Kalman Filtering for Spacecraft Attitude Estimation by Low-Cost Sensors

Henzeh Lee*, Yoonhyuk Choi and Hyochoong Bang*****
KAIST, Daejon, Korea, 305-701

Jong-Oh Park****
Dong-A University, Busan, Korea, 604-714

Abstract

In this paper, fine attitude estimation using low-cost sensors for attitude pointing missions of spacecraft is addressed. Attitude kinematics and gyro models including bias models are in general utilized to estimate spacecraft attitude and angular rate. However, a linearized model and a transition matrix are derived in this paper from nonlinear spacecraft dynamics with external disturbances. A Kalman filtering technique is applied and offers relatively high estimation accuracy under dynamic uncertainties. The proposed approach is demonstrated using numerical simulations.

Key Words : Kalman Filter, low-cost sensors, external disturbance, attitude pointing

Introduction

Recently, small satellites have entered a new era initiated by several university programs of space research exploration. Some universities have been designing nano-satellite weight class under very tight budget constraints, some universities have the challenge of designing a small scale spacecraft with specific science missions for exploration to the moon, asteroid or other planets. Since these programs are initially defined by universities, several limitations exist. The constraints may be divided by three categories : prices, weight and power. As a result, the sensor suites of the small satellites are comprised of less accurate low cost component with little heritage. The mass and constraints also discouraged the use of accurate or redundant sensors.[1]

As available sensors for spacecraft attitude determination(AD), there are several useful AD sensors. The three-axis magnetometer(TAM) is an imperative part.[2] TAM gives a continuously available two-axis attitude measurement. Even though this AD sensor is relatively low-cost element, it gives surprising accuracy. Therefore, almost every attitude controlled spacecraft has included one type of TAM. Other acceptable AD sensors for small satellites are sun and star trackers. Sun sensors based on small CMOS cameras have been developed and offer the inspiring prospect of sub-degree pointing accuracy. The CMOS star tracker is the ultimate want list of a small satellite for attitude determination.

* Post-Doctoral Researcher
** Ph.D Candidate
*** Professor
E-mail : hcbang@ascl.kaist.ac.kr Tel : 042-869-3722 Fax : 042-869-3710
**** Adjunct Professor

To accomplish fine attitude pointing mission of spacecraft by the low-cost AD sensors with relatively low accuracy have been a challenging issue in this field. A reasonable approach under the significant condition to de-tumble relying on magnetometer measurements has been published to dissipate kinetic energy adequately[3], while the filtering method based on only magnetometer gives rate estimate with proper accuracy. Predictive filtering technique based on only AD sensors was applied to estimate angular rate of spacecraft without rate measurement.[4]

The space has hardly predictable sources such as torques from the solar pressures, gravity gradient, and etc. Especially, it has been known that and the internal instability due to the distribution of static and dynamic unbalance of highly rotating reaction wheels lead to significant uncertainties for the modeling. The dynamic equations for the spacecraft attitude include many difficulties in the filter modeling.[5] It means that it is undesirable to use the dynamic model based filtering technique such as Kalman Filtering. Nevertheless, to accomplish the given tight attitude pointing budget using the low-cost sensors, a Kalman filtering approach is used to estimate spacecraft attitude. It is believed that the Kalman filtering technique including spacecraft dynamic model is applicable for attitude and rate estimation in a steady state case such as earth pointing or external galaxy observation missions. In this approach, relatively fine attitude accuracy can be obtained by estimating the external disturbance and decreasing the modeling uncertainties as well.
Attitude kinematics and gyro models including bias models are in general utilized to estimate spacecraft attitude and angular rate. However, in this paper, a linearized model and transition matrix from the spacecraft nonlinear dynamic model with external disturbance are obtained for fine attitude determination with low-cost sensors and real-time applications.

The organization of this paper proceeds as follows. First, A brief review of the dynamic model of spacecraft, sensor models is presented. Then, linearized model of spacecraft is shown. Next, Kalman filtering approach for fine attitude estimation and process covariance analysis are also shown. Finally, the proposed method is used for attitude pointing missions to demonstrate the usefulness.

Attitude Dynamics, Kinematics and Sensor Models

In this section, a brief review of attitude dynamics and kinematics of spacecraft and sensor models are shown. For equations of motion of spacecraft, a widely used Euler' s equation is[1]

$$\omega = J^{-1}(-\omega^{\times}J\omega + u + \eta_{\omega}) \tag{1}$$

where ω denotes an angular rate vector of spacecraft, u is a control input vector, $J \equiv \mathrm{diag}[J_{xx} \quad J_{yy} \quad J_{zz}]$ denotes the moment of inertia, which is assumed that the cross-product terms of spacecraft are in general very small to neglect. A skew-symmetric matrix defined as

$$x^{\times} = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix} \tag{2}$$

is the cross-product matrix, and η_{ω} is a independent white Gaussian noise process with

$$E\langle \eta_{\omega}(t) \rangle = 0 \tag{3}$$

$$E\langle\eta_{\omega}(t_1)\eta_{\omega}^{T}(t_2)\rangle = I_{3\times3}\sigma_{\omega}^{2}\delta(t_1 - t_2) \tag{4}$$

where $E\langle\cdot\rangle$ denotes the expectation, $\delta(t_1 - t_2)$ is the Dirac delta function, and σ_s is a standard deviation of η_{ω}.

The attitude kinematics equations of spacecraft are presented in this paragraph. As one of the representative attitude parameters, the well-known quaternion is defined by $q = [p^T \quad q_4]^T$ with $p = [q_1 \quad q_2 \quad q_3]^T$. The successive quaternion product operation is defined as [4]

$$[q\otimes]q' = \begin{bmatrix} q_4 I_{3\times3} - p^{\times} & p(7) \\ -p^T & q_4 \end{bmatrix} q' \tag{5}$$

$$q^{-1} = [-p^T \quad q_4]^T \tag{6}$$

$$q \otimes q^{-1} = q_I \equiv [0 \quad 0 \quad 0 \quad 1]^T \tag{7}$$

where q_I is so-called an identity quaternion, q^{-1} is an inverse quaternion of q, and $I_{n\times n}$ is a $n\times n$ identity matrix. The quaternion satisfies a constraint given by $q^T q = 1$. With $q_{\omega} = \begin{bmatrix}\omega^T & 0\end{bmatrix}^T$, the rotational kinematics equation of the quaternion is defined as

$$q = \frac{1}{2}\Omega(\omega)q = \frac{1}{2}[q_{\omega}\otimes]q \tag{8}$$

$$\Omega(\omega)q_I = q_{\omega} \tag{9}$$

A widely used sensor that measures the angular rate is a rate integrating gyros such as fiber optic gyros (FOGs). For this sensor, a commonly used model is described as [4]

$$\omega_g = \omega - b - \eta_a \tag{10}$$

$$b = \eta_b \tag{11}$$

where ω_g is an actual gyro output vector, that is, the continuous-time measured angular rate vector, b is a bias vector, and η_a and η_b are independent white Gaussian noise processes with

$$E\langle\eta_a(t)\rangle = 0 \tag{12}$$

$$E\langle\eta_a(t_1)\eta_a^T(t_2)\rangle = I_{3\times3}\sigma_a^2\delta(t_1 - t_2) \tag{13}$$

$$E\langle\eta_b(t)\rangle = 0 \tag{14}$$

$$E\langle\eta_b(t_1)\eta_b^T(t_2)\rangle = I_{3\times3}\sigma_b^2\delta(t_1 - t_2) \tag{15}$$

σ_a and σ_b are standard deviations of η_a and η_b, respectively.

To measure the attitude of spacecraft, a star tracker model is given by

$$q_s = \delta q_s \otimes q \tag{16}$$

where q_s is the star tracker output vector, namely, continuously measured quaternion vector, $\delta p_s = \eta_s$ is a independent white Gaussian noise process of the measurement noise quaternion vector with

$$E\langle \eta_s(t) \rangle = 0 \tag{17}$$

$$E\langle \mu_s(t_1)\eta_s^T(t_2) \rangle = I_{3\times3}\sigma_s^2\delta(t_1 - t_2) \tag{18}$$

σ_s is a standard deviation of η_s.

Summary

Process equations of motion to estimate interested states are summarized as follows :

$$\dot{\omega} = J^{-1}(-\omega^{\times}J\omega + u + \eta_{\omega}) \tag{19}$$

$$\dot{q} = \frac{1}{2}\Omega(\omega)q \tag{20}$$

$$\dot{b} = \eta_b \tag{21}$$

By taking expectations, predicted estimates satisfy differential equations given by

$$\frac{d}{dt}x = E\langle f(x) \rangle \equiv \hat{f}(x) \tag{22}$$

which we can write approximately as

$$\frac{d}{dt}x = f(x) \tag{23}$$

such that the predicted estimates are also summarized as

$$\frac{d}{dt}\omega = J^{-1}(-\omega^{\times}J\omega + u) \tag{24}$$

$$\frac{d}{dt}q = \frac{1}{2}\Omega(\omega)q \tag{25}$$

$$\frac{d}{dt}b = 0 \tag{26}$$

and using the predicted estimates, the predicted gyro output vector is

$$\omega_g = \omega - b \tag{27}$$

Linearization

To use the extended Kalman filter(EKF) for obtaining best estimate, linearization is required. Let us begin the linearization process with defining small variations described as

$$\delta\omega = \omega - \omega \tag{28}$$

$$\delta q = q \otimes q^{-1} \tag{29}$$

$$\delta b = b - b \tag{30}$$

One can readily obtain a linearized model by expanding the differential equation by the Taylor series and neglecting higher order terms of $O(\delta x^2)$ such as

$$\frac{d}{dt}(x+\delta x) \approx f(x) + \frac{\partial}{\partial x} f(x)\bigg|_{x=\hat{x}} \delta x \tag{31}$$

A linearized error spacecraft dynamics model is computed as

$$\delta\dot{\omega} = J_{\hat{\omega}}\delta\omega + J^{-1}\delta u + J^{-1}\eta_{\omega} \tag{32}$$

where $J_{\hat{\omega}}$ is defined as

$$J_{\hat{\omega}} = \begin{bmatrix} 0 & J_x\hat{\omega}_z & J_x\hat{\omega}_y \\ J_y\hat{\omega}_z & 0 & J_y\hat{\omega}_x \\ J_z\hat{\omega}_y & J_z\hat{\omega}_x & 0 \end{bmatrix} \tag{33}$$

where $J_x = \frac{J_{yy}-J_{zz}}{J_{xx}}, J_y = \frac{J_{zz}-J_{xx}}{J_{yy}}$ and $J_z = \frac{J_{xx}-J_{yy}}{J_{zz}}$.

The dependency of the quaternion elements described as $q^T q = 1$ makes an error covariance be singular. Therefore, even though the quaternion is bilinear, we need linearization process to obtain error quaternion representation to avoid the singularity problem. For derivation of the error quaternion, a brief review of quaternion properties is presented as follows :

$$q_1 \otimes q_2 - q_2 \otimes q_1 = 2\begin{bmatrix} p_2^{\times} p_1 \\ 0 \end{bmatrix} \tag{34}$$

$$(q_1 + q_2) \otimes q = q_1 \otimes q + q_2 \otimes q \tag{35}$$

$$\frac{d}{dt}(q_1 \otimes q_2) = q_1 \otimes q_2 + q_1 \otimes q_2 \tag{36}$$

For a reference quaternion state vector, a small variation of the quaternion is described as

$$q = \delta q \otimes q \tag{37}$$

$$\dot{q} = \frac{1}{2}\Omega(\omega) q \tag{38}$$

By inserting Eq.(43) to Eq.(44), the quaternion kinematics is given by

$$\frac{d}{dt}(\delta q \otimes q) = \frac{1}{2}\Omega(\omega)(\delta q \otimes q) \tag{39}$$

By using the property in Eq.(34) and applying $q \otimes q^{-1} = q_I$, the differential equation of δq is described as

$$\delta\dot{q} = \frac{1}{2}\Omega(\omega)\delta q - \frac{1}{2}\delta q \otimes \Omega(\omega) q_I \tag{40}$$

The angular rate vector is

$$\omega = \omega + \delta\omega \tag{41}$$

Also, note that Ω is linear in its elements,

$$\Omega(\omega + \delta\omega) = \Omega(\omega) + \Omega(\delta\omega) \tag{42}$$

Applying Eqs.(9),(34) and (35) gives

$$\delta\dot{q} = \frac{1}{2} q_{\hat{\omega}} \otimes \delta q - \frac{1}{2}\delta q \otimes q_{\hat{\omega}} + \frac{1}{2}\Omega(\Delta\omega)\delta q \tag{43}$$

$$= \begin{bmatrix} -\omega^{\times}\delta p \\ 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} \delta\omega \\ 0 \end{bmatrix}\delta q_4 + O(\delta q^2) \tag{44}$$

Assuming $\delta q_4 = 1$ and neglecting higher order terms give

$$\delta\dot{p} = \frac{1}{2}\delta\omega - \omega^{\times}\delta p + \eta_p \tag{45}$$

Note that the kinematic equation has originally no white Gaussian noise process. By the integrated angular rate due to the process noise (η_ω) and the higher terms of order two, a white Gaussian noise process, η_p, is introduced such as

$$E\langle \eta_p(t) \rangle = 0 \tag{46}$$

$$E\langle \eta_p(t_1)\eta_p^T(t_2) \rangle = I_{3\times3}\sigma_p^2\delta(t_1 - t_2) \tag{47}$$

This noise process will be discussed shortly in the next section. The linearized bias error equation of the rate gyros is simply given by

$$\delta\dot{b} = \dot{b} - \dot{\hat{b}} = \eta_b \tag{48}$$

The process equations of motion of the continuous-time error state vector are summarized as

$$\delta\chi \equiv A\delta\chi + B\delta u + G\eta \tag{49}$$

where $\delta\chi = [\delta\omega^T, \delta p^T, \delta b^T]^T \in R^9$, $\eta = [\eta_\omega^T, \eta_p^T, \eta_b^T]^T \in R^9$ and

$$A = \begin{bmatrix} J_{\hat{\omega}} & 0_{3\times3} & 0_{3\times3} \\ \frac{1}{2}I_{3\times3} & -\omega^{\times} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{bmatrix} \tag{50}$$

$$B = \begin{bmatrix} J^{-1} \\ 0_{3\times3} \\ 0_{3\times3} \end{bmatrix} \tag{51}$$

$$G = \begin{bmatrix} J^{-1} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & \frac{1}{2}I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & I_{3\times3} \end{bmatrix} \tag{52}$$

and the continuous-time covariance matrix is given by

$$Q = \mathrm{diag}\left[\sigma_\omega^2 I_{3\times3} \quad \sigma_p^2 I_{3\times3} \quad \sigma_b^2 I_{3\times3}\right] \tag{53}$$

Transition Matrix

Process models

Actually, the computation of transition matrices may be one of the important factors to implement real-time applications. A discrete version of the process model is given by

$$\delta\chi_{k+1} = \Phi_k \delta\chi_k + \Psi_k \delta u_k + w_k \tag{54}$$

where the transition matrices are simply and approximately obtained as

$$\Phi_k \approx I + A\Delta t \tag{55}$$

$$\Psi_k \approx B\Delta t + \frac{1}{2} AB\Delta t^2 \tag{56}$$

The above approximation is quite accurate most of linear systems with small time interval. However, a reasonable assumption of spacecraft' s properties can offer much more accurate transition matrix. An axis-symmetric platform of spacecraft has been adopted in most small satellites due to its simplicity. If $J_{yy} = J_{zz} > J_{xx}$, the modified momentum of inertia matrix is rewritten as

$$J_{\hat\omega} = \kappa_y \begin{bmatrix} 0 & 0 & 0 \\ \hat\omega_z & 0 & \hat\omega_x \\ -\hat\omega_y & -\hat\omega_x & 0 \end{bmatrix} \tag{57}$$

The transition matrix is given by

$$\Phi_k = \Phi(t_k + \Delta t, t_k) \approx \begin{bmatrix} \phi_k(\Delta t) & 0_{3\times3} & 0_{3\times3} \\ \frac{1}{2} I_{3\times3}\Delta t & \varphi_k(\Delta t) & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & I_{3\times3} \end{bmatrix} \tag{58}$$

where

$$\varphi_k(\Delta t) = (\cos\hat\theta) I_{3\times3} + \left(\frac{1-\cos\hat\theta}{\hat\theta^2}\right)\theta\theta^T - \left(\frac{\sin\hat\theta}{\hat\theta}\right)\theta^\times \tag{59}$$

$$\phi_k^\psi(\Delta t) = \begin{bmatrix} 1 & 0_{1\times2} \\ \left(\lambda^\dagger\theta_s + \theta_s\right)/\hat\theta_x & \lambda^\dagger \end{bmatrix}, \ \psi = 0,1. \tag{60}$$

$$\lambda^\dagger(\hat\theta_x) = \begin{bmatrix} \cos(\kappa\hat\theta_x) & \sin(\kappa\hat\theta_x) \\ -\sin(\kappa\hat\theta_x) & \cos(\kappa\hat\theta_x) \end{bmatrix} \tag{61}$$

with $\theta = [\hat\theta_x, \hat\theta_y, \hat\theta_z]^T = [\hat\theta_x, \theta_s^T]^T = \omega\Delta t$, and $\hat\theta = \|\theta\|$.

Measurement Models

The measurement output equations are rewritten as

$$\omega_g = \omega - b - \eta_a \tag{62}$$

$$q_s = \delta q_e \otimes q \tag{63}$$

The discrete−time linearized measurement error equations are given by

$$\delta y_k = H_k \delta \chi_k + v_k \tag{64}$$

$$= \begin{bmatrix} I_{3\times3} & 0_{3\times3} & -I_{3\times3} \\ 0_{3\times3} & I_{3\times3} & 0_{3\times3} \end{bmatrix} \delta \chi_k + \begin{bmatrix} \eta_{a_k} \\ \eta_{s_k} \end{bmatrix} \tag{65}$$

where $\delta y_k = [\delta \omega_{gk}^T, \delta p_k^T]^T$.

Kalman Filtering

In this section, Kalman filtering technique for spacecraft attitude determination is briefly introduced. attitude kinematics and gyro models including bias models are in general utilized to estimate spacecraft attitude and angular rate. However, the form presented in this case slightly differ from the general case, since the spacecraft nonlinear dynamic model with external disturbance and control inputs is added.

Prediction

Discrete version of the predicted estimates is given by

$$x_{k+1}^- = \Phi_k^p(t_k + \Delta t, t_k) x_k^+ + \int_{t_k}^{t_k+\Delta t} \Phi_k^p(t_k + \Delta t, \tau) B^p(\tau) u(\tau) d\tau \tag{66}$$

where $\Phi_k^p(t_k + \Delta t, t_k)$ denotes the transition matrix of Eq.(27)−(29) during the time interval, B^p is defined as $[J^{-1} \quad 0_{3\times7}]^T$, and $x_{k+1}^- \equiv [\omega_{k+1}^{-T} \quad q_{k+1}^{-T} \quad b_{k+1}^{-T}]^T$. To decrease the computational burden to obtain the predicted estimates, the second term of the left hand side of Eq.(66) is given by numerical approaches such as trapezoid rule for integration. Using the predicted estimates from the transition matrix, the predicted gyro output vector can be predicted as

$$\omega_g^- = \omega_{k+1}^- - b_{k+1}^- \tag{67}$$

Measurement Update

From the measurement vector and the predicted state vector, the residuals, $\delta y_{k+1} \equiv \left[\delta \omega_{gk+1}^T \quad \delta p_{k+1}^T\right]^T$, can be obtained as

$$\delta \omega_{gk+1} = \omega_g - \omega_g^- \tag{68}$$

$$\delta q_{k+1} = \left[\delta p_{k+1}^T \quad \delta q_{4_{k+1}}\right]^T = q_s \otimes (q_{k+1}^-)^{-1} \tag{69}$$

Therefore, the optimally estimated error state vector is

$$\delta\chi_{k+1} = K_{k+1}\delta y_{k+1} \tag{70}$$

with $\delta\chi_{k+1} = [\delta\omega_{k+1}^T \quad \delta p_{k+1}^T \quad \delta b_{k+1}^T]^T$. Next, the state vector is updated using

$$\omega_{k+1}^{+} = \omega_{k+1}^{-} + \delta\omega_{k+1} \tag{71}$$

$$q_{k+1}^{+} = \delta q_{k+1} \otimes q_{k+1}^{-} \tag{72}$$

$$b_{k+1}^{+} = b_{k+1}^{-} + \delta b_{k+1} \tag{73}$$

where $\delta q_{k+1} = [\delta p_{k+1}^T \quad \surd(1 - \delta p_{k+1}^T \delta p_{k+1})]^T$

Filtering Including External Disturbances

In this section, the spacecraft dynamics includes external disturbances. As mentioned in the introduction, the space has hardly predictable sources such as torques from the solar pressures, gravity gradient, and etc. As a result, the dynamic equations for the spacecraft attitude include many difficulties in the filter modelling. Therefore, by estimation the unpredictable external disturbance, the accuracy of the interested state can be enhanced. Overall procedure is similar with the previous case. Process models to estimate interested states are summarized as follows :

$$\dot{\omega} = J^{-1}(-\omega^{\times}J\omega + u + d + \eta_{\omega}) \tag{74}$$

$$\dot{q} = \frac{1}{2}\Omega(\omega)q \tag{75}$$

$$\dot{b} = \eta_b \tag{76}$$

$$\dot{d} = \eta_d \tag{77}$$

where $x = [\omega^T \quad q^T \quad b^T \quad d^T]^T \in R^{13}$.

The process equations of motion of the continuous-time error state vector are summarized as follows:

$$\delta\chi \equiv A\delta\chi + B\delta u + G\eta \tag{78}$$

where $\delta\chi = [\delta\omega^T \quad \delta q^T \quad \delta b^T \quad \delta d^T]^T \in R^{12}$, $\eta = [\eta_{\omega}^T, \eta_p^T, \eta_b^T, \eta_d^T]^T \in R^{12}$

$$A = \begin{bmatrix} J_{\hat{\omega}} & 0_{3\times3} & 0_{3\times3} & J^{-1} \\ \frac{1}{2}I_{3\times3} & -\omega^{\times} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \end{bmatrix} \tag{79}$$

$$B = \begin{bmatrix} J^{-1} \\ 0_{9\times3} \end{bmatrix} \tag{80}$$

$$G=\begin{bmatrix} J^{-1} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & \frac{1}{2}I_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & I_{3\times3} \end{bmatrix} \tag{81}$$

and the continuous-time covariance matrix given by

$$Q=\mathrm{diag}\left[\sigma_{\omega}^{2}I_{3\times3} \quad \sigma_{p}^{2}I_{3\times3} \quad \sigma_{b}^{2}I_{3\times3} \quad \sigma_{d}^{2}I_{3\times3}\right] \tag{82}$$

with the assumption of moment of inertia matrix (axis-symmetric case), the transition matrix is readily given by

$$\Phi_k=\Phi(t_k+\Delta t,t_k)=\begin{bmatrix} \phi_k(\Delta t) & 0_{3\times3} & 0_{3\times3} & J^{-1}\Delta t \\ \frac{1}{2}\Delta t I_{3\times3} & \varphi_k(\Delta t) & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & I_{3\times3} \end{bmatrix} \tag{83}$$

Covariance Analysis

Most of the structural disturbances and uncertainties of spacecraft are internal momentum instability due to the imperfection of the reaction wheels. In this paper, only disturbance by the reaction wheel is considered. The disturbance torque induced by the dynamic imbalance of an reaction wheel spinning ϖ rad/s could be approximated as

$$\tau_d(t)=\sum_{i=1}^{n}c_i^d\varpi^2\cos(h_i\varpi t) \tag{84}$$

where h_i is the significant harmonics generating high disturbance torques, and c_i^d is the related dynamic imbalance for the i-th harmonic. Also the disturbance force due to the static imbalance of an reaction wheel is approximately given by

$$f_d(t)=\sum_{i=1}^{n}c_i^s\varpi^2\cos(h_i\varpi t) \tag{85}$$

where c_i^d is the related static imbalance for the i-th harmonic. Therefore, the resultant disturbance torque due to the static imbalance is given by $F_d\times l$, where l is the length between the center of mass of spacecraft and the reaction wheel. Certainly, the farther is a reaction wheel placed with respect to the center of mass of spacecraft, the bigger is the impact of this disturbance force on the attitude motion of the spacecraft. The maximum resultant disturbance torque from the imbalance of an reaction wheel is approximately written as

$$\tau_{rw}=\sum_{i=1}^{n}(l_jc_i^s+c_i^d)\varpi_{max}^2 \tag{86}$$

where ϖ_{max} is a maximum wheel speed. From the internal torque disturbance in Eq.(86), we may assume that the noise covariance of the attitude dynamic motion of spacecraft installed with one reaction wheel would be $\sigma_{\omega}^2 : \tau_{rw}^2$.

The estimated angular velocity of spacecraft has been obtained directly by the gyro output data considered estimated gyro bias elements. It means that the estimated angular rate to use for an attitude control purpose of spacecraft has a noise level equivalent to the combination of the gyro angle random walk and the rate random walk. Thus, the gyro data have been not considered as measurements. Theoretically, the attitude kinematic equations itself have no noise processes. The rate random walk by the noise process of the spacecraft attitude dynamics generates angular rate uncertainties. The only integrated rate noise can make angular rate noise process such that η_p is zero if $\Delta t \rightarrow 0$. For the integration interval, let us analysis only one-axis covariance of the noise process by examining simple dynamics given by

$$\dot{\theta} = \omega \tag{87}$$

$$\dot{\omega} = \eta_{\omega} \tag{88}$$

The covariance matrix of the process model is given by

$$\Phi(t_k + \Delta t, t_k) = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \tag{89}$$

and

$$\eta_p = \int_{t_k}^{t_{k+1}} (t_{k+1} - \tau)\eta_{\omega} d\tau \tag{90}$$

finally, the covariance of the kinematic equation for a given time interval is

$$\sigma_{pk}^2 = E\langle \eta_{pk}^2 \rangle = \int_{t_k}^{t_{k+1}} (t_{k+1} - \tau)^2 \sigma_{\omega}^2 d\tau = \frac{1}{3}\sigma_{\omega}^2 \Delta t^3 \tag{91}$$

Using the trapezoid rule for integration, the discrete-time process noise covariance is obtained approximately as

$$Q_k = \begin{bmatrix} (J^{-1})^2 \sigma_{\omega}^2 \Delta t I_{3\times3} & \frac{1}{4}(J^{-1})^2 \sigma_{\omega}^2 \Delta t^2 I_{3\times3} & 0_{3\times3} \\ \frac{1}{4}(J^{-1})^2 \sigma_{\omega}^2 \Delta t^2 I_{3\times3} & \frac{1}{8}(J^{-1})^2 \sigma_{\omega}^2 \Delta t^3 I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & \sigma_b^2 \Delta t I_{3\times3} \end{bmatrix} \tag{92}$$

where $\sigma_p^2 = \frac{1}{8}(J^{-1})^2 \sigma_{\omega}^2 \Delta t^3$ consists of a noise covariance of attitude motion. As the same reason, a discrete-time measurement covariance is

$$R_k = E\langle v_k v_k^T \rangle = \begin{bmatrix} (\frac{1}{3}\sigma_b^2 \Delta t^3 + \sigma_a^2) I_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & \sigma_s^2 I_{3\times3} \end{bmatrix} \tag{93}$$

where gyro angular rate covariance for a given sampling time is modified by the bias noise process.

Simulation Study

In this section, several simulations are performed to demonstrate the proposed algorithm using realistic spacecraft model. The simulation conditions are shown in Table 1. The attitude determination hardware consists of an digital sun sensor, or coarse sun sensors, a three-axis magnetometer and gyroscopic rate sensors. TAM sensor noise is in general modeled by a zero-mean Gaussian white-noise process. With two vector observations by QUEST or TRIAD algorithms[6], quaternion measurements can be modeled. In this simulation, the quaternion measurements are directly utilized for measurement models assumed that the two-vector observation is performed using a sun sensor and a TAM. The sampling time of sensors is 0.25sec. The state prediction of filtering is also selected as 0.25sec and the moment of inertia of spacecraft is given by

$$J = \begin{bmatrix} 16.3 & 0.0869 & 0.60167 \\ 0.0867 & 36.6 & 0.13571 \\ 0.60167 & 0.13571 & 38.6 \end{bmatrix} \quad (94)$$

For realistic simulations, we added about 10% of modeling error of moment of inertia. The simulation scenario is a simple small angle attitude re-orientation of spacecraft. The first simulation illustrated in Figs. 1 and 2 is of attitude determination using only quaternion error and gyro bias estimation technique in Ref.[4] In a steady state condition after about 600sec, it is shown that the attitude errors are about 0.08, 0.08 and 0.37(3σ), respectively and bias errors are also about 0.015, 0.015 and 0.017(3σ), respectively.

Table 1. Numerical conditions

Symbol	Value
σ_s	diag$[0.045, 0.045, 0.86]$ deg
σ_a	diag$[0.025, 0.025, 0.025]$ $\text{deg}/\sqrt{\text{s}}$
σ_b	diag$[0.0011, 0.0011, 0.0011]$ $\text{deg}/\text{s}^{3/2}$
σ_ω	diag$[0.00005, 0.00005, 0.00005]$ Nm
b_0	$[1.66, -1.66, 1.099]^T$ $\text{deg}/\sqrt{\text{s}}$

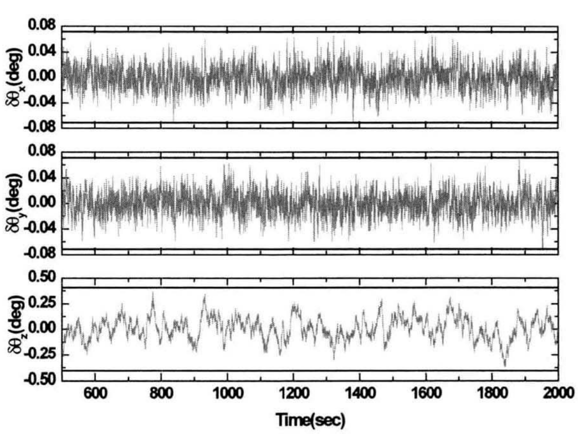

Fig. 1. Attitude error history by the general quaternion error and gyro bias estimation technique

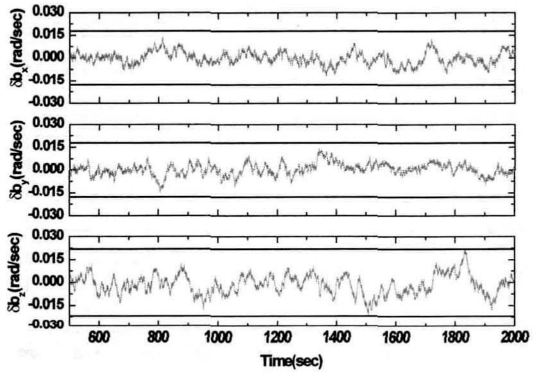

Fig. 2. Gyro bias error history by the general quaternion error and gyro bias estimation technique

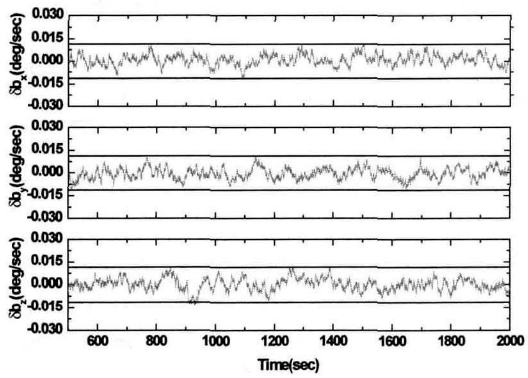

Fig. 3. Attitude error history by the proposed algorithm

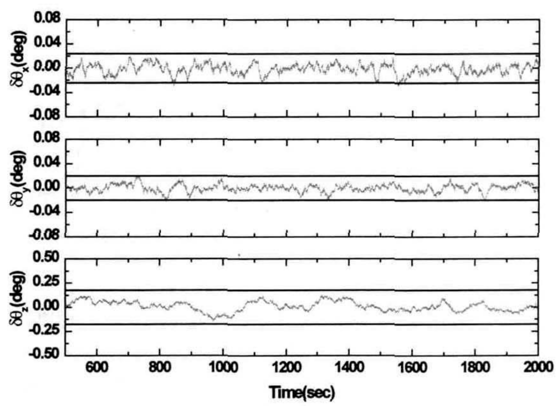

Fig. 4. Gyro bias error history by the proposed algorithm

The second simulation by the proposed fine attitude determination technique is shown in Figs. 3,4 and 5. In a steady state condition after about 600sec, it is shown that the errors of the estimation parameters are smaller than the previous case. It means that the accurate spacecraft dynamic model have an ability to enhance estimation performance. Furthermore, the angular rate of spacecraft is also directly estimated by filter formulation while the gyro angular rate of the first simulation case is estimated by only eliminating gyro bias terms.

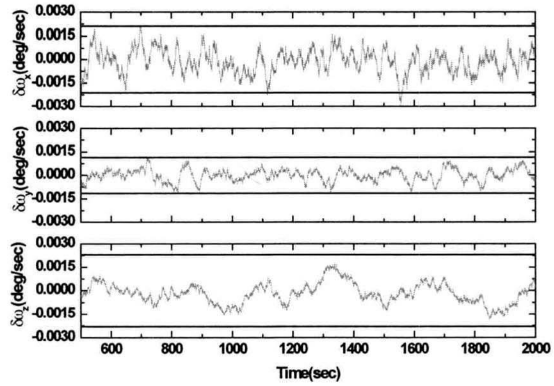

Fig. 5. Angular rate error history by the proposed algorithm

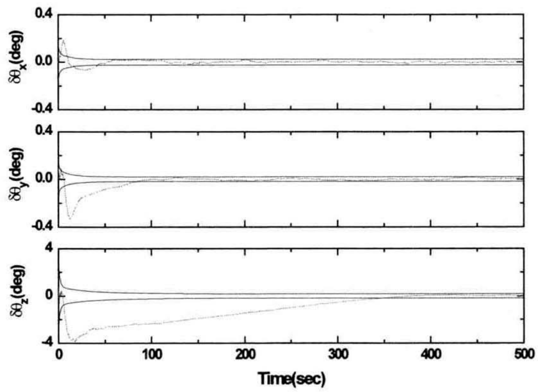

Fig. 6. Disturbance estimation history

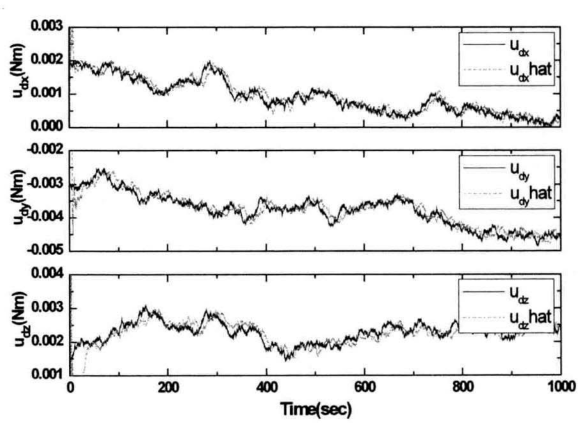

Fig. 7. Attitude history in an attitude maneuvering phase

To obtain a reliable estimation performance, very accurate process model is necessarily. It is achieved by estimating the external disturbance illustrated in Fig. 6. Nevertheless, the attitude performance shown in Fig. 7 in an attitude maneuvering phase shows somewhat undesirable results. Attitude errors do not converge to within their respective 3σ bounds due to process model error until well after 600sec. Therefore, the proposed algorithm is useful only steady state conditions such as earth pointing missions.

Conclusions

With low−cost sensors fine attitude estimation for attitude pointing missions of spacecraft is addressed. A real−time applicable linearized model and a transition matrix are derived from nonlinear spacecraft dynamics with external disturbances. A numerical simulation shows that the proposed algorithm offers relatively high estimation accuracy under dynamic uncertainties in a steady state condition. Further analysis to obtain feasible estimation performance for a maneuvering phase is required.

Acknowledgments

This research was supported by NSL(National Space Lab) program through the Korea Science and Engineering Foundation funded by the Ministry of Education, Science and Technology.

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