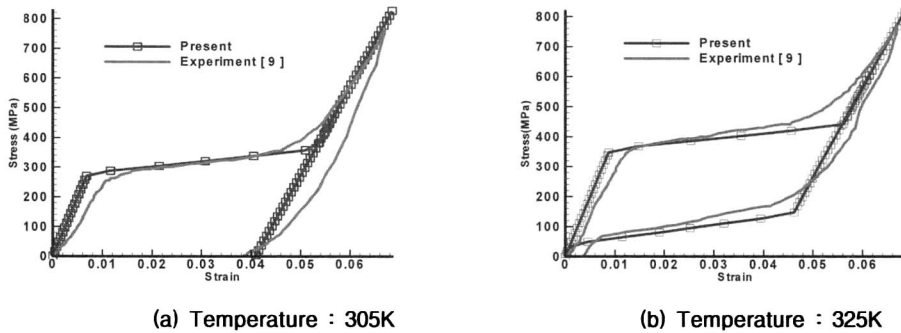


Fig. 1. Stress-strain curves of SMA thin film

### Damage analysis of composite plate

#### Failure criteria

In the impact analysis of composite structure, the consideration of impact damage is important to predict reasonable results. For the damage analysis, the failure criteria[8] which are improved from that in the DYNA3D are used. For the plane stress assumption, only fiber failure and matrix crack (crushing) failure modes are considered. The delamination failure mode is not considered because the delamination mode is related with the stresses in the thickness direction which can not be calculated in the problem assuming plane stress conditions. Also, the stresses in the thickness direction are neglected in the failure criteria.

If the failures occur, the stiffness matrix is modified to consider the effects of failure modes and the stresses are updated. In the case of fiber failure, it is assumed that the material cracks in the transverse direction of the fibers and  $\sigma_{11}$  and  $\sigma_{12}$  are updated to zero. In the case of matrix cracking, it is assumed that the material cracks in the fiber direction and  $\sigma_{22}$  and  $\sigma_{12}$  are updated to zero. For the case of matrix crushing, only  $\sigma_{22}$  becomes zero because the material can resist the shear stress in compression. If the fiber failure and matrix cracking (or crushing) occur at the same position, all the stresses become zero.

### Verification of failure analysis method

For the verification of the damage analysis algorithm, impact analyses of composite plates are performed and the results are compared with the experimental ones [11]. The size of the specimens used in the experiment are  $125 \times 75 \times 1\text{mm}$  and  $200 \times 200 \times 2\text{mm}$ . The materials of composite plate and impactor are *HTA/6376* and steel, respectively. And the radius of impactor is 6.35mm. The material properties and strengths are given in the reference [11]. The contact forces of two models are compared with the experimental results in Figure 2. The 'undamaged' means that the damage analysis algorithm is not considered.

The numerical results are comparable to the experimental results, but the maximum contact forces are smaller than the experimental ones and there are sharp oscillations in the contact forces after initial failure, because the stresses are updated to zero abruptly when the damages occur without gradual degradation. This trend is also reported in the reference [10].

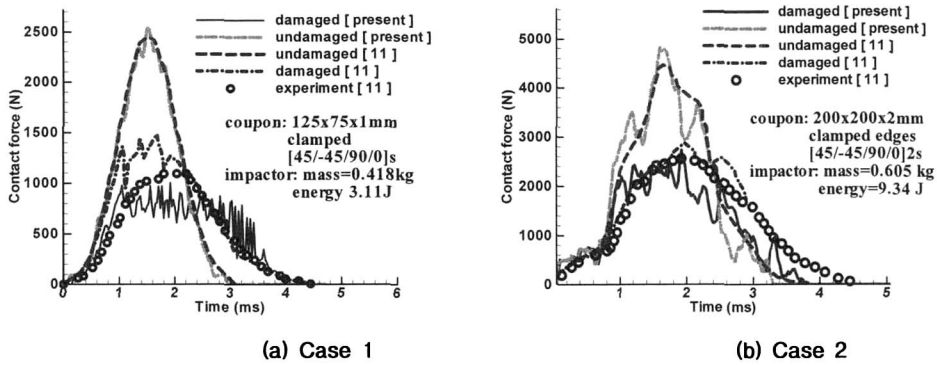


Fig. 2. History of contact forces

### Impact analysis of SMA hybrid composite plate

To enhance the impact resistance of the composite structure, SMA thin film is embedded. The density of the SMAs is about 4 times that of common laminated composites. So, strip type of SMAs is used to reduce the increase of weight. The SMA strips are embedded in the back face of the plate because the largest stresses are applied in the back side during low velocity impact. Moreover, SMA strips are embedded in the impact face to make symmetric composite plate. The numerical model is shown in the Figure 3. The size of the SMA strips are  $8 \times 200 \times 0.125\text{mm}$  and the properties of SMAs are given in the reference [6]. The impact energy and impact velocity are 9.34J and 5.56m/s, respectively. Four models, namely one base model and three SMA hybrid models, are used in the analyses. In the case of base model, all the layers are *HTA/6376* and the layup angle is  $[90^\circ/(45^\circ/-45^\circ/90^\circ/0^\circ)]_2$ . In the SMA hybrid models, 0%, 2%, 4% pre-strained SMA strips are used.

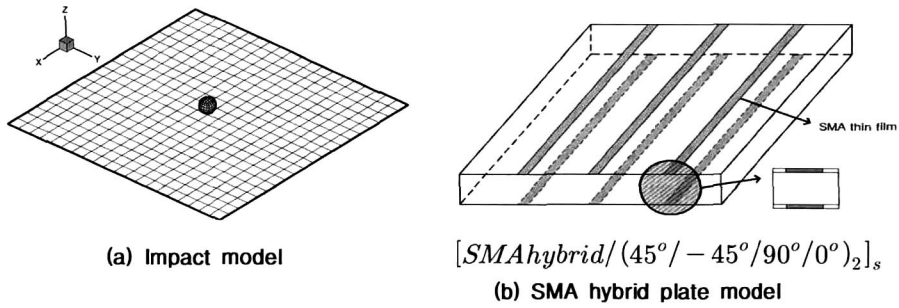


Fig. 3. Numerical model of the impact analyses

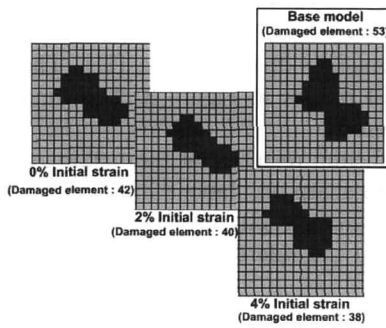


Fig. 4. Maximum damaged areas

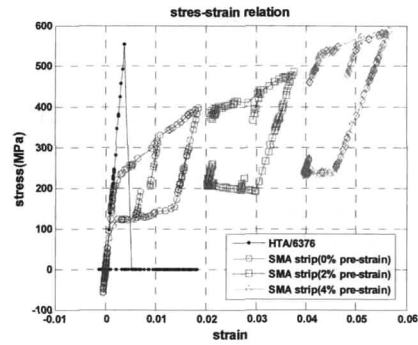


Fig. 5. Stress-strain curves

The largest damage areas are compared with each other in Figure 4. The damage areas include both the fiber breakage and matrix cracking. The maximum damaged areas of SMA hybrid models are smaller than that of the base model. The maximum damaged areas are detected in the second layer from the back face for all models. And the damaged area in each layer is smaller than that of the base model in the same position. The reason seems that the SMA strips can absorb large impact energy through phase transformation. Figure 5 shows the stress-strain curves of SMAs and *HTA/6376*. The areas of the hysteresis of SMAs mean the energy dissipated by the SMAs and the areas below the curve mean the energy that the material can absorb. The stress of *HTA/6376* becomes zero abruptly due to failure. As can be seen, SMAs can absorb a large amount of energy without damages comparing with the *HTA/6376* material.

## Conclusions

SMA modeling tool is developed for considering reasonable SMA behaviors including pseudoelastic effect and the simple damage analysis algorithm is implemented. The developed SMA modeling program and damage analysis algorithm are verified with the experimental results and there are good agreements between the numerical and experimental results. Also, numerical impact analyses are performed using ABAQUS with the developed program. From the numerical results, it can be known that considering the damage analysis is important in the impact analysis of composite structure. The impact resistance of the composite plate is effectively enhanced by embedding SMA strips, because the embedded SMAs absorbed a large amount of impact energy. Moreover, there is only 4.7% of weight increment comparing with the base model.

## Acknowledgement

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